المجلة الدولية للأداء الاقتصادي
International journal of economic performance
ISSN: 2661-7161 EISSN:2716-9073
Publications are Open Access in this journal

The non-existence of an inflation-output trade-off under the rational expectations hypothesis as the cause of the steady state situation. Overtaking attempt.

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Submitted:12/11/2022 Accepted:27/11/2022 Published:04/12/2022

Abstract

This article tries to prove the existence of an inflation-output (unemployment) trade-off, at least in the long-run, under a context of the rational expectations hypothesis REH, which makes it possible to overcome the Lucasian steady state situation. This existence of trade-off is due to the importance of saving and the accumulation of capital over time, as well as to unexpected economic policies by the agents, which is essential for economic growth. To achieve this objective, we will expose the unrealistic aspect of REH which avoids any inflation-output (unemployment) trade-off in the short-run and the long-run, and consequently it leads the economy to an infinite steady state situation.

Our work undertakes a personal philosophical and mathematical approach, which finds its origins in the pioneering works of economic growth theory, namely those of Harrod, Domar and Solow. We have tried to propose a solution for overcoming the steady state *à la Lucas*. Nevertheless, our work requires further studies to validate its theoretical and mathematical framework ... *or criticize it!*

Keywords: Rational expectations hypothesis, inflation-output trade-off, steady state, natural long-run equilibrium, super-neutrality of money, NAIRU, Phillips curve, Harrod-Domar consistency condition, saving, capital accumulation.

JEL classification codes: B22, E21, E22, E23, E24, E58

"To the soul of our colleague Merad Rachid"

"In The General Theory (Keynes 1936), expectations are driven by animal spirits which represent an independent driving force of business cycles. When Lucas introduced the concept of rational expectations into macroeconomics, he thought that he had banished the idea of animal spirits from macroeconomics. He was wrong."

Roger E.A. Farmer, Post-Keynesian dynamic stochastic general equilibrium theory, in European Journal of Economics and Economic Policies: Intervention, 2017, p 176.

1. INRODUCTION

For us; all human rationality is in irrationality! With the appearance of Lucas in the debate between Keynesians and classics in the early 1970s, the position of Keynesians was weakened and also intensified due to the lack of the mathematical and microeconomic foundations necessary to explain the general framework of Keynesian theory and especially the essential assumption of this theory; namely price rigidity and wage rigidity, and how these rigidities explain the inflation-output trade-off; specifically, the inflation-unemployment trade-off.

In his famous paper of 1972, see (Lucas, Expectations and the neutrality of money, 1972), Lucas established a new vision of macroeconomic theory by reconstructing a new vision of different economic problems that were the scene of an intellectual battle between the classics and the Keynesians, reviving the hypothesis of rational expectations that John Muth addressed in the 1960s, (Farmer, 2017, p. 176)

Indeed, Lucas made a Pandora's box for the rational expectations hypothesis, similar to the Pandora's box that Zeus gave to Pandora, Epimetheus wife, and asked her not to open it, the only difference is that Lucas asked the classics to open it. The opening of this Pandora's box released the first curse on the Keynesian theory; that there is no trade-off between inflation and output, and between inflation and unemployment.

To show that it does not exist a correlation or any kind of mechanic relationship between inflation and output, even inflation and unemployment, Lucas treated this point in his first's works by relating the aggregate demand (AD) function with monetary variables, as established by Irving Fisher and Milton Friedman later in the famous *Quantity theory of money* (QTM).

Rational expectations hypothesis (REH) is an important argument for Lucas to deny the non-existence of inflation-output trade-off, even the possibility to use it as a monetary authority target. To support his thesis, Lucas gives us a solid theoretical explanation of the way that economic equilibrium can be realized with rational expectations of agents. Indeed, economic equilibrium is achieved by equalization the aggregate supply (AS) with the aggregate demand function. Hence, the AD function describes the production (supply or output) as an increasing function with the money supply (M^s) and decreasing with the price level (p_t) or inflation (π_t) (Mignon, 2010, p. 28). This relationship between production as a real variable, on one side and the money supply and the price level, as a nominal variable, in the other side; demonstrates the existence of correlation or trade-off between inflation and output. *That would have been correct if there were no absolute rational expectations of agents according to Lucas philosophy; rational expectations à la Lucas!*

The hypothesis of (absolute) rational expectations *à la Lucas* is a theoretical hypothesis much more than a realistic hypothesis; authorities cannot cheats agents (we will adopt the term cheat as is used in the economic literature) see (Barro & Gordon, Rules, discretion and reputation in a model of monetary policy, 1983) with systematic monetary policy, because they anticipates precisely any action of the monetary authority, each expansion in M^s will be reflected in the price level only (Fischer, 1977, p. 194). With this statement, Lucas breaks the chain between inflation, production and unemployment, and at the same time he gives us, indirectly, an exception: *the trade-off may exist; only if the monetary authority can cheat agents with using an unexpected (unusual) monetary policy, which means a shock of the unsystematic monetary policy.*

According to Lucas and the New Classical Macroeconomics NCM, the trade-off cannot exist in short-run, even in the long-run. In fact, in his model, Lucas eternizes the hypothesis of classical neutrality of money when he denied this trade-off, and after him the new classical macro-econometricians such as (Sargent & Wallace, 1974, p. 233). They insisted that there is no possibility of arbitration in their model. For this school of thought and his leaders and disciples, the hypothesis of the super-neutrality of money is so important to explain how M^s, and thus π_t, does not affect the real side of economy, like consumption, product, and capital (Wickens, 2010, p. 231).

Thus, the aggregate supply (AS) curve will be vertical and stationary, because it will take the same position in short-run and long run under the model of rational expectations. Therefore, output (Y_t) will always be at its natural level, we can call (Y_n) as natural output. So, if that is absolutely true, in the long-run, output tends towards the steady state point of the economy.

Indeed, when expectations are rational, the only level the economy can reach in a long-run, and even in a short-run, is the steady state, with AS curve vertical, output also called income will be in his natural level, and the equilibrium in the labour market will be at the natural rate of unemployment (u_n), too (Maddock & Carter, 1982, p. 42).

Well, it is all? The trade-off, does not exist anymore? Is the steady state like a dead point where investment equals the capital depreciation (ϖK_t), and where everything is in the stationary mode and cannot move on? What goes do the monetary authority if he aims to increase the output beyond the natural rate and decrease unemployment? We can open some little of hope if we interpret in other way the final result of rational expectations, but it will make a kind of sensitivity for the NCM. The output can move away from its natural level, up or down, if the gap between the effective M^s and the expected M^s component is not equal to zero (Romer, 2012, p. 297) . In other words, can be exists a money effect on real side of the economy, what we call the unsystematic monetary policy or the unexpected monetary policy.

Now *we have a trade-off between the unexpected money supply and output*. For (Gordon, 1977, p. 9), the increase in employment depends on the unexpected supply of money by the public, so the monetary policy cannot influence the output and unemployment in the long-run and in the short-run, unless his policy is irregular and unexpected. That exception was mentioned by Barro and Gordon (Barro & Gordon, 1983, p. 590); when the growth of money supply is unexpected, even the inflation will be unexpected, and so there will be an influence on output, and therefore on unemployment, like showed in the Phillips curve.

At last but not least, still we have some questions needed to be answer by the REH. What if there was no confidence between government and the public (households, business sector), can expectations still rational? Do public have confidence in the future directions of government economic policies, especially the future directions of monetary authority? Well; for most underdeveloped and emerging economies, the public do not trust in the authorities because their failures and ambiguity in the management of different economic policies. For that we think the possibility of the trade-off can be functioned, especially, in those countries.

Furthermore, the possibility to make rational expectations is so difficult for the public, so they think that future monetary policy will be unsystematic and unstable, therefore they fail to expect the monetary shocks in advance, this situation creates a monetary effect on real side of the economy.

Actually, should public trust in the policymakers? Should they expect a stable monetary policy? (Laidler, 1982, p. 21). For Kydland and Prescott; the optimal policy is inconsistent and unstable (Kydland & Prescott, 1977, p. 480).

Moreover, if expectations are rational, it is clear that kind of expectations will pull eventually, in the long run, the economy towards a steady state, where there is no monetary shock happens, and therefore the natural output will be at its long run level (Kempf, 2001, p. 308). This situation prompts us to ask the principal question of our work: *under the rational expectations hypothesis, how can the output move on beyond the steady state with using an unsystematic monetary policy?*

Obviously, the principal question connects the growth theory with the unexpected inflation as a result of the unexpected part of the monetary policy, in the model of REH *à la Lucas*, as we know it. In order to answer this question, we are started by giving an algebraic model of rational expectations that determines non-existence of any kind of trade-off between inflation and output. Secondly, we exposed the idea of an infinite steady state as an ultimate result of REH. Then we tried to theorize a solution to go beyond the Lucasian steady state, by (*re*)-establishing the importance of capital accumulation through saving, as well as the importance of unexpected financial policy by economic agents. All this to end with results and remarks important enough to open an intellectual and scientific debate on the importance of small shocks that can push the economy beyond the Lucasian steady state.

2. *AB ORIGINE FIDELIS*: ALGEBRAIC MODEL OF THE REH

Before starting the algebraic demonstration of the REH model, it is clear that when expectations are rational, that means the price level (inflation) in the time (t) is the same expected price level (expected inflation) in the time $(t-1)$: $(p_t = p_t^e)$.

When we read the works of Lucas and any others works of the NCM, immediately we notice the devotion of those works to the old classical theories; they stay *Ab origine fidelis*. Indeed, we can find the demand equation in the Lucas's model by using the all most popular equation cited in the all manuals of macroeconomics, the Fisher's equation, see (Levačić & Rebmann, 1982) and (Wickens, 2010) for more readings. This equation was, and will stay, the angular stone for the economics thought, especially for the Classical thought (classical economy), the Neoclassical, and the NCM. However, according to Fisher's "*renovated*" equation *à la Friedman* (Wickens, 2010, p. 223):

$$M_t V_t = P_t Y_t, \quad \dots (1)$$

where M_t = the money supply, V_t = the velocity of money. However, from Fisher's equation, the output takes the following form:

$$Y_t = \frac{M_t}{P_t} V_t. \quad \dots (2)$$

In this form, the Fisher's equation does not reflect the relationship between the output, as the real side of the economy, and the two monetary variables. For that, it is necessary to add two parameters to equation (2):

$$Y_t = \frac{M_t^{\varphi}}{P_t^{\delta}} V_t, \quad \varphi > 0, \delta > 0. \quad \ldots (3)$$

Now, the demand side in the Lucas's equation is obtained by the log-linearization of equation (3):

$$\log Y_t = \log \left[\frac{M_t^{\varphi} V_t}{P_t^{\delta}}\right]$$

$$\log Y_t = \log\left(M_t^{\varphi} V_t\right) - \log P_t^{\delta}$$

$$\log Y_t = \log M_t^{\varphi} + \log V_t - \log P_t^{\delta}$$

$$y_t = \varphi m_t - \delta p_t + v_t, \quad \dot{V}_t = \frac{dV_t}{dt} = 0$$

$$y_t = \varphi m_t - \delta p_t. \quad \ldots (4)$$

Equation (4), is the same equation given by Lucas in his original paper (Lucas, Some international evidence on output-inflation tradeoffs, 1973, p. 328)

$$y_t = x_t - p_t.$$

Where $x_t = m_t$ = the money supply, in the Lucas model. For Lucas; x_t represents an exogenous variable which changes over time, and which is expressed in logarithmic form.

To complete the model, we need the Lucas supply function, with the following formula, as mentioned in Lucas' article (Lucas, Some international evidence on output-inflation tradeoffs, 1973, p. 328):

$$y_t = y_t^n + \theta\gamma(p_t - \bar{p}_t) + \lambda\left[y_{t-1} - y_{n,t-1}\right] \quad \ldots (5)$$

Where $\bar{p}_t = p_t^e$. Well; according to the concept of REH à la Lucas, we can put:

$$p_t = p_t^e. \quad \ldots (6)$$

$$p_{t-1} = p_{t-1}^e. \quad \ldots (7)$$

From the seventh equality, we get the following result:

$$y_{t-1} = y_{t-1}^n.$$

To simplify the solution, we assume that $\gamma = 1$ in (5); and the aggregate supply function (5) becomes:

$$y_t = y_t^n + \theta(p_t - p_t^e). \quad \text{... (8)}$$

Where θ is the ratio that takes the following form (Lucas, Jr, 1983, pp. 275-276):

$$\theta = \frac{\tau^2}{\sigma^2 + \tau^2}.$$

Where τ^2 describes the variability of relative prices in the economy. σ^2 is the variance of the general price level or inflation about its expected level.

Now according to (6), the output or the supply side in (8) becomes:

$$y_t = y_t^n. \quad \text{... (9)}$$

Indeed, as mentioned above, every single time when the expectations are rational the only level the output (the economy) can reach in a long run, and even in a short-run is the steady state (the natural level), and the equilibrium in the labour market will be at the natural rate of unemployment (u_n).

The Lucas model of REH is based on definite characteristics, because it is a model designed in discrete time, and the agents have all the information necessary to make their decisions rationally in time $(t-1)$, with a lag of one-period information about last period's prices across markets (Barro R. J., Rational expectations and the role of monetary policy, 1976, p. 3).

Obviously, the first price realized across markets is the *ex-ante* price before agents make their price expectations. This price is that of the Walrasian equilibrium which allows the realization of *an imaginary* Walrasian general equilibrium; in other words, it is the algebraic model of the neoclassical general equilibrium (Théoret, 1978, p. 386), also based on the existence of *a fictitious* auctioneer.

Therefore, to find this price we need the price expectation formula. The formula is made according to the REH philosophy. Thus, to expect the price level (p_t^e) (inflation rate (π_t^e) in the time (t) we need all the efficient information disposable (Ω_{t-1}) in the time $(t-1)$ which is a conditional probability distribution (mathematical expectation) for (p_t):

$$p_t^e = E[p_t \setminus \Omega_{t-1}]. \quad \text{... (10)}$$

Indeed, according to REH à la Lucas; the differences between the realized and expected values of the price level (inflation rate) are unknown (Burda & Wyplosz, Macroéconomie, 2003, p. 105):

$$p_t - p_t^e = \varepsilon_t.$$

Where ε_t is prevision's error and its take a random walk, called the white noise. The mathematical expectation $E(\varepsilon_t) = 0$, because agents efficiently use all available information in order to expect the price level.

However, we can rewrite (8), after adding the random error of all variables affecting (y_t), but with little effect:

$$y_t = y_t^n + \theta(p_t - p_t^e) + \varepsilon_t. \quad \text{... (11)}$$

Thus, by equalizing (4) and (11), we can find the *ex-ante* equilibrium price in the Lucasian model:

$$y_t^d = y_t^s$$

$$\varphi m_t - \delta p_t = y_t^n + \theta(p_t - p_t^e) + \varepsilon_t$$

$$\delta p_t = \varphi m_t - y_t^n - \theta(p_t - p_t^e) - \varepsilon_t$$

$$\delta p_t = \varphi m_t - y_t^n - \theta p_t + \theta p_t^e - \varepsilon_t$$

$$\delta p_t + \theta p_t = \varphi m_t - y_t^n + \theta p_t^e - \varepsilon_t$$

$$p_t = \frac{1}{\delta + \theta}(\varphi m_t - y_t^n + \theta p_t^e - \varepsilon_t), \quad \delta, \theta \; and \; \varphi > 0. \quad \text{... (12)}$$

Now, by substitution (12) into (10) and after the policymakers determine the monetary policy for the time (t), the *ex-post* price with rational expectations becomes:

$$p_t^e = E\left[\frac{1}{\delta + \theta}(\varphi m_t - y_t^n + \theta p_t^e - \varepsilon_t)\right]$$

$$p_t^e = \frac{1}{\delta + \theta}(\varphi E m_t - E y_t^n + \theta E p_t^e - E\varepsilon_t)$$

$$p_t^e = \frac{1}{\delta + \theta}(\varphi E m_t - y_t^n + \theta p_t^e). \quad \text{... (13)}$$

Where the best output the economy can reach is natural output $Ey_t^n = y_t^n$, and it is also clear that the best price agents can expect is the expected price $Ep_t^e = p_t^e$.

Obviously, from (13) the natural output becomes:

$$\frac{1}{\delta + \theta} y_t^n = \frac{1}{\delta + \theta} (\varphi E m_t + \theta p_t^e) - p_t^e$$

$$y_t^n = \varphi E m_t - \delta p_t^e. \quad \text{... (14)}$$

Therefore, (14) it relates natural production (the supply side of the economy) to expected money supply and price level expectations. If the government and the policymakers want to reduce unemployment by increasing natural production, the only possibility offered in the Lucas model is unsystematic monetary policy or unpredictable monetary policy. Indeed, by substituting (12) and (13) into (11) this relation can be further clarified (We have reduced some algebraic proof steps):

$$y_t - y_t^n = \theta \left[\left(\frac{1}{\delta + \theta} \right) (\varphi m_t - y_t^n + \theta p_t^e - \varepsilon_t) - \left(\frac{1}{\delta + \theta} \right) (\varphi E m_t - y_t^n + \theta p_t^e) \right] + \varepsilon_t$$

$$y_t - y_t^n = \frac{\theta}{\delta + \theta} \varphi m_t - \frac{\theta}{\delta + \theta} \varphi E m_t - \frac{\theta}{\delta + \theta} \varepsilon_t + \varepsilon_t$$

$$y_t - y_t^n = \frac{\theta \varphi}{\delta + \theta} (m_t - E m_t) + \varepsilon_t \left(\frac{\theta}{\delta + \theta} \right). \quad \text{... (15)}$$

Although equation (15) shows us that even though we have a correlation between the real variables (production and unemployment) and the nominal variables (money supply), this correlation does not explain any direct correlation between these variables. But on the other hand, equation (15) *opens a small crack* in the Lucasian theory. A link or correlation can exist between output and the money supply if the policymakers manage to implement *an unsystematic monetary policy* or discretionary policy, even *by cheating and tempting agents* (Barro & Gordon, Rules, discretion and reputation in a model of monetary policy, 1983, pp. 106-107).

Suppose now that the policymakers undertake and commit to follow a systematic monetary policy according to a rule, in the following form:

$$m_t^s = \delta_0 + \delta_1 y_{t-1} + \delta_2 p_{t-1} + \xi_t. \quad \text{... (16)}$$

$$E m_t^s = \delta_0 + \delta_1 y_{t-1} + \delta_2 p_{t-1}. \quad \text{... (17)}$$

By subtracting (17) from (16), we get:

$$m_t^s - Em_t^s = \xi_t. \quad \dots (18)$$

Indeed, (18) explains that there is a minimal effect of unsystematic monetary policy on output, at least in the short run. We substitute (18) into (15) in order to have a final result (the output gap):

$$y_t - y_t^n = \frac{1}{\delta + \theta}(\theta\varphi\xi_t + \theta\varepsilon_t). \quad \dots (19)$$

Obviously (19) clearly shows that the deviation of the output from its natural level is only related to random factors, and therefore any systematic policy will be recognized and neutralized by the agents and exerts no impact on the economy.

Through our study of the question of arbitration between inflation and output (unemployment) in a universe of absolute rationality, we have reached a very important conclusion; which is the absence of any possibility of arbitration in a context of systematic policies and the absolute rationality of individuals. This result leads us to ask, *again*, the following question: *how can we move away from the steady state in a universe of rational expectations?*

3. FROM THE SHORT-RUN TO THE LONG-RUN: MULTIPLE STEADY STATES

Thus, Lucas created a Pandora's box in which he established the non-existence of any inflation-output trade-off. Indeed, Lucas made a plan of no escape from the non-existence of any inflation-output trade-off, in the short run even in the long-run. In fact, this plan leads the economy in the long-run towards a situation of *multiple steady states*.

Obviously, the steady state situation has been mentioned a lot in the economic literature, as a long run state in which no shocks or surprises happens and therefore the natural output will be at its long-run level (Kempf, 2001, p. 308). Also one of the best explanations, is that of Solow in his reference work on *growth theory*; for him, in this type of economy, output, employment, and capital stock increase or grow exponentially, in which the capital/output ratio for that economy is constant (Solow, 1988, p. 4).

The absolute rationality of individuals, to which NCM thought is "constrained", represents an intellectual and scientific obstacle as much as a practical one, because it makes it difficult to answer the following question: *is the national product (output) always oriented towards its natural state, and therefore it always tends towards the steady state?*

In this philosophical and ideological context, it is impossible, for example, to follow a systematic economic policy to influence savings, unless the policymakers choose to cheat economic agents. Thus to place our work in context, we have defined the multiple steady states situation as the situation in which the NCM's AS curve intersects the savings curve, an intersection that reflects two different markets; goods and services market, and capital market. The first market can be expressed as reflecting the new investments that are linked with the savings offered on the second market, it is therefore a matter of an equilibrium between investment and savings. Therefore; it is a situation where any systematic policy has no effect on the real side of the economy. This situation reflects the infinite horizon of output in the short-run as cumulative process of y_t, as showing in (9). Let it $y_t = (y_0, y_1, y_2, \dots, y_t)$ be a sequence of output for a period 1 to *t* (it can be an infinite horizon, y_∞) and $y_t^n = (y_0^n, y_1^n, y_2^n, \dots, y_t^n)$ be the corresponding sequence for natural output (it can be an infinite horizon too, y_∞^n), (see figure 1).

3.1. *Short-run: "Wicksell-Friedman" non-neutrality of money vs. Lucas's neutrality*

Undoubtedly, in the economic literature, it is evident that *Friedman's natural rate of unemployment* is the copy of *Wicksell's natural rate of interest* (i_n), as developed in Wicksell's monetary theory. Indeed, *a bove majore discit arare minor*; Friedman particularly thanked Wicksell for his clarification about the natural rate, and for his clarification about the discrepancy that may exist between a normal rate and a market rate (Friedman, 1968, p. 7). *Perhaps*, Friedman was deeply inspired by the idea of the natural rate and Wicksell's monetary theory to the point of proclaiming the non-neutrality of money in the short-run.

Friedman's contribution to the renovation and reinterpretation of the original Phillips relationship (the original Phillips curve) is fundamental, it has allowed us to better (without being the most precise) understand the relationship between the variables of the Phillips curve. His criticism focused on two fronts:

1. NAIRU (Non Accelerating Inflation Rate of Unemployment) (u_n);
2. In the long-run, "*Phillips' relationship disappears*".

However, since the appearance of REH à la Lucas, *Friedman's adaptive expectations* have become more and more unattractive to policymakers. Before this eclipse, Friedman broke away from some old monetary theories by his work, of the 1960s, on the non-neutrality of money in the short-run, *What makes him like a excommunicado from the NCM sanctuary*, and by introducing adaptive expectations into the original Phillips curve, to give us what we call the augmented Phillips curve, or the expectations-augmented Phillips curve (EAPC), following the simplified form (Ball & Mazumder, 2015, p. 6):

$$\pi_t = \pi_t^e + \beta(u_t - u_n) + \varepsilon_t. \quad \beta < 0. \quad \dots (20)$$

Where $\pi_t = \frac{\dot{P}_t}{P_t}$, and $(u_t - u_n)$ is the unemployment gap used as a measure of excess demand which can also be represented by the output gap, cited above in (19) (Karanassou, Sala, & Snower, 2006, p. 11).

Obviously, Friedman distinguishes between the short-run and the long-run analysis; hence Friedman lays out two Phillips curves, one for the short-run (short-run expectations-augmented Phillips curve SREAPC) and the other for the long-run (long-run expectations-augmented Phillips curve LREAPC), respectively as follows (Mignon, 2010, p. 22):

$$\pi_t = \pi_t^e + \alpha - \beta u_t - \frac{\dot{Y}_t}{Y_t}. \quad \dots (21)$$

When $(\pi_t = \pi_t^e)$ become identical and equal in the long-run, (21) becomes (Mignon, 2010, p. 22):

$$\alpha - \beta u_t - \frac{\dot{Y}_t}{Y_t} = 0, \quad \dots (23)$$

In the long-run $(u_t = u_n)$, therefore (23) becomes:

$$u_n = \frac{\alpha - \left(\frac{\dot{Y}_t}{Y_t}\right)}{\beta}. \quad \dots (24)$$

Where $\left(\frac{\dot{Y}_t}{Y_t}\right)$ is the growth rate of output, i.e. (g_{y_t}).

Absque argento omnia vana; for Friedman, the non-neutrality of money in the short-run plays an important role in explaining the existence of trade-off. Friedman's view of QTM is interesting in explaining the rise in inflation; for him, inflation increases only if the growth rate of the money supply (the rate of monetary expansion) increases more than the growth rate of production; let's put $\left(\frac{\Delta M_t}{M_t} = \frac{\dot{M}_t}{M_t} = g_{m_t}\right)$ the growth rate of money supply. The link between money supply and inflation is clearly demonstrated under certain hypothesis; Y_t is assumed to be constant in short-run, empirical studies confirm this (Abouderaz, Réflexion sur l'inflation algeriénne (2000-2012): étude empirique, 2013, pp. 43-44), and V_t is assumed to be constant since the old view of Fischer's QTM, empirical studies confirm this. Indeed, the constancy of V_t

has not been unanimous between economists, and that for a long time, different empirical studies show that V_t is stable over time, see (Tétinier, 1943), (Parizeau, 1957) and (Abouderaz, Réflexion sur l'inflation algeriénne (2000-2012): étude empirique, 2013).

In order to demonstrate the above, we use (4), with some modifications:

$$g_{y_t} = \varphi g_{m_t} - \delta g_{p_t}$$

$$g_{y_t} = \varphi g_{m_t} - \delta g_{\pi_t}. \quad \dots (25)$$

$$\varphi g_{m_t} = \delta g_{\pi_t}. \quad \Leftrightarrow g_{y_t} = 0. \quad \dots (26)$$

Obviously, (26) established Friedman's cause-effect; *inflation is always a monetary phenomenon*.

- *Ex-ante situation*, when $\pi_t \neq \pi_t^e \therefore g_{m_t} = g_{y_t}$:

To demonstrate the impact of money on the real side of the economy (output and unemployment) in the short-run, we start with log-linearization of (1):

$$\log(M_t V_t) = \log(P_t Y_t)$$

$$y_t = p_t - m_t. \quad \dots (27)$$

Where p_t is the price level, or rate of inflation written as π_t, hence (27) can be (*re*)-written as:

$$y_t = \pi_t - m_t. \quad \dots (28)$$

By substitution of (21) in (28), the SREAPC becomes (some algebraic proof steps have been reduced):

$$y_t = (\pi_t^e + \alpha - \beta u_t - g_{y_t}) - m_t. \quad \dots (29)$$

We assume that the simplified (reduced form) algebraic function of the unemployment can take the following form, *ceteris paribus* neglected in this framework, for now:

$$U_t = f(Y_t;\, \Psi), \quad \Psi = (\alpha, \beta), \alpha > 0, 0 < \beta < 1 \; and \; \frac{dU_t}{dY_t} < 0, \forall_t$$

The equation:

$$U_t = \alpha - Y_t^{\beta}. \quad \dots (30)$$

After log-linearization of the unemployment equation:

$$u_t = -\beta y_t. \quad 0 < \beta < 1. \quad \dots (31)$$

By substitution of (31) in (29), SREAPC becomes:

$$y_t = u_t = \frac{\pi_t^e - \mathcal{g}y_t - m_t + \alpha}{1 - \beta^2}. \quad \ldots (32)$$

- *Ex-post situation*, when $\pi_t = \pi_t^e \therefore \mathcal{g}_{m_t} > \mathcal{g}_{y_t}$:

Obviously, according to (20) and (26); in the long-run $\pi_t = \pi_t^e$ and $u_t = u_n$, as a consequence of adaptive expectations and the monetary cause of inflation. Thus, from (32) LREAPC becomes:

$$y_n = u_n = \frac{\pi_t - \mathcal{g}y_t - m_t + \alpha}{1 - \beta^2}$$

$$y_n = u_n = \frac{\alpha - \mathcal{g}y_t}{1 - \beta^2}. \quad \ldots (33)$$

From (33), it is easy to deduce the (*super*)-neutrality of money in the long-run, natural unemployment and natural output will only be driven by the real variable ($\mathcal{g}y_t$); no effect of money supply on the real side of the economy in the long-run.

3.2. *long-run: "Friedman-Lucas" super-neutrality of money*

In the study of two-time economy and generally in intertemporal analysis, this necessitates a distinction between neutrality and super-neutrality of money (Kempf, 2001, p. 187). We can explain the super-neutrality of the currency by the nominal shocks on the economy (unexpected expansionary monetary policy) in the form of monetary shocks. This explains the absence of effect of relative variations in nominal cash balances, and therefore of inflation, on the real variables of the economy such as consumption, production or capital (Wickens, 2010, p. 233).

Now, if we set $\beta u_n = \alpha - \left(\frac{\dot{Y}_n}{Y_t}\right)$ and substitute it in (21), it will be is easy to deduce the algebraic form (without the random shock) of the EAPC in (20):

$$\pi_t - \pi_t^e = \beta u_n - \beta u_t$$

$$\pi_t - \pi_t^e = -\beta(u_t - u_n). \quad \beta > 0. \quad \ldots (34)$$

Where the coefficient β measures the sensitivity of inflation to cyclical unemployment, in other words; if ($\beta = 0.3$), this means that a drop in unemployment of one point relative to the natural rate causes inflation to rise to a rate of 0.3% relative to the expected rate of inflation.

According to (34) and Friedman, when agents adjust (adapt) their long-term expectations, the unemployment rate will settle at its natural rate and the only result that can be achieved is higher inflation. This result is the same as that claimed by Lucas and Sargent, which recognizes the super-neutrality of money in the short and long-run, except that Friedman remains *as "a short-run excommunicado"*.

4. BEYOND THE STEADY STATE, CAPITAL STOCKS AS A SOLUTION: AN ATTEMPT AT THEORIZATION

The idea that the interest rate (i) was an absolute monetary phenomenon and influencing real economic variables was not unanimously accepted by most economists; for example, it wasn't the case for *Knut Wicksell*. This is what *Trygve Haavelmo* tried to refute in his work, and therefore, trying to prove the existence of a relationship between the interest rate and investment (I) (Haavelmo, 1960, p. 157).

For Wicksell there is an interest rate related to loans that does not affect the prices of goods "*commodity prices, as Wicksell cited in his work*" by making them raise or lower, because it is neutral; which can be interpreted as *the current value of the natural rate of interest on capital* (Wicksell, Interest & prices, 1962, p. 102). Wicksell tried, through the idea of the natural rate of interest, to show that if the real activity of the economy in the short-run is affected by the money supply (supply of bank money, loans to the economy), which is contrary to the hypothesis of the classical dichotomy of course, it will not be the case in the long-run, and therefore the money will return to its usual neutrality (Lavialle, 2014, p. 8).

Indeed, if we back to the *Wicksell's monetary theory*; for him there is a natural rate, and it is not fixed or unalterable in magnitude. There are also variables that affect the natural interest rate: the efficiency of production, the available amount (value) of fixed and liquid capital, the supply of labour and land (Wicksell, Interest & prices, 1962, p. 106). The notion of natural rate can be described in a simpler way; *"as the rate of interest that equates saving with investment"* (Amato, 2005, p. 3). *Now, how is the Wicksellian natural interest rate created?*

Wicksell answers this question in one of his masterpiece (Lectures on political economy, Volume II: money), explaining how the economy works at the natural rate; at this rate, the demand for capital loans corresponds to the supply of savings (S), as well as to the (more or less) expected (anticipated) return on the newly created capital. For Wicksell, future prospects (or anticipations) concerning capital investment play a very important role in the evolution of this rate; if these prospects tend to improve, demand will increase and exceed supply. In this case, the interest rate rises, further stimulating savings, at the same time as investor demand falls, until the new equilibrium is reached at the point where a slight rise in the interest rate is reached (Wicksell, Lectures on political economy, Volume II: money, 1978, p. 193).

Wicksell's proposal includes the concept of marginal productivity of capital (mP_K); *which means the additional output obtained through an increase in the stock of capital* (Samuelson & Nordhaus, 2005, p. 277). Moreover, the notion of internal rate of return, or the marginal efficiency of capital (ρ), *which is a Keynesian term*. The concept of internal return is consistent with the concept of rate of return on investment; It is the discount rate that makes the present value of the net flows expected from an investment equal to the price paid for the acquisition of the investment equipment. This rate is obtained, if for example the net variation of flows (ΔN_t) in an infinite horizon (∞), by the following formula (Levačić & Rebmann, 1982, pp. 231, 239) (We have avoided mathematical proofs so as not to disturb the reader with mathematical equations and intuitive proofs):

$$\rho = \frac{\Delta N}{pkI}. \quad , \overline{\Delta N} \neq 0$$

Where, $pkI = \sum_{t=0}^{\infty} \frac{\Delta N_t}{(1+\rho)^t}$ is the initial cost of the capital equipment, it is also called cash flow for the acquisition of new fixed assets (capital).

Compared to the notion of Wicksell natural rate of interest, we note that the notion of internal return or marginal efficiency of capital corresponds to the notion of Wicksell's rate of interest. We can therefore consider the Wicksell natural rate as the marginal productivity of invested capital or the marginal efficiency of investment (replacement of capital by investment), which prevails if and only if the market for goods and services is in equilibrium (Chauveau, 2001, p. 181).

Hence, we will simplify the idea by starting from the point where we left off, which is the emergence of the natural rate when the market for goods and services is in equilibrium; in this case the net investment is equal to the net saving:

$$I_t = S_t \Rightarrow i_t = i_n.$$

Where I_t and S_t are the net investment and saving, consecutively.

Indeed, when the market for goods and services is in equilibrium due to the equalization between the supply of capital (saving) and the demand for capital (investment) in the long-run, the Wicksellian natural interest rate will exist. Obviously, the equalization will be the combination between the sum of the savings and the sum of the investments, which represents the points (combinations) of the long-run equilibrium, and at the same time they are the points (combinations) of the short-run equilibrium. The question now is: *What is the source of long-run capital accumulation?*

Obviously, the new capital accumulates in the capital market, due to the accumulation of long-run savings which turn into investments, then new capital is formed and so on; therefore, it is a situation of permanent equilibrium in the long-run, which we can call *"the natural long-run equilibrium (NLRE)"*. This equilibrium situation results in a natural rate of interest, equal to the monetary (effective) rate of interest. Short-run interest rates are considered as monetary (nominal) interest rates; in other words, as interest rates for discontinuous periods. Arrived at the period $(t + 1)$, the natural interest rate will reflect the accumulation of short-run interest rates. We shall assume that the long-run interest rate tends to move in the same direction as the short-run monetary interest rate.

As we saw above indeed; in the short-run, bank loans affect the real side of the economy (contrary to the classic dichotomization), but this effect will disappear in the long-run, because of the equality between the rate of return on investments with the bank interest rate. This equality will generate the following equilibrium:

$$I_t = S_t. \quad \forall_t. \quad \ldots (34)$$

Now, taking (34), we can arrive at the same view of Solow's growth. Starting from the equality between the sum of aggregate investment and the sum of aggregate saving:

$$\sum I_t = \sum S_t. \quad \forall_t.$$

From Solow's original growth model (Solow M, 1956, pp. 66-67):

$$Y = F(K, L). \quad \ldots (35)$$

$$\dot{K}_t = sY_t. \quad \forall_t. \quad \ldots (36)$$

Where $\dot{K}$ is the rate of increase of capital stock; $\dot{K}_t = \frac{dK}{dt}$, this rate represents the net investment I_t and s is the rate of saving or the marginal propensity to save.

We put the rate of increase of capital stock following this form (Niehans, 1963, p. 358):

$$\frac{dK}{dt} = \dot{K} = s(y - m_k)L. \quad \ldots (37)$$

Where $y = \frac{Y}{L}$ is the real income (the output) per capita, and m_k is the minimum level of capital and finally, L is the labour.

Obviously, (37) shows us the rate of increase of capital stock is correlated to saving, real income per capita, the minimum level of capital which insensitive to the monetary interest rate, and finally to labour.

The process of capital accumulation is considered as a continuous process over time, knowing that net investment is the sum of total investment minus a part of the depreciation or deterioration of the capital stock, the mathematical identity follows the form:

$$\frac{dK}{dt} \equiv I_t. \quad \dots (38)$$

Furthermore, according to the definition above, the net investment is:

$$I_t = I_g - \varpi K. \quad \dots (39)$$

Where I_g is the gross investment.

By substituting (38) in (37), the net investment becomes:

$$I_t = \dot{K} = s(y - m_k)L. \quad \dots (40)$$

Using (36),

$$I_t = sY_t. \quad \forall_t. \quad \dots (41)$$

What we can also explain in terms of time, as:

$$dK \equiv I_t\, dt = sY_t. \quad \dots (42)$$

And:

$$K_t = \int I_t\, dt = \int \frac{dK}{dt} = \int dK$$

$$K_t = \int dK. \quad \dots (43)$$

Indeed, (42) clearly explains that (dK), the accumulation of K, depends not only on I_t, but also on (dt) which is the time it takes. This mathematical explanation leads us to ask the following question: *What is the size of the capital accumulated during a period of time between* $[0, t]$*?*

The answer takes the following form:

$$\int_0^t I_t\, dt = K_t$$

$$\int_0^t I_t\, dt = [K_t]_0^t = K_t - K_0$$

$$K_t = K_0 + \int_0^t I_t\, dt. \quad \dots (44)$$

Obviously, (44) reflects the size of capital over different time periods; which is equal to the initial capital plus the capital accumulated to date (t). The search for the accumulation of capital leads us to ask another question*: what will be the extent of this accumulation during an infinite horizon of time; Or in a different sense, what is the size of the accumulation over a continuous period of time, between the interval* $[0, \infty]$*?*

The answer is contained in Wicksell's monetary theory, as well as in the definition of (pkI), exposed in this article! Wicksell suggested that the natural rate of interest does not remain fixed, and it is adjustable (alterable) in its level; it depend on efficiency of production, on the available amount of fixed and liquid capital on the supply of labour and land (Wicksell, Interest & prices, 1962, p. 106). Indeed, in the same sense as explained before, forecasts and prediction on the placement (investment) of capital in the future, play an important role in the variations of the natural rate of interest (Wicksell, Lectures on political economy, Volume II: money, 1978, p. 193).

Furthermore, the amount of savings in (t) is the fixed rate s deducted from the real income Y_t, also called the rate of production; so we can write the following:

$$S_t = sY_t$$

$$\Rightarrow sY_t = I_t$$

$$\sum sY_t = \sum I_t. \quad \dots (45)$$

Clearly, (45) shows that the equilibrium of the goods and services market depends on the constant rate of savings s deducted from Y_t, where the real income value evolves according to two factors: (i) the rate of inflation prevailing during the period (t), which positively affects production or supply (the AS curve). (ii) the volume of investment (demand for new capital for investment) which is linked to the natural long-run interest rate or the bank interest rate at which acceptance is achieved between the savings suppliers (providers) and those who demand this savings, moreover the volume of investment which is affected by the rate of inflation prevailing at the period (t).

By definition, the saving size cannot be the same for each period, it will be different from period to period, which can be explained mathematically as:

$$S_t = \int \mathscr{s}Y_t\, dt = \int \dot{K}\, dt = \int I_t\, dt = \int \frac{dK}{dt}\, dt$$

$$S_t = \int dK. \quad \dots (46)$$

Now we can write the saving as:

$$S_t = \frac{dS}{dt} \equiv Y_t. \quad \dots (47)$$

Indeed, (46) and (47) show the amount of saving in period (t), considering that saving is the amount of capital accumulation. It is also the accumulation of total savings over time; which is by definition, the saved part of the real income (output), growing over time. Mathematically, saving takes the following form:

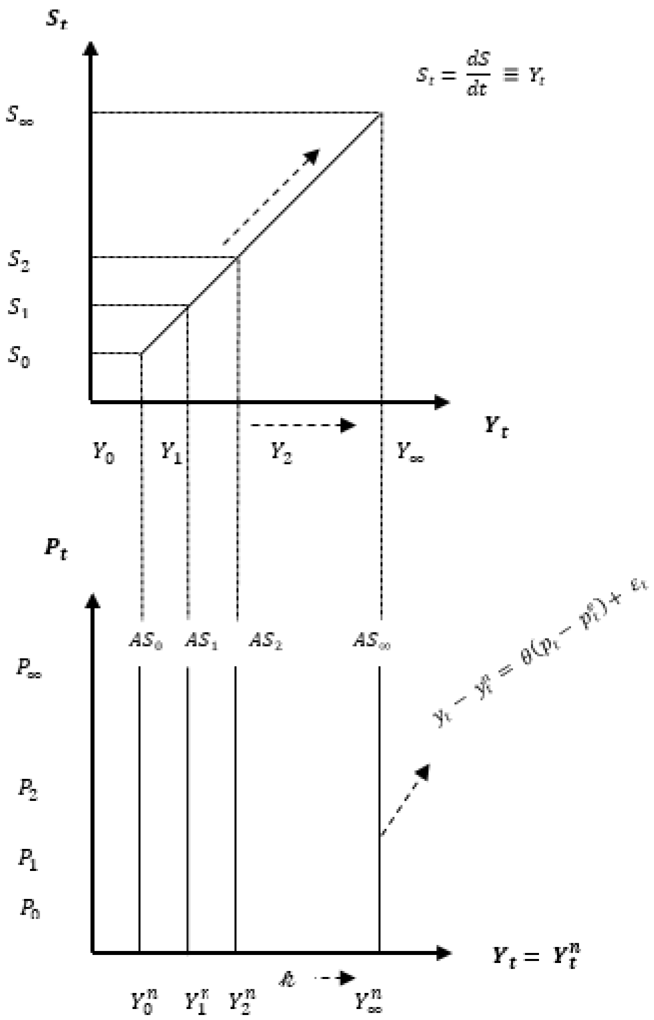
$$\int_0^t [\mathscr{s}Y_t]_0^t = \mathscr{s}y_t - \mathscr{s}Y_0$$

$$\mathscr{s}Y_t = \mathscr{s}Y_0 + \int_0^t \mathscr{s}Y_t\, dt$$

$$S_t = S_0 + \int_0^t S\, dt. \quad \dots (48)$$

Now it is easy to know, according to (43), (44), (47) and (48) , that the amount of savings for each period, which corresponds to changes in real income (the output), and the following figure illustrates this:

Fig.1: Accumulation of aggregate savings over time due to rising real income and growing natural output



Source: Own work, based on the result of equations (9), (11), (47) and (48).

Indeed, fig.1 clearly illustrates how saving can accumulate and rise simultaneously each time real income rises, and that over an infinite horizon. In theory, this could happen; indeed, savings can grow proportionally with real income, but not in the same proportion as that of real income (Keynesian marginal propensity to save).

To illustrate the idea of moving the economy from a steady state to another with a systematic economic policy, in a growing economy, where super-neutrality of money is absent, we will use Solow's growth theory and the Harrod-Domar consistency condition. Assuming the government increases the growth rate of the nominal stock of money (public debt), i.e. g_{m_t}; therefore, as a direct consequence, the public debt grow at a faster rate. We shall assume that the economy will reach a new steady state point, but this is an unjustifiable situation at this time. Well, how does the new steady state differ from the previous steady state? (Solow, 1988, pp. 68-

69). The answer is contained in (28) and in Friedman's monetary theory. So far we have discussed the effect of higher money supply on the inflation rate; the increase in the new inflation rate will be equal to the new increase in g_{m_t}, comparative to the old one; this is the same conclusion reached by Friedman in the QTM. We can explain the new inflation rate as follows (Solow, 1988, p. 69):

$$\pi_t = g_{m_t} - g_{y_n}. \quad ... (49)$$

Where g_{y_n} is the natural rate of the growth or the steady state output growth rate.

It would be overhasty to conclude the final result! Furthermore, we still have one missing question; *is that all that will happen?*

To answer the question, let us use the Harrod-Domar consistency condition for the non-monetary economy (Solow, 1988, p. 62):

$$g = \frac{s}{v}. \quad ... (50)$$

where $g = g_{y_n}$ is the natural rate of growth and $v = \frac{K}{Y}$ is the capital requirement per unit of output (capital/output ratio).

As shown in (50), the natural rate of growth is exactly the rate of growth of the stock of capital $\frac{s}{v}$. We can reformulate (50) in another more expressive form:

$$g = \frac{s}{\frac{K}{Y}}$$

$$g = g_{y_n} = \frac{sY_t}{K}. \quad ... (51)$$

Now, using the algebraic steps above and especially equations (44) and (48), we can (re)write a new form of (51):

$$g_{y_n} = \frac{sY_0 + \int_0^t sY_t \, dt}{K_0 + \int_0^t I_t \, dt}. \quad ... (52)$$

$$g_{y_n} = \frac{S_0 + \int_0^t S \, dt}{K_0 + \int_0^t I_t \, dt}. \quad ... (53)$$

Obviously, (51), (52) and (53) shows that the rate of natural growth is the part that is saved and invested for one unit of capital as a rate of units of capital used in production; this becomes clearer with the log-linearization of (51):

$$Y_n = \frac{\mathcal{s}Y_t}{K}$$

$$\log Y_n = \log\left(\frac{\mathcal{s}Y_t}{K}\right)$$

$$y_t - y_n = \mathcal{k}. \quad \dots (54)$$

Clearly, (54) explains that the difference between y_t and y_n, reflects the growth rate of the capital stock. If $\mathcal{k}$ ceases to grow and increase, due to low or zero savings, the economy will stabilize in steady state, i.e. $y_t = y_n$. *But, how can the stock of capital rise if expectations are rational, therefore?*

Now we want to argue heuristically! By equalization between (11) and (54):

$$\theta(p_t - p_t^e) + \varepsilon_t = \mathcal{k}. \quad \dots (55)$$

Because expectations are rational, (55) becomes:

$$\mathcal{k} = \varepsilon_t. \quad \dots (56)$$

Indeed from (56) we deduce an important result: *the increase in capital stock can result from economic policies (monetary or financial) to which agents do not attach much importance; "Unexpected economic policy actions by economic agents", which will increase output in a new steady state, with more inflation, growing output and less unemployment.*

Henceforth we will try to establish a relationship between the slight part (ε_t) of economic policy and the real side of the economy, *at least in the long-run*. Well, in this point, Solow exposes the importance of the absence of neutrality of money in a long-run growing economy, and how the real side of the economy could be affected by an unexpected fiscal policy, affecting this small part of the model.

Due to the financial (fiscal) policy assumed by the government above, we have seen that the rate of inflation increases as a direct consequence of this policy. A higher inflation rate increases the opportunity cost of holding money; this creates a decrease in real demand for money corresponding to each value of $\mathcal{v}$, this can be attributed to its lower real yield (real interest rate). In this situation, the new steady state has a higher ratio v, with a lower ratio of money supply to national product than the old one. This situation creates a superficial paradox; a faster increase in the nominal money supply moves the economy towards the state of *full employment* that would prevail in a non-monetary economy. The effect of an increase in the nominal money supply on the economy may occur, due to the flight of agents from money as a monetary asset due to the higher rate of inflation as the ratio of monetary assets to monetary income decreases (Solow, 1988, p. 69).

In other words; an increase in the nominal money supply increases the rate of inflation, thus the real interest rate which is the difference between the nominal interest rate and the expected rate of inflation will decrease. Therefore, savings do not bring much interest. Thus, agents demand money for transactions and reserves, which creates new demand (by the households and investors) which shifts the economy to the right to a new steady state (see Figure 1), but with higher inflation than before, all in the long-run. Here we have an important result: *money was not created by a systemic monetary policy with the instrument of open market operations, but by government budget deficit* (Solow, 1988, p. 69).

Henceforth, perhaps now it is acceptable to say that equation (56) is the result of this type of government policy or government action to affect output or growth and hence unemployment.

5. CONCLUSIONS AND REMARKS

The remarkable mathematical and philosophical solidity of the rational expectations hypothesis, makes it difficult to go beyond the Lucasian model. When Lucas built his sanctuary of rational expectations, his goal was to demonstrate the non-existence of any inflation-output (unemployment) trade-off, and also to prove the failure of the standard Keynesian model. With this quest; the NCM is demarcated from other economic thought and has banished others, like Friedman, at least in the short-run.

In this article we have explored the inflation-output trade-off and the steady state within the context of REH. Our main objective was to try to prove the existence of the inflation-output (unemployment) trade-off which allows the overcoming of the infinite steady state, under REH. Our important findings are as follows. First, we concluded that REH is more scientific and mechanical than a realistic hypothesis. Second, we found Friedman's adaptive hypothesis more realistic than REH *à la Lucas*, in the short-run. Third, REH eliminates any possibility for policymakers to control inflation or unemployment under REH through money. Fourth, REH brings the economy to an infinite steady state situation. Five, under REH; saving and capital accumulation can move the economy from a steady state situation to another in long-run, using economic policies (monetary or financial) to which agents do not attach much importance; *"Unexpected economic policy actions by economic agents"*, which will increase output in a new steady state, with more inflation, growing output and less unemployment.

Accordingly, we argue that policymakers should continue to pay attention to inflation-output trade-off.

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