Modeling Algeria's broad money supply using ARIMA-EGARCH model

نمذجة عرض النقود بمعناها الواسع في الجزائر باستخدام نموذج-ARIMA EGARCH

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Abstract

To model and forecast Algeria's broad money growth rate, a hybrid model was used based on autoregressive moving average ARIMA (0, 1, 1) where the Akaike information criterion (AIC) is used to examine the model's goodness of fit and an asymmetric GARCH model (EGARCH (1, 1)) to model the conditional variance where the log likelihood was behind our choice, while the percentage error of forecast was used to assess the predicting performance.

Keywords ::ARIMA-EGARCH, broad money,short-term forecast.

ملخص

لنمذجة وتوقع معدل نمو النقود بمعناها الواسع في الجزائر، تم استخدام نموذج هجين مع المتوسط المتحرك المتكامل الذاتي(ARIMA (0, 1, 1 حيث يتم استخدام معيار معلومات Akaike (AIC) لفحص ملاءمة النموذج ونموذج GARCH غير المتانظر(EGARCH (1, 1)) لنمذجة التباين الشرطي بحيث احتمال المعقولية كانت وراء خيارنا هذا ، بينما تم استخدام النسبة المئوية للخطأ في التنبؤ لتقييم الأداء التنبؤي.

الكلمات المفتاحية: نموذج ARIMA-EGARCH، عرض النقود بمعناها الواسع، التنبؤ قصير المدى.

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1. Introduction

Given a univariate time series $(Y_t)_{t \le n}$, the classic estimation of ARIMA (autoregressive integrated moving average) forecast models is based on modeling the mean conditionally on past information $E(Y_t / I_{t-1}) / I_{t-1} = Y_{t-1} - Y_{t-2} - \ldots$, and thus neglects the modeling of the local variance by considering it as constant, which is not always verified, especially in the case of financial chronicles that are characterized by a volatile variance. ARCH / GARCH (autoregressive conditional heteroskedasticity / generalized autoregressive conditional heteroskedasticity) estimation is used to model time series with unstable conditional variability.

To be able to integrate the estimation of the conditional mean and variance within the same model, a more intricate methodology of the ARIMA-ARCH type takes into account the information contained in residuals via an ARCH(P) model, or its GARCH(P, Q) generalization to improve the accuracy of short-term forecasting. Following that, this work is organized as follows: Part 2 presents linear modeling of the conditional mean (ARIMA), Part 3 illustrates the theoretical framework of an ARCH (P) model, Part 4 provides a list of the most commonly used forecast accuracy measures, Part 5 summarizes the empirical study undertaken in this work, and finally, Parts 6and 7 that summarize the main findings of the study.

2. Conditional Mean modeling ARIMA (p, d, q)

This class of models was developed by Box and Jenkins (1976) and is founded on three parameters: « p » expressing the order of the autoregressive process (AR), « d » representing the order of integration (I), and finally « q » representing the order of moving averages (MA). Unlike prevailing methods, ARIMA methodology presupposes the presence of innovations ε_t that affect the time series values.

2.1. Auto-regressive process AR (p)

This isbased on the assumptionthateach value at the instant t is a linearcombination of the p values passed to a nearby innovation:

$$Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \ldots + \varphi_p Y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim BB(0;\sigma_\varepsilon^2)$$

Using the backshiftoperator, thiscanberewritten:

$$(1-\varphi_1 B^1 - \varphi_2 B^2 - \ldots - \varphi_p B^p) Y_t = \varepsilon_t$$

$$\varphi_p(B) Y_t = \varepsilon_t$$

2.2. Integrated process I (d)

Nonstationarity in this case isstochastic and resultsfrom the accumulation of innovations (shocks), which tends to increase the variance, whichis no longer constant, but the seriesdifferentiated by order of integration d must bestationary.In the case of an order $d = 1$, wecanspeak of a pure randomwalk :

$$Y_t = Y_{t-1} + \varepsilon_t$$

This term refers to the act of entering the value of the instant t, beginning with the value of the instant $t-1$ and proceeding on a completely random trajectory of innovation ε_t (Desbois, 2005).

2.3. Moving average process MA (q)

In this case, the value of the seriesis a weighted moving average of previous innovations (Terraza and Bourbonnais, 2010); an MA (q) processisthenwritten: $Y_t = \theta_q(B)\varepsilon_t$. In contrast to integratedprocess, whichisdistinguished by the persistence of shocks, the effect of an innovation in a movingaverageprocess fades after a few observations

(Desbois, 2005). As a result, the linear ARIMA model (p, d, q) is a combination of the threepreviouslydiscussedprocesses:

$$\varphi_p(B)(1-B)^d Y_t = \theta_q(B)\varepsilon_t$$

3. Conditional variance modeling ARCH (P)

As stated by (Engle, 1982), the purpose of this type of processes was to model chronicles with unstable conditional variance, with highs and lows intervals of euphoria and calm , but in an ARCH error model, the instantaneous variance is a function of the set of information contained in the residuals $V(\varepsilon_t / \xi_{t-1}) = h_t^2 / \xi_{t-1} = \sigma\{\varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$ (Terraza and Bourbonnais, 2016):

$$V(\varepsilon_t / \xi_{t-1}) = h_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + ... + \alpha_P \varepsilon_{t-P}^2$$

With $\varepsilon_t = \eta_t h_t$ / $\eta_t \sim N(0,1)$

It is important to note that although the ARIMA model's residuals are serially uncorrelated, this does not imply the absence of heteroscedasticity; for this, we must test the correlation of the residuals at their second moment using the Lagrange multiplier (LM) test (Rublikova and Lubyova, 2013). In the case of coefficient significance beyond order 4, the presence of a GARCH (P, Q) (GARCH(P, Q)$\sim$ ARCH(∞)) process, which is a generalization of the ARCH (P) process due to Bollerslev (1986) is justified.

$$V(\varepsilon_t / \xi_{t-1}) = h_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{Q} \beta_j h_{t-j}^2$$

Following that, the ARMA-GARCH model class is defined by:

$$Y_t = \sum_{i=1}^{p} \varphi_i Y_{t-1} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-1} + \eta_t h_t$$

Given that the GARCH process is based on the assumption of a symmetry effect on the conditional variance, more sophisticated processes, such as the GJR GARCH and EGARCH models, attempt to capture the effect of asymmetry for negative and positive shocks. (Andersson and Haglund, 2015)

4. Accuracy

When assessing the accuracy of forecasts used on univariate time series, many measures have been presented (Hyndman and Koehler, 2006), with $err_t = Y_t - \hat{Y}_t$:

$$ME(\text{mean error}) = \frac{\sum_{t=1}^{n} err_t}{n}$$

$$RMSE(\text{root mean squared error}) = \sqrt{\frac{\sum_{t=1}^{n} err_t^2}{n}}$$

$$MAE(\text{mean absolute error}) = \frac{\sum_{t=1}^{n} |err_t|}{n}$$

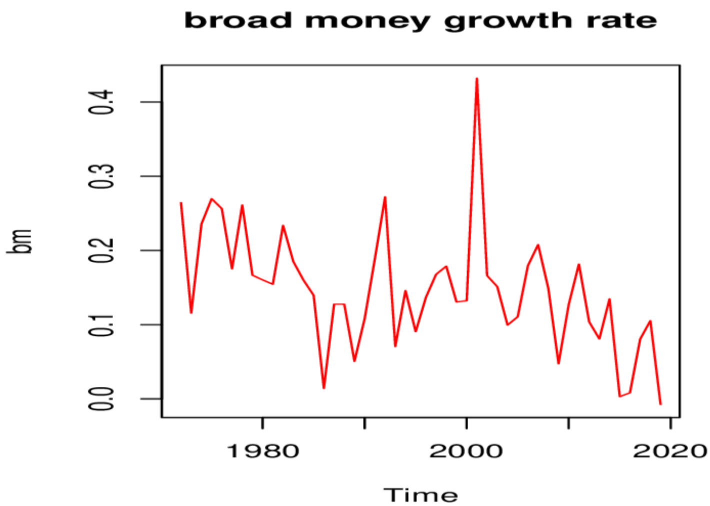
$$p_t(\text{percentage error}) = \frac{100.err_t}{Y_t}$$

5. Experimental

The annual data concern the Algeria's broad money supply (current LCU) between 1971 and 2019[1]. Growth rate was obtained by taking the log difference of the series $\log(Y_t) - \log(Y_{t-1})$.

[1]World Development Indicators (World Bank)

Fig.1. Annual broad money growth rate in Algeria



Source: Outputs of R

The ADF test was used to examine the null hypothesis of non stationarity of broad money growth rate series and yielded a nonsignificant p-value. The differentiated series reveals a significant p-value which leads us to accept the alternative hypothesis of stationary. Table 1 displays the results.

Table 1.ADF test results

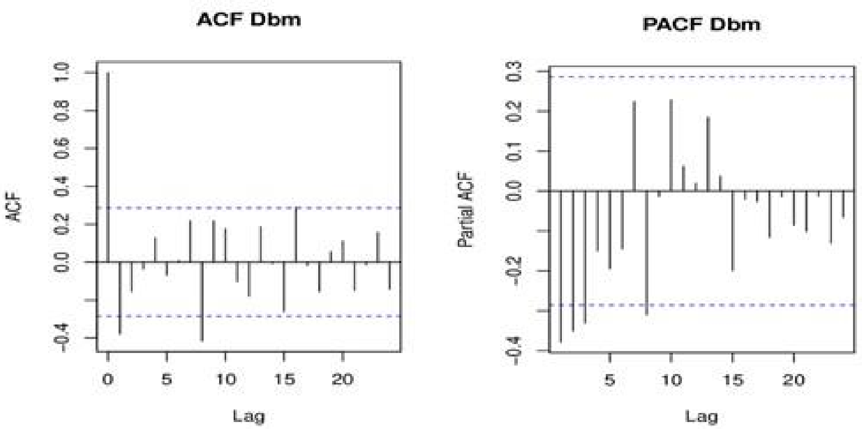
	no drift no trend	**with drift no trend**	**with drift and trend**
Broad money growth rate	-1.39	-1.76	-2.38
p-value	0.1779	0.4182	0.4120

Differentiated(Broad money growth rate)	-5.19	-5.30	-5.23
p-value	0.01	0.01	0.01

Source: Prepared by the researcher based on the outputs of R

Then we plotted the correlograms of the differentiated series to determine the order of the delays (p, q) and used the information criterion to select the best model ARMA (0, 1), with the following parameter estimation. (See table 2)

Fig.2. Differentiated broad money growth rate correlograms



Source: Outputs of R

The table below shows that the p-value of the estimated coefficient is less than 0.05, indicating that the moving average coefficient polynomial meets the parameter significance requirements.

Table 2: Model regression results of the ARMA (0, 1) model

Variable	Coefficient	Standard Error	t statistic	P-value
MA(1)	-0.7602	0.097	-7.8371	0 .0000

Log likelihood	54.01
Akaike Info criterion	-104.02

Source: Prepared by the researcher based on the outputs of R

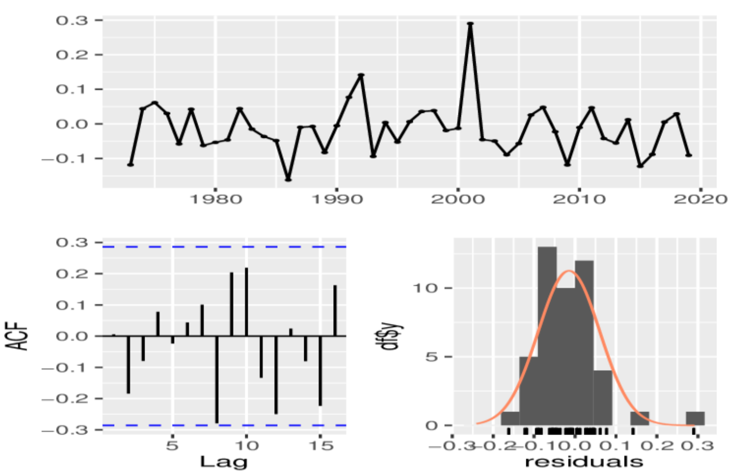
The p-value of the Q statistic (Box-Ljung test) of the first 20 residuals in the ARMA (0, 1) model is substantially more than 0.05, suggesting that the residual demonstrates pure randomness (Liu and Wang, 2019). The jarquebera test, on the other hand, reveals that the residuals normality is rejected with a kurtosis >3 and a skewness>0, since the residuals are fat tailed a stylized fact of varying volatility (Tsay, 2005, p115) the ARCH LM test must be conducted. (See figure 3)

Table 3: Residuals diagnostic

	Statistic value	DF	Pvalue
Box-Ljung	27.015	19	0.1043
Jarquebera	48.289	2	3.267e-11
Skweness	1.31852		
Kurtosis	7.207642		

Source: Prepared by the researcher based on the outputs of R

Fig.3. Residuals distribution



Source: Outputs of R

We perform an ARCH LM test using the Lagrangien multiplier method. The results show that there is a clear ARCH effect (significant p-value). The ARCH effect exists when the lag is greater than 3, implying that the residual sequence is heteroscedastic. In addition, an EGARCH model is developed in this study.

Table4. ARCH LM test results

Lags	LM	P-value
4	51.12055	4.61e-11
8	15.73339	2.77e-02
12	6.14563	8.63e-01
16	2.42280	1.00e+00
20	0.60048	1.00e+00

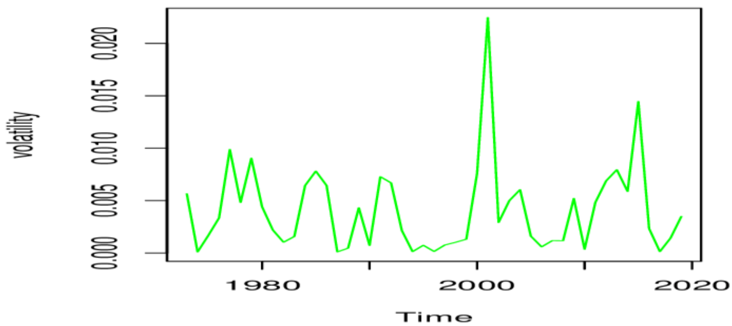
Source: Prepared by the researcher based on the outputs of R

6. Results and Discussion

Based on the log likelihood results, an optimal model was chosen in the study of symmetric and asymmetric GARCH models. As a result, the paper proposes using ARIMA-EGARCH to analyze the covered series. Where the Exponential-GARCH model (EGARCH (p, q)) is a modified form of GARCH, this allows the model to account for the asymmetric influence of negative and positive shocks on conditional variance (Nelson, 1991). The asymmetry can be explained by the fact that negative shock affects volatility more than positive shock. (Berlinger et al, 2015) The equation for an EGARCH (1, 1) model is given as:

$$\log(h_t^2) = \omega + \beta \log(h_{t-1}^2) + \alpha \left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}}$$

Fig.4. Conditional variance



Source: Outputs of R

Table5. Estimation of ARMA-EGARCH model parameters

Variable	Coefficient	Standard Error	t statistic	Pvalue
Mu	-0.002668	0.000001	-4247.9	0.0000
MA(1)	-0.797262	0.000412	-1933.56	0.0000
Omega	-3.292080	0.013595	-242.15	0.0000
Alpha	0.741553	0.000485	1530.26	0.0000
Beta	0.425859	0.001484	286.89	0.0000
Gamma	-1.984336	0.002827	-701.92	0.0000
Log likelihood	72.85			

Source: Prepared by the researcher based on the outputs of R

The alpha parameter value is interpreted as a measure of past innovation effect on volatility, whereas beta is interpreted as an impact of past volatility value on today's volatility. Gamma (the leverage effect) is less than zero; it means that negative shocks have a significant impact on volatility than positive ones.

The residuals of ARMA (0, 1) +EGARCH (1, 1) model are evaluated for serial correlation and ARCH effect and return nonsignificant p-values. (See table 6)

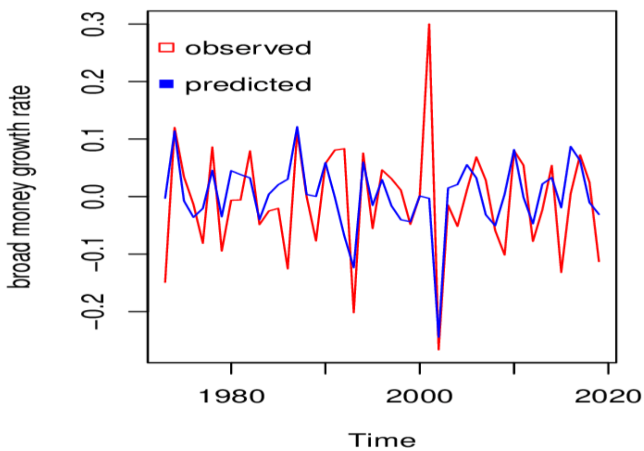
Table6. ARMA (0, 1) + EGARCH (1, 1) residuals diagnostic

	Statistic value	P-value

Box-Ljung(lag 1)	0.0041	0.9489
Box-Ljung(lag 2)	0.6093	0.9326
Box-Ljung(lag 5)	1.4270	0.8641
ARCH Lag[3]	0.2845	0.5937
ARCH Lag[5]	0.3616	0.9235
ARCH Lag[7]	1.0650	0.9027

Source: Prepared by the researcher based on the outputs of R

Fig.5. Predicted versus observed values



Source: Outputs of R

In-sample Forecasting: Figure 5 shows that the in-sample predicted values fit the growth rate curve very well. This is confirmed by the accuracy criteria with low ME, indicating low bias in forecasting. The RMSE and MAE values are very close, indicating that there are no large gaps in the forecasting error.

Table7. In-sample forecasting accuracy

	ME	RMSE	MAE
accuracy	0.00851	0.07516	0.05297

Source: Prepared by the researcher based on the outputs of R

Out-sample Forecasting: The gap between the projected and actual value (out of sample forecast) is minimal (less than 10%), indicating that the ARIMA with EGARCH model has a better prediction effect on Algeria's broad money growth rate in the short term.

Table8. Forecast percentage error

	ARIMA (0,1,1) forecast	ARIMA (0,1,1)+ EGARCH (1,1) forecast	Actual value	Percentage error of ARIMA (0,1,1) forecast	Percentage error of ARIMA (0,1,1)+ EGARCH (1,1) forecast
2020	0.06151412	0.06277	0.0685786	10.30%	8.46%

Source: Prepared by the researcher based on the outputs of R

7. Conclucion

The aim of this contribution was to model the broad money growth rate from 1971 to 2019. We used the best ARIMA(0,1,1) model among several candidates, where the choice was established on the information criterion, The residuals of the regression revealed the presence of an ARCH effect, leading us to model conditional variance using an asymmetric GARCH (EGARCH(1,1)) model. This choice was based on the log likelihood. The lack of serial correlation and ARCH effect on the residuals of the hybrid model lead us to make a short-term forecasting. The percentage error between the forecasting and the real value was less than 10%, indicating that the ARIMA with EGARCH model has a better prediction effect on Algeria's broad money growth rate.

8. Bibliography:

1. Andersson, O., &Haglund, E. (2015). Financial Econometrics: A Comparison of GARCH type Model Performances when Forecasting VaR.
2. Berlinger, E., Illés, F., Badics, M., Banai, Á., Daróczi, G., Dömötör, B., ... &Vidovics-Dancs, Á. (2015). Mastering R for quantitative finance. Packt Publishing Ltd.
3. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of econometrics, 31(3), 307-327.
4. Bourbonnais, R., &Terraza, M. (2016). Analyse des séries temporelles-4e éd.: Cours et exercices corrigés-Applications à l'économie et à la gestion. Dunod.
5. Desbois, D. (2005). Une introduction à la méthodologie de Box et Jenkins: l'utilisation de modèles ARIMA avec SPSS. La revue MODULAD.
6. Engle, R. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. Journal of economic perspectives, 15(4), 157-168.
7. Hyndman, R. J., & Koehler, A. B. (2006). Another look at measures of forecast accuracy. International journal of forecasting, 22(4), 679-688.
8. Liu, X., & Wang, L. (2019). Predictive Analysis of China's Broad Money Supply (M2) Based on ARIMA-GARCH Model.
9. Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica: Journal of the Econometric Society, 347-370.
10. Rublikova, E., &Lubyova, M. (2013). Estimating ARIMA-ARCH model rate of unemployment in Slovakia. Forecast. Pap. Progn. Pr, 5, 275-289.
11. Tsay, R. S. (2005). Analysis of financial time series (Vol. 543). John wiley& sons.

9. Appendices. R code for automatic modeling and forecasting

```
data<-ts(wdi, start=1971,end=2020,freq=1)# convert a numeric vector into an R time series object.
library (forecast) # Loading forecast package
logdata<-log(data)
bm<-diff(logdata,diff=1) #calculating the growth rate
Dbm<-diff(bm,diff=1) # differentiated series
data.train<-window (Dbm, start=1973, end=2019, freq=1) # split the data
fit<-auto.arima(data.train, trace=TRUE,test="kpss", ic="bic") # automatic estimation of a general ARIMA model
checkresiduals(fit)
Box.test(resid(fit),type="Ljung",lag=20,fitdf=1) # autocorrelation test for residuals
jarque.bera.test(fit$residuals) # residual normality test
library(aTSA) # loading aTSA package
arch.test(arima(data.train, order=c(0,0,1)) ,output=TRUE) # examine for the presence of an ARCH effect
library(rugarch) # loading rugarch package
egarchsnp.spec = ugarchspec(variance.model=list(model="eGARCH",garchOrder=c(1,1)), mean.model=list(armaOrder=c(0,1)))
mod= ugarchfit(egarchsnp.spec, data.train) #modeling
ugarchforecast(mod, n.ahead=1) #forecasting
```