

Optimal Level and Base Mother Wavelet Selection for Wavelet Analysis in Wind Turbine Health Monitoring

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ABSTRACT

Wind turbines are gaining global attention as a renewable energy solution. However, issues with unexpected component faults in their powertrains are common. Vibration monitoring is commonly used to detect early signs of bearing and gear failures, aiming to improve productivity. Research shows that bearings are more prone to failures compared to gearbox gears. The wavelet transforms, particularly the discrete wavelet form, has recently become popular for analyzing non-stationary signals. Nevertheless, its effectiveness depends on choosing the right decomposition level and mother wavelet. The primary challenges of discrete wavelet transform (DWT) involve selecting the suitable mother wavelet for bearing signal analysis, as different choices lead to diverse results, and determining the optimal decomposition level to extract valuable features for anomaly identification. This study focuses on vibration signals from inner race faults and proposes an optimization approach for selecting the best decomposition level and mother wavelet based on criteria like Shannon entropy, energy-to-Shannon-entropy ratio, and reconstruction quality (root-mean-square error RMSE). The validity of the method is confirmed through correlation coefficient calculation and signal-to-noise ratio (SNR) analysis on the denoised vibration signal. These results prove the validity and robustness of the proposed method for the bearing fault signal analysis.

I. Introduction

The exponential growth of manufacturing has been primarily responsible for the enormous increase in global energy consumption over the past few years [1]. The world must adopt energy sources that are environmentally friendly to satisfy carbon-lowering goals [2]. Wind power has emerged as a prominent renewable energy source, showing significant advancements in clean electricity generation in recent years [3,4]. However, wind turbine components face premature breakdowns, leading to costly unplanned downtime [5]. Such turbine drive systems frequently encounter untimely component failures, with multi-stage malfunctions constituting around 20% of overall turbine downtime [6]. Bearings, crucial rotating machinery components, have been identified as a key factor [7]. Research highlights that gearbox bearing failures contribute to approximately 76% of total failures, surpassing gear failures at 17% [8]. To address this, defect identification technology plays a pivotal role in prognostics and health management (PHM) [9]. Utilizing vibration signals to detect, identify, and locate bearing faults in wind turbines is critical for effective maintenance and avoiding substantial financial losses [10]. Given

that bearing vibration signals exhibit non-stationary and non-linear characteristics, it becomes imperative to extract their distinct attributes prior to diagnosing any issues [11].

Fault diagnosis relies on three primary vibration processing techniques: time domain analysis [12], frequency domain analysis, and time-frequency analysis methods [13]. Recognizing the challenge of appraising non-stationary signals within either the time or frequency domain, the time-frequency method, which amalgamates the two domains, proves to be a viable choice [14]. The wavelet transforms, a commonly employed time-frequency analysis tool, is particularly suited for processing non-stationary data and offering comprehensive insights [15]. This transform effectively localizes signals in terms of both time and frequency, rendering it ideal for identifying local characteristics [16]. Nonetheless, the drawbacks linked to parameter selection in wavelet analysis have implications for the efficiency of the wavelet transform. In the context of wavelet-based data analysis, identifying the optimal decomposition level assumes critical importance. This question has garnered attention from diverse researcher perspectives in recent years. In a study [17], the use of autocorrelation aided in determining the optimum wavelet decomposition level for noise reduction. Minimal wavelet decomposition has a negligible effect on signal processing, whereas excessive decomposition can obscure valuable information in the signal. Additionally, alternative criteria such as Shannon entropy [18] and signal-to-noise ratio (SNR) [19] are employed to establish the optimal decomposition level.

Wavelet analysis provides numerous varieties of mother wavelets, with the key to successful application lying in the selection of an appropriate wavelet, a task that proves difficult. When different mother wavelets analyze the same signal, outcomes can diverge. To tackle this, previous research introduced both qualitative and quantitative methods for choosing the optimal mother wavelet for specific applications [20]. Qualitatively, wavelet characteristics, including orthogonality, compact support, symmetry, and their resemblance to the signal, influence the choice of the mother wavelet. In the quantitative approach, wavelet selection rests on the computation of various criteria. Reference [21] proposed employing permutation entropy to identify the optimal wavelet for defect detection and classification. In [22], the author found the maximum energy- to-Shannon-entropy ratio to be the superior criterion for selecting the best mother wavelet for machining fault identification. [23] presented an approach based on the root-mean-square difference (error) between the original and reconstructed signals to select suitable mother wavelets for power system fault transients. Conversely, for diagnosing and classifying bearing faults, the Daubechies wavelet db3 is commonly considered the optimal choice [24].

This work introduces diverse methods for selecting the optimal decomposition level and the suitable mother wavelet to analyze vibration signals originating from gearbox-bearing faults. From the three available wavelet families (Daubechies, Coiflets, and Symlets) in Matlab, we randomly select the mother wavelets. The outcomes of this investigation will be evaluated and compared through denoising operations. The proposed approach is divided into two sections: Section (1) outlines the methodology for calculating Shannon entropy and RMSE. The minimum Shannon entropy value is chosen as the primary parameter for optimal level selection, while the mother wavelet exhibiting the highest energy-to-Shannon-entropy ratio and the lowest RMSE values is selected. Section (2) involves evaluating the suggested algorithm, focusing on determining correlation coefficients and signal-to-noise ratio (SNR) during denoising via the wavelet transform. Evaluation parameters employed to gauge denoising operation quality are applied across different decomposition levels and various mother wavelets.

II. Gearbox Bearing Dataset Description

The experimental data from the SKF Bearing Data Center, supplied by Case Western Reserve University (CWRU) [25], is frequently utilized in this study, while actual bearing fault signals for wind turbine gearboxes predominantly stem from private commercial datasets.

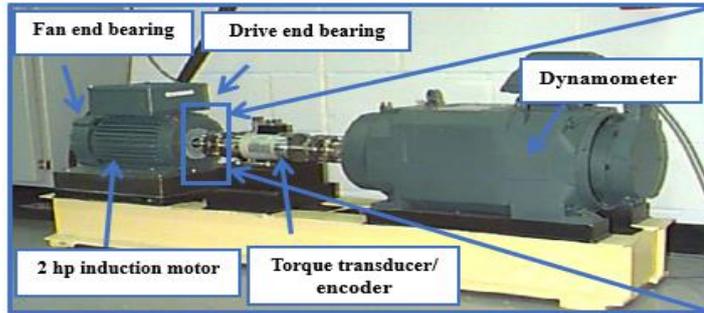


Figure 1. CWRU test rig [26]

The test platform illustrated in figure (1) comprises an electronic control unit, an accelerometer, an electric driving motor, and a 2 hp loading motor. Electro-discharge machining (EDM) was utilized to gather vibration signal data under four distinct experimental conditions: (a) normal state, (b) inner ring fault, (c) outer ring fault, and (d) ball fault. Each defect type encompasses four varying levels for the fault bearings, with fault diameters of 0.007 inches, 0.014 inches, 0.021 inches, and 0.028 inches, respectively. Vibration data were recorded through an accelerometer at a sampling rate of 12 kHz and 48 kHz for drive-end bearing faults. All data files were stored in Matlab (.mat) format. The bearing vibration signals were captured for motor loads ranging from 0 to 3 hp, at motor rotation velocities that varied between 1797 rpm and 1730 rpm. In our study, we have chosen a sampling frequency of 12 kHz for the inner race fault under the maximum shaft speed (1797 rpm) and with no load. Figure (2) represent the vibration signal of bearing inner ring fault.

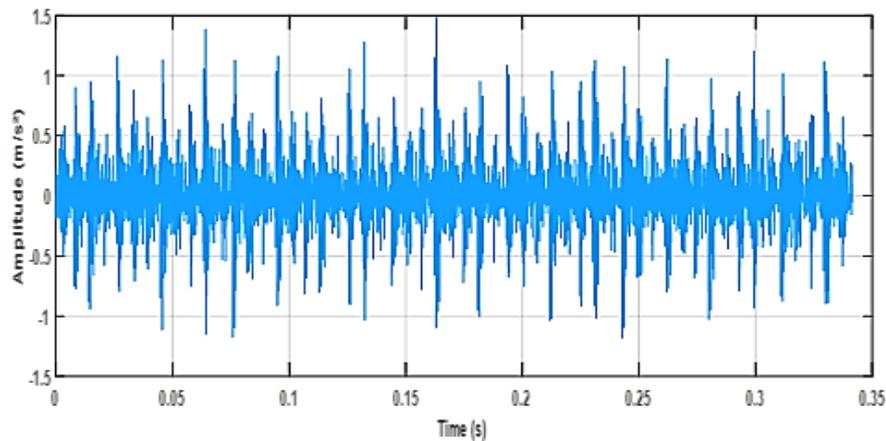


Figure 2. Vibration Signal (0 hp, 0.007 inches) of inner race fault.

III. Overview of Wavelet Transform Theory

In this context, the utilization of wavelet transform proves more effective in identifying peaks and irregularities in signal processing for fault diagnosis than other signal processing techniques. For specialized applications, the Fourier transform and the short Fourier transform have been merged to create the wavelet transform, which offers insights in both the time and frequency domains [27]. Through a mathematical process called wavelet analysis, data is divided into various frequency components, each of which is then examined with a resolution tailored to its scale [20]. The analysis employs numerous orthogonal basis functions, termed mother wavelets, which are scaled and translated into the time domain for this purpose [28]. Wavelet transforms exist in two distinct forms: continuous wavelets (CWT) and discrete wavelets (DWT).

The continuous wavelet transform is defined as follows:

$$CWT(a, b) = 1/\sqrt{a} \int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-b}{a}\right) dt \quad (1)$$

Where ψ , a and b denote the basic mother wavelet, the scaling parameter and the translation parameter respectively.

The discrete wavelet is the discretization of the CWT and can be expressed as:

$$DWT(j, k) = 1/\sqrt{2^j} \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{t-2^j k}{2^j}\right) dt \quad (2)$$

Where, $a=2^j$ and $b=2^j k$.

The ability for multi-resolution analysis makes it an optimal tool for extracting fault-related features from non-stationary signals recorded in rotating machinery. DWT functions as a filter bank, encompassing high-pass and low-pass filtering operations. The outcomes of each filtering stage yield both high-frequency and low-frequency components known as detail coefficients (cD) and approximation coefficients (cA), respectively.

IV. Shannon Entropy Criteria for the Optimum Decomposition Level and Mother Wavelet Selection

IV.1. Selection of Optimum Decomposition Level

Based on Mallat's 1988 algorithm [29], the wavelet transform involves multiple levels for signal decomposition, with the highest-level contingent on the dataset's sample count. The signal with a length N, the highest level of decomposition j_{max} can be determined by:

$$j_{max} = \log_2 N \quad (3)$$

The selection of the decomposition level is guided by the application's goal, as signal characteristics fluctuate according to the specific context [30]. Precise level assignments dictate suitable scales and bands for precise signal analysis and efficient noise removal. Various techniques exist to optimize the decomposition level, and among these is Shannon entropy. In the realm of mathematics, Shannon entropy serves as a common measure to quantify the extent of uncertainty or randomness within a process [31]. Elevated entropy values correspond to heightened randomness within a system, whereas lower values indicate greater signal regularity and reduced complexity. The Shannon entropy is presented as [32]:

$$H(x) = \sum_{i=0}^{N-1} P_i \log_2 P_i \quad (4)$$

The original signal $x(k)$ can be represented by the sum of all wavelet components:

$$x(k) = \sum_{j=1}^{j_{max}} cD_j(k) + cA_{j_{max}}(k) \quad (5)$$

The energy spectrum of wavelet coefficients at decomposing scale j after DWT analysis, can be defined as follows:

$$E_j = \sum_k |C_{j,k}|^2 \quad (7)$$

The total energy wavelet spectra of the $x(k)$ signal at maximum level j_{max} are presented as follows:

$$E_T = \sum_{j=1}^{j_{max}} E_j \quad (8)$$

The Shannon entropy of wavelet coefficients at level j is given by:

$$S(j) = - \sum_k \frac{|C_{j,k}|^2}{E_j} \log_2 \left(\frac{|C_{j,k}|^2}{E_j} \right) \quad (9)$$

For successful feature extraction, the spectral distribution of energy must be taken into account.

$$P_j = \frac{E_j}{E_T} \quad (10)$$

The wavelet energy entropy (WEE) is therefore described as follows:

$$WEE = - \sum_j P_j \log_2 P_j \quad (11)$$

IV.2. Selection of the Appropriate Mother Wavelet

The primary and pivotal step in wavelet analysis involves the selection of the optimal mother wavelet. Basic wavelets typically possess diverse attributes, including orthogonality, symmetry, and compact support. Grasping these characteristics aids in choosing a prospective basic wavelet for analyzing a given signal. A limitation of qualitative approaches is the challenge of visually interpreting the shape of the signal corresponding to the basic wavelet. Commonly employed criteria for mother wavelet selection encompass energy and Shannon entropy. The energy-to-Shannon-entropy ratio is a criterion rooted in a combination of the wavelet coefficients signal's energy level and entropy [33]. The mother wavelet that best fits the data is presumed to exhibit the highest energy-to-Shannon-entropy ratio. Equation (12) [34] was used to obtain the energy ratio as illustrated below:

$$EE_{ratio} = \frac{E_T}{WEE} \tag{12}$$

In the present study, various mother wavelets are employed for DWT, namely Daubechies (db), Symlets (sym), and Coiflets (coif). Similar findings from prior research indicate that using low-order db and sym wavelets is frequently recommended and appropriate for analyzing non-stationary signals [35]. Furthermore, one of the challenges in wavelet analysis pertains to reconstruction accuracy. The optimal reconstruction of bearing vibration signals forms the foundation for selecting an appropriate wavelet. For achieving the most accurate reconstruction, the disparity between the original signal $x(t)$ and the reconstructed signal $\hat{x}(t)$ should be minimized. The root-mean-square error (RMSE) is an indicator used to quantify the precision of the wavelet reconstruction. The following formula is used to determine RMSE:

$$RMSE = \sqrt{1/N \sum_{i=1}^N (x_i - \hat{x}_i)^2} \tag{13}$$

V. Results and Discussion

In this experiment, the foundational wavelet ensemble for processing vibration signals comprises fifteen frequently employed wavelets, spanning the Daubechies wavelet family (db2, db3, db4, db5, db7, db9), the Symlets family (sym4, sym5, sym7, sym8, sym9), and the Coiflets series (coif1, coif3, coif4, coif5). Our approach involves two key parameters: the decomposition level and the mother wavelet. Initially, the vibration signal associated with inner ring defects undergoes discrete wavelet decomposition, extending up to the maximum decomposition level of 11. Subsequently, the Shannon entropy values for each set of wavelet coefficients are computed using the equations provided in the preceding section. Table 1 illustrates the Shannon entropy values for each decomposition level pertaining to the vibration signal attributed to inner ring faults.

Table 1. Shannon Entropy Values

Mother Wavelet	Decomposition Level										
	1	2	3	4	5	6	7	8	9	10	11
db2	0.3598	0.3667	0.2331	0.1142	0.0212	0.0123	0.0029	0.0043	0.0069	0.0117	0.0198
db4	0.3532	0.3679	0.2417	0.1217	0.0132	0.0104	0.0005	0.0010	0.0016	0.0026	0.0048
db5	0.3599	0.3665	0.2431	0.1238	0.0128	0.0106	0.0003	0.0004	0.0007	0.0012	0.0022
db7	0.3664	0.3622	0.2445	0.1265	0.0109	0.0110	0.0020	0.0044	0.0072	0.0117	0.0205
db9	0.3583	0.3678	0.2488	0.1277	0.0132	0.0159	0.0006	0.0028	0.0064	0.0122	0.0210
sym4	0.3668	0.3610	0.2420	0.1188	0.0169	0.0113	0.0020	0.0043	0.0072	0.0127	0.0240
sym5	0.3643	0.3635	0.2405	0.1241	0.0128	0.0108	0.0006	0.0013	0.0020	0.0037	0.0070

sym7	0.3505	0.3675	0.2493	0.1279	0.0101	0.0109	0.0008	0.0011	0.0021	0.0036	0.0063
sym8	0.3659	0.3625	0.2462	0.1279	0.0126	0.0111	0.0010	0.0014	0.0025	0.0041	0.0096
sym9	0.3636	0.3652	0.2455	0.1284	0.0135	0.0116	0.0018	0.0041	0.0019	0.0040	0.0086
coif1	0.3666	0.3600	0.2329	0.1151	0.0245	0.0131	0.0030	0.0044	0.0074	0.0130	0.0225
coif3	0.3654	0.3628	0.2441	0.1260	0.0111	0.0113	0.0020	0.0037	0.0027	0.0045	0.0086
coif4	0.3652	0.3637	0.2459	0.1298	0.0120	0.0115	0.0015	0.0039	0.0032	0.0056	0.0112
coif5	0.3650	0.3640	0.2486	0.1312	0.0133	0.0132	0.0009	0.0012	0.0016	0.0028	0.0053

Shannon entropy values serve as indicators for evaluating the preservation of relevant information across various decomposition levels. In our investigation, we establish the optimal level by comparing criteria values. If the partitioning results in a reduction in entropy, which is intriguing, it signifies the necessity for further partitioning in the subsequent level. The minimum entropy level emerges as the optimum choice. As per Table 1, the lowest Shannon entropy value is observed at level 7 across all wavelet families. Following this point, an increase in Shannon entropy values becomes evident, reflecting an amplified degree of randomness in the signal and signaling the termination of the decomposition process. Consequently, the optimal level for inner race vibration signal analysis is determined to be level 7.

Figures 3 and 4 show the energy to Shannon entropy ratio and RMSE values, respectively for the inner race fault vibration signal.

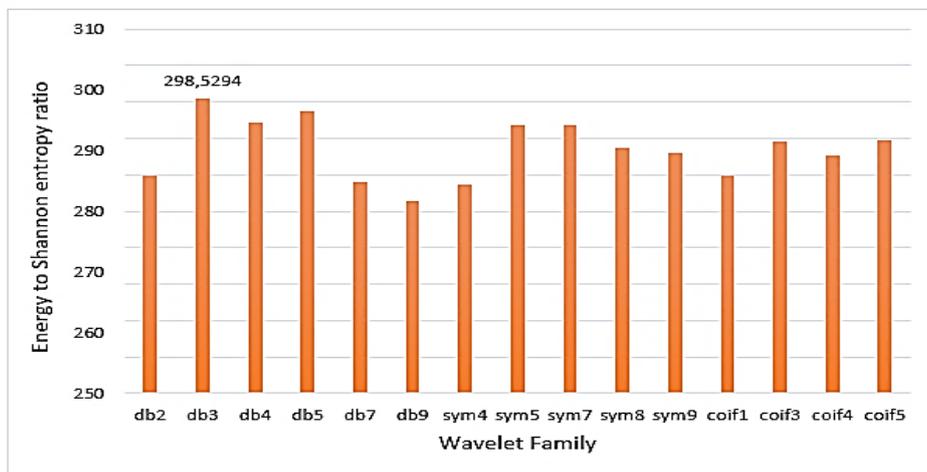


Figure 3. Values of energy to Shannon entropy ratio for different mother wavelets.

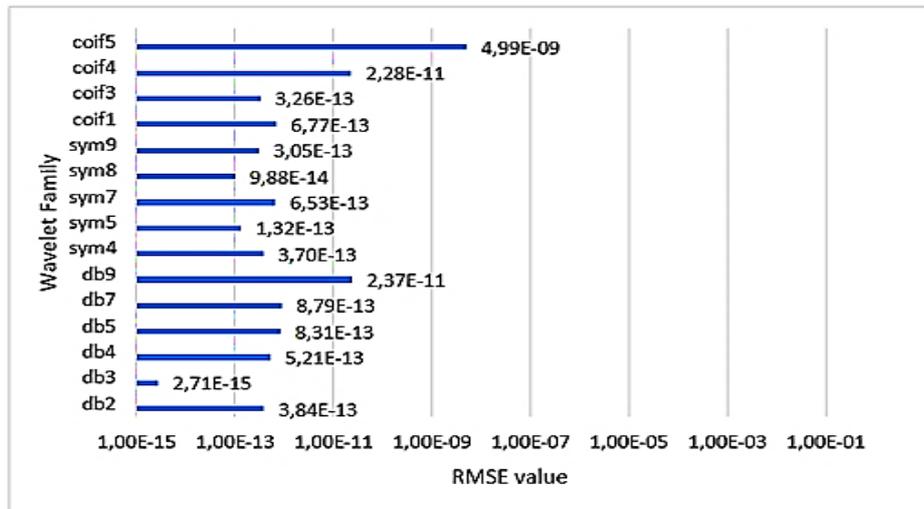


Figure 4. Values of RMSE for different mother wavelets.

In certain scenarios, qualitative criteria for selecting the optimal mother wavelet may not be met. To address this issue, our study proposes two comprehensive criteria: the energy-to-Shannon-entropy ratio and the RMSE. Following the selection of the optimal level using Shannon entropy across the 15 diverse wavelet families, the wavelet family with the highest energy ratio and the lowest RMSE value is identified as the most suitable for vibration signal analysis. As shown in Figures 3 and 4, it's apparent that db3 exhibits the highest energy-to-Shannon-entropy ratio and the lowest RMSE values. This observation underscores that the db3 mother wavelet facilitates enhanced energy distribution and reconstruction performance. Consequently, db3 emerges as the optimal mother wavelet for analyzing gearbox bearing vibration signals.

The evaluation of the suggested method is conducted through denoising noisy vibration signals. The effectiveness of noise reduction aids in establishing the ideal decomposition level and the suitable mother wavelet essential for isolating the noise. The validation process hinges on the correlation between the original signal and the denoised signal, a criterion used to identify the optimal mother wavelet. The signal-to-noise ratio (SNR) serves as the metric for assessing noise reduction at each decomposition level. To introduce noisy signal, White Gaussian noise was added to the inner race fault vibration signal.

The noisy signal is presented as follow:

$$Y[n] = W[n] + x[n] \tag{14}$$

Where $W[n]$ is the White Gaussian noise, and $x[n]$ is the original signal.

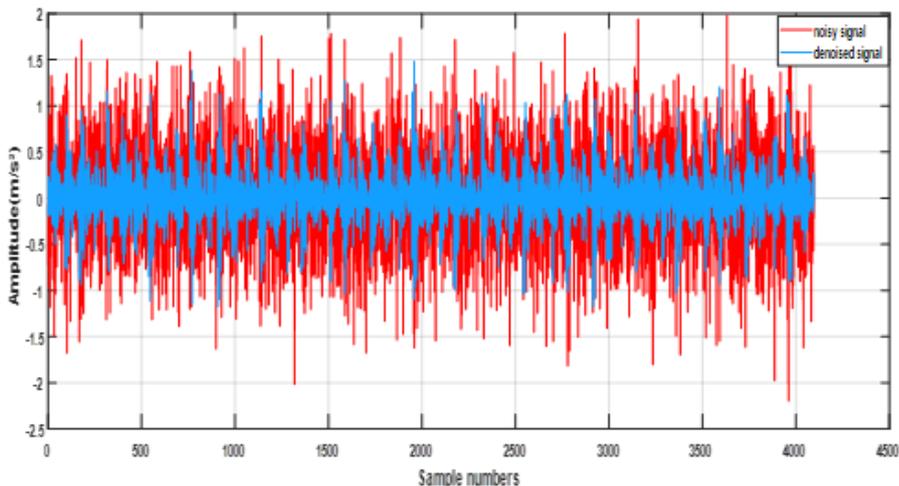


Figure 5. Shows the noisy signal, and denoised signal (SNR=6 dB)

Wavelet denoising operation consists of three primary steps: Signal decomposition, thresholding, and reconstruction. For the denoising process, specific threshold values are required. In our study, we employ soft thresholding.

V.1. Evaluation process

- 1) **SNR:** It is a widely used measure for assessing signal quality in the presence of noise and for evaluating the performance of noise reduction techniques. [17]. SNR is defined by:

$$SNR = 10 \log_{10} \left(\frac{Var(x)}{Var(x-\hat{x})} \right) \quad (15)$$

Where, Var is variance, x is the original signal, and \hat{x} is the filtered sub-signal.

- 2) **Correlation coefficients:** The cross-correlation function is a measure that quantifies the disparity in quality between the original signal and the denoised signal [19].

$$r = (Cov(x, \hat{x})) / \sqrt{(Var(x) - Var(\hat{x}))} \quad (16)$$

With, Cov is covariance, and \hat{x} is the denoised signal.

Figure 6 and 7 display the values of correlation coefficients and SNR, respectively.

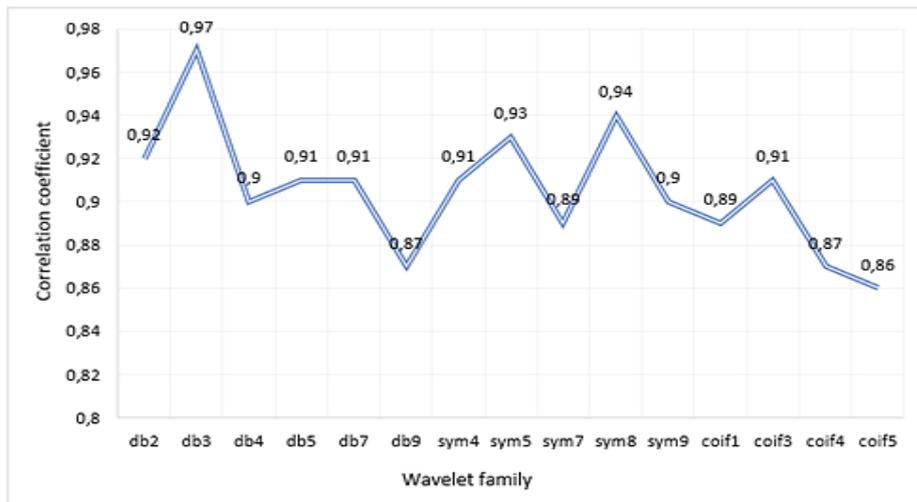


Figure 6. Correlation coefficient values for different mother wavelets.

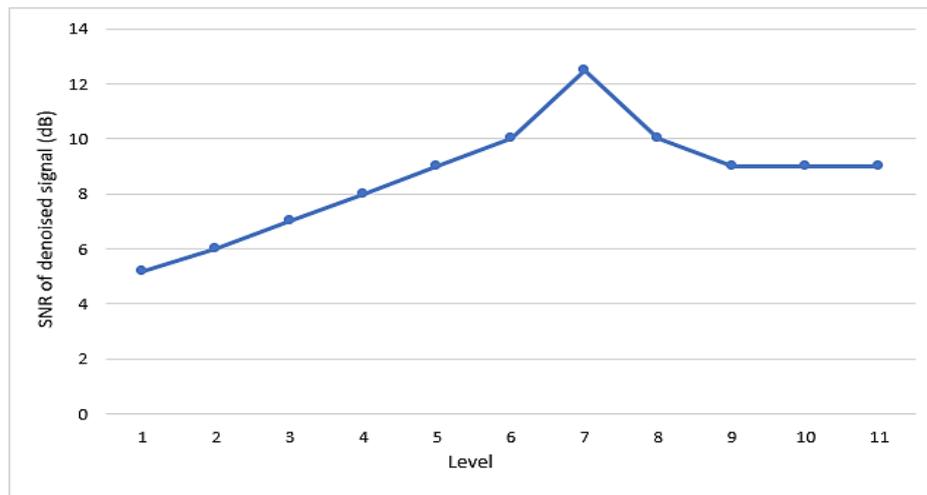


Figure 6. SNR values of denoised signal using db3 at different decomposition levels.

As depicted in Fig. 6, the correlation coefficients between the denoised signal and the original signal, reveal that the Daubechies family with order 3 yields the most effective denoising outcome ($r = 0.97$). This suggests a striking resemblance between the denoised and original signals. Fig. 7 illustrates that the most substantial denoising occurs at decomposition level 7. Beyond this level, the enhancement in denoising quality remains unaffected by increasing the decomposition level, owing to the minimal and imperceptible presence of noise. In a broader sense, the experimental outcomes demonstrate that the seven-level decomposition yields superior noise reduction, as indicated by the SNR. In summation, the obtained results substantiate the success of the proposed method in selecting the optimal level and the optimal mother wavelet for analyzing bearing vibration signals.

VI. Conclusion

The selection of the optimal level and the best mother wavelet poses the most significant challenge in wavelet analysis for a given signal. Importantly, the optimal level and wavelet selection can hinge on the signal's unique characteristics and the prevailing noise. This paper introduces a proposed approach for determining the optimal decomposition level and the most suitable mother wavelet for bearing vibration signal analysis. The method centers on the concept of Shannon entropy, crucial for identifying the optimal wavelet decomposition level. Additionally, two criteria guide mother wavelet selection: the energy-to-Shannon-entropy ratio and root-mean-square error. The optimal decomposition level is derived from the minimum Shannon entropy value, while the optimal wavelet choice stems from the maximum energy-to-Shannon-entropy ratio and the minimum RMSE value. Noise quality assessments validate the efficacy of the proposed technique. Conclusions drawn from both criteria affirm level 7 as the optimal decomposition level and db3 as the apt mother wavelet. These findings underscore the effectiveness of this technique in mother wavelet selection, given db wavelets' established prominence in analyzing bearing fault signals.

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