

# Regression of NRTL Parameters from liquid–liquid equilibria for water + ethanol + solvent (dichloromethane, diethyl ether and chloroform) using particle swarm optimization and discussions at $T = 293.15\text{ K}$

Sabrina Hebboul<sup>1</sup>; Mourad Korichi<sup>2</sup>; Amel Hebboul<sup>3</sup>

1: Department of Process Engineering. Dynamics, Interaction and Reactivity of Systems laboratory, University of Kasdi

Merbah, Ouargla, Algeria, [sabrinahebboul30@gmail.com](mailto:sabrinahebboul30@gmail.com)

2: Department of Process Engineering. Dynamics, Interaction and Reactivity of Systems laboratory, University of Kasdi

Merbah, Ouargla, Algeria, [Mourad.Korichi@gmail.com](mailto:Mourad.Korichi@gmail.com)

3: Department of Mathematics and Computer science. High Normal School Assia Djebbar, Ali Mendjli. Automatic and

Robotic Laboratory, University of Frères Mentouri, Constantine, Algeria, [ahebboul@yahoo.fr](mailto:ahebboul@yahoo.fr)

Received: 29/11/2021 Accepted: 23/02/2022 Published online: 15/03/2022

## Abstract

*Experimental data taken from previous scientific studies of the liquid-liquid equilibrium (LLE) for ternary and quaternary systems {water + ethanol + dichloromethane (DCM), chloroform (CHCl<sub>3</sub>) or diethyl ether (DEE)} were correlated using the nonrandom two-liquid (NRTL) model. In Chemical Engineering, metaheuristic methods like the Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO) are used to obtain binary interaction parameters for this model. In addition, the quality of the NRTL model predictions was assessed by calculating the variance of Root-Mean-Square Error (RMSE) between the experimental equilibrium and predicted data. As a result, the PSO RMSE values were better than the GA ones. Therefore, the PSO was the best optimization method for these systems.*

**Keywords:** Genetic Algorithm, Liquid-Liquid Extraction, Nonrandom Two-liquid model and Particle Swarm Optimization.

## I. Introduction

Liquid–liquid extraction (LLE) is a separation technique and is a fundamental mass transfer operation between two liquid phases which may be totally or partially miscible or immiscible. Typically water is the diluents. It is a polar compound and is generally a good solvent to extract polar compounds. The extraction process consists in extracting one or more constituents (solutes)

from a solution by dissolution in an organic solvent where they are more soluble than in the initial liquid medium. Then the two obtained liquid phases are separated by decantation, achieving a solvent extraction process [1].

In fact a liquid-liquid extraction process requires three successive operations. First the two liquid phases are put into an intimate contact in special device (ampoules, columns, mixers) to encourage mass exchange [1]. The higher the extraction rates

are, the greater the differences in solute concentration of the two phases in contact, and the larger the exchange surface between the two phases. Then a sufficient time is allowed to obtain equilibrium of the system, which is governed by the laws of diffusion and solubility such as partition coefficient of the solute from one phase to another. For the liquid-liquid system, the distribution coefficient (D) and separation factor (S) are fundamental and essential parameters that are always determined [1]. The distribution coefficient also known as the distribution ratio or partition coefficient is a quantitative factor which is used to assess the dispersion of the components dispersed in the equilibrium phase. Regarding the separation factor, it is used to assess the solvent extraction capacity [2, 3]. Finally, after their contact, under the effect of natural gravity (gravimetric) or decantation, a mechanical separation is performed between two liquid phases [4]. The densities of the two outgoing liquid phases are different. The solute rich phase is the extract whereas the solute poor phase is the raffinate (R) [4, 5]. The final concentration of the solute in each phase depends on its solubility in the involved liquids [1].

In liquid-liquid extraction (LLE), an activity coefficient thermodynamic model is required. Most of these models were developed from vapor-Liquid data and their application extended to liquid-liquid systems for phase equilibria predictions [6]. The usual procedure is based on the minimization of Gibbs free energy. The determination of the phase equilibria consists in solving the isoactivity equations along with the mass balance constraints [7]. In addition the required interaction parameters for the used model are obtained by minimizing a well defined objective function involving the system compositions [8]. Therefore one can see how important phase equilibrium calculations are in process modeling and simulation.

In fact thermodynamic modeling of liquid-liquid equilibria (LLE) is an important and difficult step in solvent extraction. One of the most significant issues is the estimation of the adjustable parameters required by the activity coefficient model. There are several activity coefficient models, such as NRTL (Non-Random Two-Liquids), UNIQUAC (UNIversal QUasi Chemical) and UNIFAC (UNIversal Functional Activity Coefficient) [9].

Metaheuristic approaches have been used to calculate interaction parameters for vapor-Liquid [10] and liquid-liquid equilibrium [11] using these models. One of the the most used technique is

based on the Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) [12]

Consequently in this paper, the NRTL model, the GA and the PSO methods were a priori defined. Then, the proposed approach to solve the isoactivity equations was detailed. In addition, the GA and the PSO have been used to regress the parameters of the NRTL model in liquid-liquid equilibrium data for the ternary and quaternary presented in [13] using the minimization of solving the isoactivity equations following mass balance constraints.

## II. Modeling of liquid-liquid equilibrium

### A. NRTL model

Non-Random, Two-Liquids (NRTL) is an activity coefficient model, was proposed by Renon and Prausnitz in 1968 [14], based on Wilson's local composition theory [15] and Scott's two-liquid solution theory [16]. The activity coefficient ( $\gamma$ ) expression for a binary system is shown as [17]:

$$\ln \gamma_i = \frac{\sum_{j=1}^N \tau_{ji} \cdot G_{ji} \cdot x_j}{\sum_{k=1}^N G_{ki} \cdot x_k} + \sum_{j=1}^N \frac{x_j \cdot G_{ij}}{\sum_{k=1}^N G_{kj} \cdot x_k} \left( \tau_{ij} - \frac{\sum_{l=1}^N x_l \cdot \tau_{lj} \cdot G_{lj}}{\sum_{k=1}^N G_{kj} \cdot x_k} \right) \quad (1)$$

With  $G_{ij}$  and  $\tau_{ij}$  defined as follows [14, 17]:

$$\tau_{ij} = \frac{g_{ij} - g_{ji}}{R \cdot T} = \frac{b_{ij}}{R \cdot T} \quad (2)$$

$$G_{ij} = \exp(\alpha_{ij} \cdot \tau_{ij}) \quad (3)$$

Where:

$g_{ij}$  is the energy interaction between i and j molecules,

$\alpha_{ij}$  is the non-randomness factor in the mixture, with  $\alpha_{ij} = \alpha_{ji}$ ,

$R$  is the universal gas constant in  $[J \cdot mol^{-1} \cdot K^{-1}]$ ,

$T$  is the mixture temperature [K].

Three adjustable parameters  $\alpha_{ij}$ ,  $a_{ij}$  and  $\alpha_{ji}$  of the NRTL model are calculated by regression of experimental equilibrium data for a specific system [18]. The parameters account simultaneously for mixed liquid interactions ( $g_{ij}$  and  $g_{ji}$ ) and pure-

component liquid interactions ( $g_{ii}$  and  $g_{jj}$ ). The non-randomness factor in the mixture ( $\alpha_{ij}$ ) varies from 0.2 to 0.47 [19] but for LLE systems is commonly set to a constant value of 0.2 [20].

## B. Optimization methods:

The GA method was used to determine the interaction parameters of the NRTL model in [13, 21]. However in this study, the PSO method was used. In addition a comparative study was carried out when using the GA and the PSO methods for the interaction parameters determination.

### B.1. Genetic algorithm (GA):

The genetic algorithm is an evolutionary optimizer (EO) [22], developed by John Holland and his collaborators in the 1960s and 1970s [14, 23]. It is a randomized search algorithm that was developed in an effort to simulate the mechanisms of natural selection and natural genetics or abstraction of biological evolution inspired by Charles Darwin's theory of natural selection. Genetic algorithms (GA) operate on string structures, like biological structures, which are evolving in time according to the rule of survival of the fittest by using a randomized and structured information exchange by genetic operators (reproduction (mutation, crossover) and new gene selection) [22]. This notion can be applied for a search problem and select the set of best solutions (see Figure 1).

### B.2. Particle Swarm Optimization (PSO):

Shi, Kennedy and Eberhart created the PSO algorithm [25, 26] for simulating social behavior [27]. It is a meta-heuristic intelligent optimization method for nonlinear functions by modeling the behavior of the biological swarms, such as fish swarm and bird flock looking for food and can search very large spaces of candidate solutions. PSO has been widely employed in many optimization issues due to its flexibility and ease of use. It has attracted significant research from a wide range of applications and has undergone several

algorithm revisions and updates [28, 29].

The algorithm of PSO was simplified showed to be of an optimizing nature, as well reported in Kennedy and Eberhart book when they discussed philosophical elements and aspects of PSO and swarm intelligence [30]. Poli made an extensive survey of PSO applications [31, 32]. Bonyadi and Michalewicz recently released a comprehensive review on theoretical and experimental works on PSO [33].

By iteratively enhancing new solutions (particles) in the search space, the PSO enhances their positions, and the particle position vectors are considered candidate optimal solutions to the optimization issue. PSO can quickly explore the search space and converge to a global or local optimal solution by surfing in the search space of the optimization problem (see Figure 2). The following equations update the PSO algorithm's velocity and position vectors [30]:

$$v_i = \omega \cdot v_i + c_1 \cdot rand_1 \cdot (p_{best} - x_i) + c_2 \cdot rand_2 \cdot (g_{best} - x_i) \quad (4)$$

$$x_i = x_i + k_{PSO} \cdot v_i \quad (5)$$

Where:

$v_i$  is the moving velocity of the particle,

$x_i$  is the current location,  $\omega$  is inertia weight,

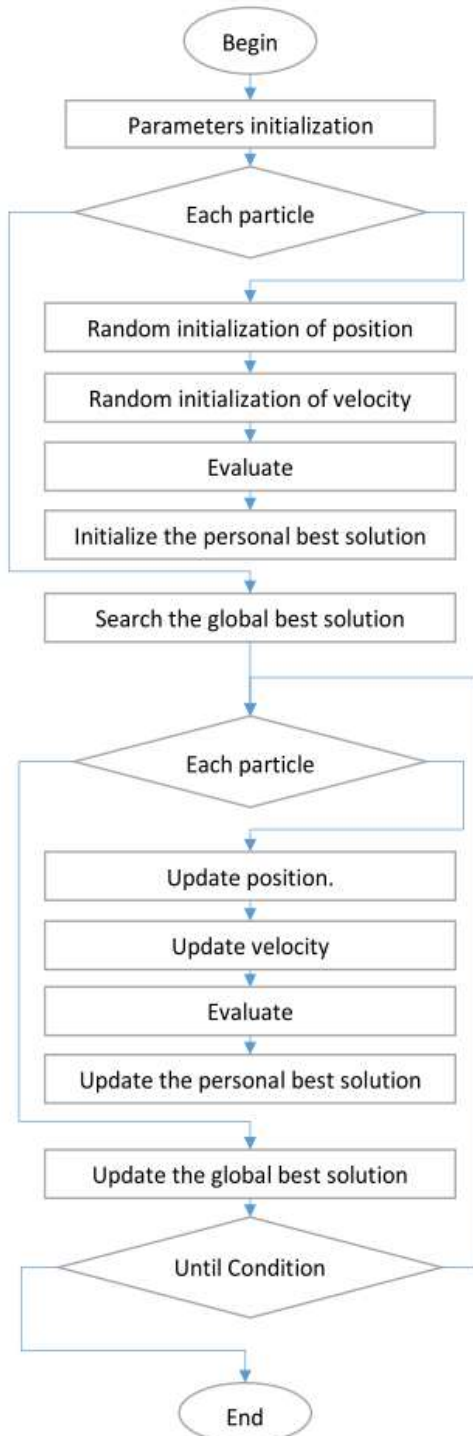
$c_1$  and  $c_2$  are positive acceleration factors,

$rand_1$  and  $rand_2$  are the two random numbers distributed in the range of [0, 1],

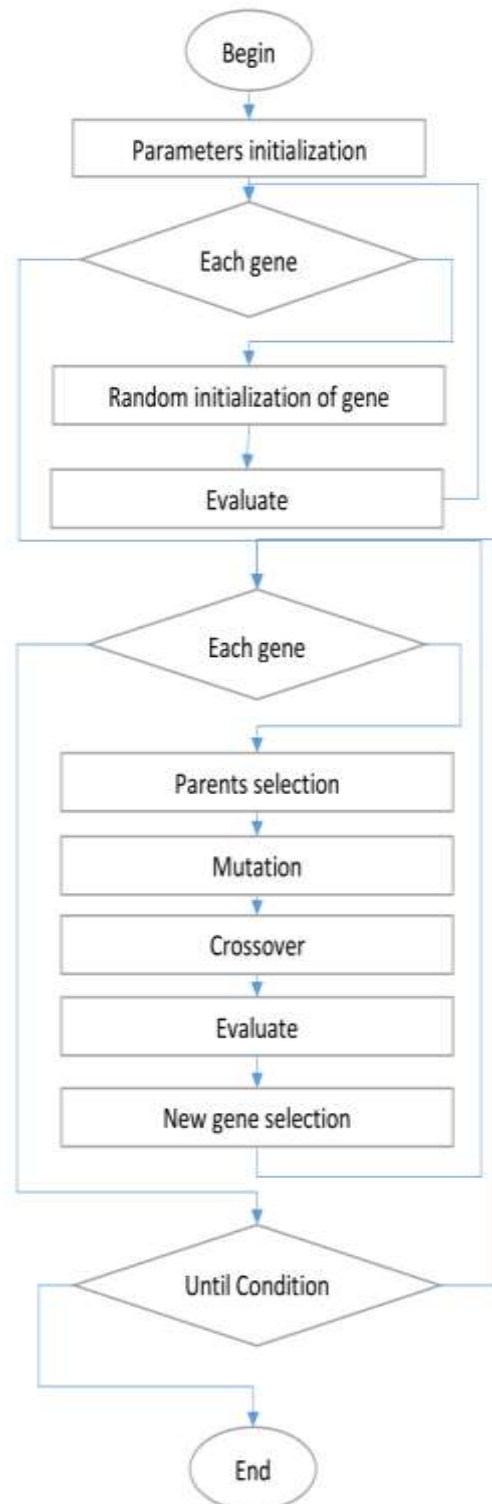
$k_{PSO}$  is the constriction factor,

$p_{best}$  is the termed as personal best at the  $t^{th}$  iterations, which refers to the best position found so far by the  $i^{th}$  particle,

$g_{best}$  is the termed as local best, which refers to the position found so far by the members in the  $i^{th}$  particle's neighborhood that has the best fitness value.



**Figure 1:** Flowchart of the genetic algorithm



**Figure 2.** Flowchart of the particle swarm optimization algorithm

### III. Equation solving isoactivity approach

In Equilibrium conditions between two phases or more, chemical potential allows to study of the phenomenon of diffusion of matter and to define the conditions of equilibrium phases. In the equilibrium condition in the case two phases, the chemical potential is equal that can be written as [8]:

$$\mu_i^I = \mu_i^{II} \quad (6)$$

Where:

$\mu_i^I$ ,  $\mu_i^{II}$  are the chemical potentials of component  $i$ , in phase  $I$  and  $II$  respectively.

Then, the Equation solving isoactivity approach utilizes uniformity of chemical potential of a component in all the phases. Fugacity defined from the chemical potential, which is the isothermal variation at constant composition of the chemical potential of a substance  $i$  pure or in mixture as a function of the pressure. The notion of fugacity allows in particular the calculation of phase equilibria that is given by the relation [34, 35]:

$$d\mu_i = R.T. d \ln f_i \quad (7)$$

Where:

$\mu_i$  is the chemical potential of a component  $i$ ,

$R$  is the universal gas constant ( $\text{cal K}^{-1} \text{mol}^{-1}$ ),

$T$  is the mixture liquid temperature (K),

$f_i$  is the fugacity of  $i$  component  $i$ .

The fugacity is equal in two phases can be written as [8]:

$$f_i^I = f_i^{II} \quad (8)$$

Where:

$f_i^I$  and  $f_i^{II}$  are the fugacities of component  $i$ , in phase  $I$  and  $II$  respectively.

The fugacity expression of a component  $i$  in phase can be written as [36]:

$$f_i^I = f_i^{Ri} \cdot \gamma_i^I \cdot x_i^I \quad (8.a)$$

$$f_i^{II} = f_i^{Ri} \cdot \gamma_i^{II} \cdot x_i^{II} \quad (8.b)$$

Where:

$f_i^{Ri}$  is the fugacity of a component  $i$  in a real solution,

$\gamma_i^I$  and  $\gamma_i^{II}$  are the activity coefficient of a component  $i$ , in phase  $I$  and  $II$  respectively,

$x_i^I$  and  $x_i^{II}$  are the mole fraction of a component  $i$  in phase  $I$  and  $II$  respectively.

The activity coefficient is defined by the ratio between the real fugacity and the fugacity in an ideal solution that is given by the relation:

$$\gamma_i = f_i / f_i^{\text{id}} \quad (9)$$

Where:

$\gamma_i$  is the activity coefficient of a component  $i$ ,

$f_i^{\text{id}}$  is the fugacity in an ideal solution.

Chemical activity is defined by the ratio of the real fugacity and in the standard state at the same temperature as the real state that is given by the relation [37]:

$$a_i = f_i / f_i^\circ \quad (10)$$

Where:

$a_i$  is the chemical activity of component  $i$

$f_i^\circ$  is the fugacity in the standard state of a component  $i$ .

In addition, chemical activity is could to define [8]:

$$a_i = x_i \cdot \gamma_i \quad (11)$$

Therefore the equality of chemical potential can be replaced by equality of fugacities, which could be further more simplified to be the isoactivity equation as given in Eq. (12) [8]:

$$a_i^I = a_i^{II} \quad (12)$$

Where:

$a_i^I$ ,  $a_i^{II}$  are the chemical activities of component  $i$  in phase  $I$  and  $II$  respectively.

The equation (12) allows us to conclude that the equilibrium state of a system is reached only when the chemical potential of each of the constituents is identical in both phases. In this study, the equation (12) is the basis of all phase equilibrium calculations.

The isoactivity equations for LLE have been given in the equations (6)-(12), which take the following form in Equation (13) [38]:

$$x_i^I \gamma_i^I = x_i^{II} \gamma_i^{II} \quad (13)$$

The material balance must constrain these isoactivity equations, which can be calculated by entering the total amount of each component in the following equation [6]:

$$n_i^I + n_i^{II} = n_i \quad (14)$$

Where:

$n_i^I$  and  $n_i^{II}$  are the mole number of component  $i$  in phase  $I$  and  $II$  respectively,

$n_i$  is the total amount of component  $i$  in the system.

Normalization of mole fraction requires two equations for equilibrium systems. Therefore the mass balance for a two-phase LLE system constraints can be written as [38]:

$$\sum_{i=1}^N x_i^I = 1 \quad (15.a)$$

$$\sum_{i=1}^N x_i^{II} = 1 \quad (15.b)$$

The isoactivity equations under mass balance constraints is the basic equations for both correlation and prediction of LLE system. For calculation of LLE systems present in [13, 21], the last mentioned operations are to be solved with a specified concentration of one component in one phase. With an initial estimation, a solution can be found by numerically solving the equations.

#### IV. Estimation of NRTL model interaction parameters

The parameter identification in local composition models is usually performed using LLE experimental data via the minimization of a suitable objective function. The objective function is defined in this study as following equation [39]:

$$\min F = \sum_{k=1}^m \sum_{j=1}^2 \sum_{i=1}^c w_{ik}^j (x_{ik}^{cal}(j) - x_{ik}^{exp}(j))^2 \quad (16)$$

Where:

$m$  is the number of experimental tie-lines used in the correlation procedure,

$x_{ik}^{cal}(j)$  and  $x_{ik}^{exp}(j)$  are calculate and experimental the mole fractions respectively of component  $i$  at liquid phase  $j$  at tie line  $k$ ,

$w_{ik}$  is the weight associated with component  $i$  in phase  $j$  at tie line  $k$ , and in addition all the weights were set to one.

When employing the AG and the PSO approaches, the quality of the model NRTL predictions is evaluated by calculating the deviation of the RMSE between the experimental and predicted data, which is described as [6, 21]:

$$RMSE = \left[ \frac{-F}{2 \cdot m \cdot n} \right]^{0.5} \quad (17)$$

Where:

$m$  is the total number of tie lines,

$n$  is the total number of components.

#### V. Correlation model and binary interaction parameters

The energy parameters ( $\tau_{ij}$ ) are searched in the range of  $[-15, 15]$  and all the non-randomness parameters ( $\alpha_{ij}$ ) equal 0.2. The metaheuristic methods was further tested by regressing the ternary and quaternary systems have presented in [13], the objective function in terms of mole fractions was minimized by 1000 iterations and using 200 particles, that are generated randomly, each particle is a group of six energy parameters of the NRTL model.

The determination of the binary interaction parameters and RMSE of the NRTL model using the PSO algorithm of MATLAB optimization toolbox in this present study, also these results were compared with literature results when using the GA algorithm, they were presented in [13], all the results are shown in tables 1, 2 and 3.

**A. Separation of the mixture {Water + Ethanol} using mixed solvents (DEE + DCM):** see Table 1

**B. Separation of the mixture {Water + Ethanol} using mixed solvents (DEE + CHCl<sub>3</sub>):** see Table 2

**C. Separation of the mixture {Water + Ethanol} using organic solvents (DEE, DCM or CHCl<sub>3</sub>):** see Table 3

**Table 1.** The NRTL binary interaction parameters and RMSE values for {water + ethanol + solvent (DEE, DCM or CHCl<sub>3</sub>)} ternary system.

	Water + Ethanol + DEE			Water + Ethanol + DCM			Water + Ethanol + CHCl <sub>3</sub>		
i-j	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE
Literature results [1]									
1-2	803.4	-494.7	1.9 %	3480.4	-522.9	1.58 %	-9.7331	2.2233	2.01 %
1-3	1553.7	618.9		2034.9	1750.9		6.018	4.7287	
2-3	-4.29.7	368.2		-200.5	3371		2.3244	-9.1458	
Results of the PSO method									
1-2	0.0352	3.5913	0.0112%	8.5366	1.9053	0.0161 %	10.3143	1.8460	0.0137%
1-3	6.1661	4.7208		4.2771	4.4592		-8.4507	5.9755	
2-3	3.1735	15		1.9498	7.6684		-13.5031	2.8034	

**Table 2.** The NRTL binary interaction parameters and RMSE values of quaternary systems { Water + Ethanol + Mixed solvent (CHCl<sub>3</sub> + DEE)}.

Water + Ethanol + mixed solvents (..% DCM + ..%DEE)									
	+25% DCM +75%DEE			+50% DCM +50%DEE			+75% DCM +25%DEE		
i-j	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE
Literature results [1]									
1-2	-195.7	-624	1.09 %	2786.2	-536.7	1.01%	3242.9	-456.2	1.04%
1-3	1606.7	1529.7		1580.2	2197.1		1909.7	2108.7	
2-3	-535.4	-457		-156.5	2571.9		-152.4	3305.2	
Results of the PSO method									
1-2	15	15	0.0181%	4.70.68	10.1229	0.0324 %	15	3.7369	0.0214 %
1-3	-10.7364	0.7642		3.0007	4.6614		6.8511	4.2202	
2-3	-15	3.0295		10.6621	4.5641		2.133	8.0408	

**Table 3.** The NRTL binary interaction parameters and RMSE values of quaternary systems { Water + Ethanol + Mixed solvent (CHCl<sub>3</sub> + DEE)}.

	Water + Ethanol + mixed solvents (..%CHCl <sub>3</sub> +..%DEE)								
	+ (25%CHCl <sub>3</sub> +75%DEE)			+ (50%CHCl <sub>3</sub> + 50%DEE)			+ (75%CHCl <sub>3</sub> +25%DEE)		
i-j	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE	b <sub>ij</sub>	b <sub>ji</sub>	RMSE
	Literature results [1]								
1-2	3501.5	106	1.69 %	3729.8	-339.9	1.55 %	2661.8	-439.4	1.23 %
1-3	1807.2	196.7		2804	1274.1		1499.7	929.8	
2-3	94.5	1224.4		-297.7	2432.2		-56.7	2562.1	
Results of the PSO method									
1-2	15	15	0.0361 %	15	4.5306	0.0575 %	10.8656	3.7098	0.0085 %
1-3	-9.7999	14.165		7.9338	3.9851		9.4421	5.5779	
2-3	-15	2.9384		2.5301	8.3620		6.2931	1.6828	

## VI. Results and discussion

In **ternary system**, the smallest RMSE value was 1.58 for individual solvent DCM and 0.0112 for individual solvent DEE when using the GA method and the PSO method respectively.

In **quaternary system**, the smallest RMSE value was 1.01 for mixed solvent (50% DCM +50%DEE), and 1.23 for mixed solvent (75%CHCl<sub>3</sub> +25%DEE) when using the GA method, but when using the PSO method, the value of RMSE was 0.0181 for mixed solvent (25% DCM +75%DEE) and 0.0085 for mixed solvent (75%CHCl<sub>3</sub> + 75%DEE).

Finally, in the liquid-liquid equilibrium (LLE) for ternary and quaternary systems {water +ethanol + dichloromethane (DCM), chloroform (CHCl<sub>3</sub>) or diethyl ether (DEE)}, the RMSE values of the PSO method are smaller than the RMSE values of the GA method. Therefore, the best method is PSO. In addition, the best individual solvent is DEE in ternary system, and the best-mixed solvent is (25% DCM +75%DEE) and (75%CHCl<sub>3</sub> +25%DEE) in quaternary system, because they have had the lowest RMSE values.

## VII. Conclusion

In this study, the PSO and the GA are optimization methods and the minimization by the objective function have been used to determine the interaction parameters of NRTL model in the liquid-liquid equilibrium data of the ternary and quaternary chemical systems have presented in [13].

The RMSE of PSO is smaller than the RMSE corresponding to the GA results. Therefore, PSO method is more successful in the determination of the NRTL parameters for liquid-liquid equilibria data of the considered systems in [13].

**Acknowledgment:** In the beginning, we want to thank our **ALLAH** for giving us the force to write scientific research paper, we hope to achieve advance about this topic in next studies. Therefore, we aim to consideration a more in-depth and extensive study. Then, we also thank the Department of Processes Engineering, Faculty of Applied Sciences, Kasdi Merbah University, Ouargla, Algeria. I also add a special thanks to the University of Constantine Salah BOUBNIDER, Algeria, and I hope this journal will be classified. Finally we hope performing scientific research in Algerian universities.

### List of Symbols

LLE	Liquid-Liquid Equilibrium
PSO	Particle Swarm Optimization
NRTL	Nonrandom Two-Liquid
RMSE	Root-Mean-Square Error
$D$	Distribution coefficient
$S$	Separation factor
$T$	Temperature of mixture [K]
$R$	Universal gas constant [J·mol <sup>-1</sup> ·K <sup>-1</sup> ]
$G$	Energy interaction
$a$	Activity
$\alpha$	Non-randomness factor in the mixture
$\tau$	Energy parameters
$\gamma$	Activity coefficient
$v_i$	Moving velocity of the particle
$x_i$	Current location
$\omega$	Inertia weight
$c_1, c_2$	Positive acceleration factors
$rand_1, rand_2$	Two random numbers distributed in the range of [0, 1]
$k_{PSO}$	Constriction factor
$p_{best}$	Personal best
$g_{best}$	Local best
$\mu_i^I, \mu_i^{II}$	Chemical potentials of component $i$ , in phase $I$ and $II$ respectively.
$\mu$	Chemical potential
$f_i$	Fugacity
$f_i^I, f_i^{II}$	Fugacities of component $i$ , in phase $I$ and $II$ respectively.



$f_i^{Ri}$	Fugacity of a component $i$ in a real solution,
$\gamma_i^I, \gamma_i^{II}$	the activity coefficient of a component $i$ , in phase $I$ and $II$ respectively,
$x_i^I, x_i^{II}$	Mole fraction of a component $i$ , in phase $I$ and $II$ respectively.
$f_i^\circ$	Fugacity in the standard state of a component $i$ .
$a_i^I, a_i^{II}$	Activities of component $i$ , in phase $I$ and $II$ respectively .
$n_i^I$ and $n_i^{II}$	Mole number of component $i$ , in phase $I$ and $II$ respectively,
$n_i$	Total amount of component $i$ , in the system.
$w_{ik}$	Weight associated with component $i$ in phase $j$ at tie line $k$ ,
$m$	Number of experimental tie-lines used in the correlation procedure,
$n$	Number of components
$x_{ik}^{cal}(j)$	Calculate mole fractions of components
$x_{ik}^{exp}(j)$	Experimental mole fractions of components
$i$	Component
$j$	Liquid Phase
$k$	Number of tie lines

### Reference:

- [1] S. Hebboul and M. Korichi, "Extraction liquide-liquide: types, facteur influençant," *Algerian Journal of Engineering Architecture and Urbanism*, vol. 5, no. 5, pp. 390–403, 2021.
- [2] T. M. Letcher, G. G. Redhi, S. E. Radloff, and U. Domańska, "Liquid– liquid equilibria of the ternary mixtures with sulfolane at 303.15 K," *Journal of Chemical & Engineering Data*, vol. 41, no. 3, pp. 634–638, 1996.
- [3] T. M. Letcher and G. G. Redhi, "Phase equilibria for liquid mixtures of (butanenitrile+ a carboxylic acid+ water) at 298.15 K," *Fluid phase equilibria*, vol. 193, no. 1–2, pp. 123–133, 2002.
- [4] E. Koller, *Aide-mémoire de génie chimique-4e éd.* Dunod, 2013.
- [5] D. Morvan, *Génie Chimique Les opérations unitaires: Procédés industriels, Cours et exercices corrigés.* Ellipses, 2009. [Online]. Available: <https://books.google.dz/books?id=wNVrPgAACAAJ>
- [6] Z. Li, K. H. Smith, K. A. Mumford, Y. Wang, and G. W. Stevens, "Regression of NRTL parameters from ternary liquid–liquid equilibria using particle swarm optimization and discussions," *Fluid Phase Equilibria*, vol. 398, pp. 36–45, 2015.
- [7] Z. Li, K. A. Mumford, K. H. Smith, J. Chen, Y. Wang, and G. W. Stevens, "Reply to 'Comments on "Analysis of the Nonrandom Two-Liquid Model for Prediction of Liquid–Liquid Equilibria,"'" *Journal of Chemical & Engineering Data*, vol. 60, no. 5, May 2015, doi: 10.1021/acs.jced.5b00130.
- [8] J. M. Sørensen, T. Magnussen, P. Rasmussen, and A. Fredenslund, "Liquid—liquid equilibrium data: Their retrieval, correlation and prediction Part II: Correlation," *Fluid Phase Equilibria*, vol. 3, no. 1, pp. 47–82, 1979.
- [9] Y. Moudjari, W. Louaer, and A.-H. Meniai, "Modeling of solid-liquid equilibrium using a modified group contribution based NRTL model," *Fluid Phase Equilibria*, vol. 492, pp. 118–136, 2019.
- [10] E. Cortés-Toro, B. Crawford, J. Gómez-Pulido, R. Soto, and J. Lanza-Gutiérrez, "A New Metaheuristic Inspired by the Vapour-Liquid Equilibrium for Continuous Optimization," *Applied Sciences*, vol. 8, no. 11, Oct. 2018, doi: 10.3390/app8112080.
- [11] H. Zhang, A. Bonilla-Petriciolet, and G. P. Rangaiah, "A review on global optimization methods for phase equilibrium modeling and calculations," *The Open Thermodynamics Journal*, vol. 5, no. 1, 2011.

- [12] M. Timedjeghdine, M. Khechai, A. Hasseine, A. Merzougui, and I. Bechka, "Measurements And Correlation Of Liquid-liquid Equilibrium Of Water + Carboxylic Acid Or Alcohol + 1,8-cineole Ternary Systems," *Algerian Journal of Engineering Research*, vol. 1, no. 2, pp. 37–41, Dec. 2017.
- [13] O. Bacha, "Etude Experimentale et Modelisation des Equilibres Liquide-Liquide des Systemes Ternaires et Quaternaires," Biskra, 2016.
- [14] H. Renon and J. M. Prausnitz, "Local compositions in thermodynamic excess functions for liquid mixtures," *AIChE journal*, vol. 14, no. 1, pp. 135–144, 1968.
- [15] G. M. Wilson, "Vapor-liquid equilibrium. XI. A new expression for the excess free energy of mixing," *Journal of the American Chemical Society*, vol. 86, no. 2, pp. 127–130, 1964.
- [16] R. L. Scott, "Corresponding states treatment of nonelectrolyte solutions," *The Journal of Chemical Physics*, vol. 25, no. 2, pp. 193–205, 1956.
- [17] A. Merzougui, N. Labed, A. Hasseine, A. Bonilla-Petriciolet, D. Laiadi, and O. Bacha, "Parameter Identification in Liquid-Liquid Equilibrium Modeling of Food-Related Thermodynamic Systems Using Flower Pollination Algorithms," *The Open Chemical Engineering Journal*, vol. 10, no. 1, Apr. 2016, doi: 10.2174/1874123101610010059.
- [18] S. Gebreyohannes, B. J. Neely, and K. A. M. Gasem, "Generalized nonrandom two-liquid (NRTL) interaction model parameters for predicting liquid-liquid equilibrium behavior," *Industrial & Engineering Chemistry Research*, vol. 53, no. 31, pp. 12445–12454, 2014.
- [19] J. M. Prausnitz, R. N. Lichtenthaler, and E. G. de Azevedo, *Molecular thermodynamics of fluid-phase equilibria*. Pearson Education, 1998.
- [20] W. Arlt, M. E. A. Macedo, P. Rasmussen, and J. M. Sorensen, *Liquid-liquid equilibrium data collection*. Dechema, 1979.
- [21] O. Bacha, A. Hasseine, and M. Attarakih, "Measurement and correlation of liquid-liquid equilibria for water + ethanol + mixed solvents (dichloromethane or chloroform + diethyl ether) at  $T = 293.15$  K," *Physics and Chemistry of Liquids*, vol. 54, no. 2, Mar. 2016, doi: 10.1080/00319104.2015.1079193.
- [22] Y. Rahmat-Samii and E. Michielssen, *Electromagnetic Optimization by Genetic Algorithms*. New York: Wiley, 1999.
- [23] J. H. Holland, *Adaptation in natural and artificial systems*. University of Michigan, 1975.
- [24] K. A. de Jong, *An analysis of the behavior of a class of genetic adaptive systems*. University of Michigan, 1975.
- [25] J. Kennedy and R. Eberhart, "Particle swarm optimization," 1995. doi: 10.1109/ICNN.1995.488968.
- [26] Y. Shi and R. Eberhart, "The 1998 IEEE International Conference on Evolutionary Computation Proceedings," 1998.
- [27] J. Kennedy, "The particle swarm: social adaptation of knowledge," in *Proceedings of 1997 IEEE International Conference on Evolutionary Computation (ICEC'97)*, 1997, pp. 303–308.
- [28] R. Poli, J. Kennedy, and T. Blackwell, "Particle swarm optimization," *Swarm Intelligence*, vol. 1, no. 1, Oct. 2007, doi: 10.1007/s11721-007-0002-0.
- [29] Y. Shi, "Particle swarm optimization: developments, applications and resources," in *Proceedings of the 2001 congress on evolutionary computation (IEEE Cat. No. 01TH8546)*, 2001, vol. 1, pp. 81–86.
- [30] J. Kennedy, R. C. Eberhart, and Y. Shi, "Swarm Intelligence Morgan Kaufmann Publishers," San Francisco, 2001.
- [31] R. Poli, "An analysis of publications on particle swarm optimization applications." London, UK: Essex, department of computer science, university of Essex, 2007.
- [32] R. Poli, "Analysis of the publications on the applications of particle swarm optimisation," *Journal of Artificial Evolution and Applications*, vol. 2008, 2008.
- [33] M. R. Bonyadi and Z. Michalewicz, "Particle swarm optimization for single objective continuous space problems: a review," *Evolutionary computation*, vol. 25, no. 1, pp. 1–54, 2017.

- [34] G. N. Lewis, "The Law of Physico-Chemical Change," *Proceedings of the American Academy of Arts and Sciences*, vol. 37, no. 3, 1901, doi: 10.2307/20021635.
- [35] G. N. Lewis, "A New Conception of Thermal Pressure and a Theory of Solutions," *Proceedings of the American Academy of Arts and Sciences*, vol. 36, no. 9, 1900, doi: 10.2307/20020988.
- [36] A. Boultif, A. Kabouche, and S. Ladjel, "Application of Genetic Algorithms (GA) and Threshold Acceptance (TA) to a Ternary Liquid-Liquid Equilibrium System," *International Review on Modelling and Simulations (IREMOS)*, vol. 9, no. 1, Feb. 2016, doi: 10.15866/iremos.v9i1.8029.
- [37] G. N. Lewis and M. Randall, *Thermodynamics and the free energy of chemical substances*. McGraw-Hill, 1923.
- [38] S. Regabe, A. Merzougui, A. Hasseine, D. Laiadi, and H. Bouredji, "Experimental Data and Modeling of Salt Effect on Liquid-Liquid Equilibrium of the Ternary (Water+ 1-Propanol+ Hexane) System at 298K," *Iran. J. Chem. Chem. Eng. Research Article Vol*, vol. 39, no. 6, 2020.
- [39] N. Bouneb and A. H. Meniai, "Temperature effect on the interaction parameters of GC-NRTL model using the genetic algorithm," *Chemical Engineering Transactions*, vol. 35, pp. 1273–1278, 2013.