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Institute of Electrical and Electronic Engineering ------ University of Boumerdes ------

# ALGERIAN JOURNAL OF SIGNALS AND SYSTEMS

## **ISSN: 2543-3792**

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Volume : 3 Issue : 3 (September 2018)

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# Combining MUSIC Spatial Sampling and Bootstrap to Estimate Closed Space DOA for Few Samples

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**Abstract:** DOA estimation in array processing uses MUSIC (Multiple Signal Classification) algorithm, mainly. It's the most investigated technique and is very attractive because of its simplicity. However, it meets drawbacks and fails when only very few samples are available and the sources are very close or highly correlated. In these conditions, the problem is more intricate and the detection of targets becomes arduous. To overcome these problems, a new algorithm is developed in this paper. We combine bootstrap technique to increase sample size, spatial sampling and MUSIC method to improve resolution. Through different simulations, the performance and the effectiveness of the proposed approach, referred as spatial Sampling and Bootstrapped technique "SSBoot", are demonstrated.

Keywords: DOA, Array processing, spatial sampling, MUSIC method, Bootstrap technique.

#### 1. INTRODUCTION

Detection the number of sources correctly is often the first step in array processing algorithms. When two sources are in the ambiguity range (very closed in space), the radar detects them like one target. The spatial resolution limits for two closely spaced sources in the context of array processing still an active research [1, 2, 3]. In fact, there has been a tremendous involvement in the investigation of how many source (target) can be detected. Most of them, [3, 4], exploit high resolution methods like Multiple Signal Classification (MUSIC) or Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT), and detect sources using eigenvalues obtained from covariance of samples. However, the main issue of high resolution method for DOA's estimation is the predetermination of model order. Because it's imperative for these techniques to know the number of sources, first. This estimation is based on information theoretic criteria like AIC (AKAIKE) and Rissanen's minimum description length criterion (MDL) algorithms to estimate the number of source [2, 4, 5]. In other hand, the performance of these techniques stays very poor for low samples, low SNR, correlated source signals and presence of impulsive white noise. To improve the resolution, it's applied spatial downsampling technique based on application of interleaving method leading to L subarray outputs, and on each subarray (trial) is applied MUSIC, the number of peaks corresponds to the number of sources [3, 6, 7].

Unfortunately, the most existing methods are less efficient and lost large performance or even breakdown when only few samples are available. To reduce this hurtful effect and improve the robustness of the covariance estimator, the authors in [7, 8] proposed a robust non-parametric bootstrap method estimator to obtain, by randomly sampling n times with replacement from the original data samples and to estimate its sampling distribution without any model assumption.

Reference [9] was our first work where we have combined spatial smoothing and MUSIC method to estimate DOA for very closed sources in space.

But in this work, we propose an algorithm which combines spatial sampling, high resolution method (MUSIC) and bootstrap technique to estimate closely spaced number of source and their direction of arrivals (DOA) when only very few samples are available. First, bootstrap method is used to estimate the covariance matrix, then the spatial sampling consist to curve up the array network of antenna, into L sub-networks; in each sub-network, it's applied Music algorithm to estimate the number of closely spaced source and their Directions of Arrival (DOA). A Comparison with classical method and numerical simulations are given to assess the performance of the technique.

The paper is organized as follows. The data, array model and music description are introduced in section 2, followed by spatial sampling model description in section 3, then the bootstrap is section

4. The proposed algorithm described in section 5. Simulation results are given in section 6, before discussion and conclusion in section 7.

#### 2. PROBLEM FORMULATION

Just to simplify the notation, we assume a ULA composed of M sensors, with equip-spacing d= $\lambda/2$  as shown in fig.1; where  $\lambda$  is the wavelength of the source signal. Consider a K narrowband far-field uncorrelated source impinging on the array with (M > K), such that sources have a direction of arrival (DOA)  $\theta_k$ , with k=1... K.

#### 2.1. Array Signal Model

The received snapshots at this array, at instance t are given by [1, 2, 3, 9, 10, 11, 12].



Fig 1. Localization of two closely spaced sourcesusing ULA

The received signal is corrupted by additive white Gaussian noise and is expressed as [2, 12, 13, 14]:

$$y(t) = A s(t) + n(t)$$
(1)

Where

 $A = \begin{bmatrix} a_1 & \dots & a_k \end{bmatrix}^T$  is the steering matrix (MxK) full rank,

 $a_k = \begin{bmatrix} 1 & a_k & a_k^2 & \dots & a_k^{(M-1)} \end{bmatrix}^T$  where  $a_k = e^{(-j 2\pi (d/\lambda) \sin \theta_k)}$ ; Superscript (.)<sup>T</sup> presents the transpose operation.

Where  $y_k$  (t) denotes the output of  $k^{th}$  sensors, and  $n_k(t)$  is a stationary model, temporally white, zero-mean Gaussian random process independent of the source signals. The covariance of receiving data is:

$$R_{yy} = E\left[Y.Y^{H}\right] = AR_{S}A^{H} + \sigma^{2}I$$
(2)

Where  $R_S = E\left\{S(t)S^H(t)\right\}$ .

The superscript (.)<sup>H</sup> stands for the conjugate transposition,  $\sigma^2$  is variance and **l** indicates the identity matrix.Furthermore, the covariance matrix is estimated by [2, 3, 14, 15, 16, 17]:

$$R_{yy} = \frac{1}{N} Y \cdot Y^{H}$$
(3)

The eigenvalues are given as follows:

$$\rho_1 \geq \rho_2 \geq \dots \geq \rho_k \geq \rho_{k+1} = \rho_M = \sigma^2$$

where the first K eigenvalues belong to the source signal, and the last (k-M) to the noise. MUSIC plots the pseudo-spectrum:

$$V_{Music} = \frac{1}{a^H(\theta)E_n E_n^H a(\theta)}$$
(4)

#### 2.2. The Spatial Downsampling

Spatial subsampling consists to divide the whole network array antenna into L subarray, by application interleaving method as shown in fig.2. L corresponds to downsampling factor, with L must be L > 1. [3, 5, 18].



Fig 2.A ULA antenna is divided into L=2 sub array, each sub array contain  $$\rm M\!/L$$  sensors.

Let Consider L=2, in this case we use two subarray 0 and 1. The receiving signal on subarray0 can be presented as fellow:

$$Y = A(\theta)S + N_0 \tag{5}$$

In case of using subarray 0:

$$Y_{0} = \begin{bmatrix} y_{1}(0) & \dots & y_{1}(T) \\ y_{L+1}(0) & \dots & y_{L+1}(T) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ y_{((N_{L}-1)L+1)}(0) & \dots & y_{((N_{L}-1)L+1)}(T) \end{bmatrix}$$
(6)

with T is snapshot number, and  $N_L = M/L$ , in this case:

$$A_{0}(\theta) = \begin{bmatrix} 1 & \dots & 1 \\ e^{j\pi \sin \theta_{1}} & \dots & e^{j\pi \sin \theta_{q}} \\ e^{j\pi (L+1)\sin \theta_{1}} & \dots & e^{j\pi (L+1)\sin \theta_{q}} \\ \vdots & \vdots & \vdots \\ e^{j\pi (N_{L}-1)(L+1)\sin \theta_{1}} & \dots & e^{j\pi (N_{L}-1)(L+1)\sin \theta_{q}} \end{bmatrix}$$
(7)

The same for the output for subarray sign 1:

$$Y_{1} = \begin{bmatrix} y_{2}(0) & \dots & y_{2}(T) \\ y_{L+2}(0) & \dots & y_{L+2}(T) \\ \vdots & \vdots & \ddots \\ y_{((N_{L}-1)L+2)}(0) & \dots & y_{((N_{L}-1)L+2)}(T) \end{bmatrix}$$

$$A_{1}(\theta) = \begin{bmatrix} 1 & \dots & 1 \\ e^{j\pi L \sin \theta_{1}} & \dots & e^{j\pi L \sin \theta_{q}} \\ e^{j\pi (L+2) \sin \theta_{1}} & \dots & e^{j\pi (L+2) \sin \theta_{q}} \\ \vdots & \vdots \\ e^{j\pi (N_{L}-1)(L+2) \sin \theta_{1}} & \dots & e^{j\pi (N_{L}-1)(L+2) \sin \theta_{q}} \end{bmatrix}$$
(8)
$$(9)$$

We consider the assumption that  $N_{\text{L}}\text{>}$  q (the number of sensor is greater than the number of sources).

Let note, that spatial downsampling, improves angular resolution because of the increasing of the imaginary part in elements matrix output in equations (7) and (9). If we assume two closely spaced source where their DOA are  $\theta_1$  and  $\theta_2$  such as:

$$\theta_1 = \theta_2 + \delta\theta \tag{10}$$

With  $\delta\theta \prec 5^0$ 

This has the advantage to increase "virtually" the angle difference by a factor of L which leads to a better resolution of two angles. [5]

#### 2.3. Bootstrap Replication

In this section, non-parametric bootstrap resampling techniques are presented, designed for independent and identically distributed data. However, the assumption of iid data can break down during operation either because data are not independent or because data are not identically distributed [7, 8].

If the original data points :

$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \dots \mathbf{x}_n)$$
(11)

These points are random drawn from an unspecified distribution *F*. A practical implementation of the non-parametric bootstrap is given in figure 3. Where  $\theta$  denote an unknown characteristic of *F*, it could be a mean, variance or even the spectral density function. The problem to solve is to find the distribution of  $\hat{\theta}$ , an estimator of  $\theta$ , derived from the original sample x. The bootstrap suggests that we resample from a distribution chosen to be close to *F* n some sense. With the non-parametric bootstrap, we simply use the random sample x and generate a new sample by sampling with replacement from x. This new sample is called bootstrap sample [6,7].



Fig 3.Principal of non-parametric bootstrap [6]

A bootstrap sample X\* is obtained through replacement of original data points by random sampling (n times) [7, 8]. Some bootstrap samples can be:

$$\mathbf{x}^{(1)} = (\mathbf{x}_2, \mathbf{x}_5, \mathbf{x}_n \dots \mathbf{x}_1) \mathbf{x}^{(2)} = (\mathbf{x}_1, \mathbf{x}_4, \mathbf{x}_1 \dots \mathbf{x}_n)$$

$$\mathbf{x}^{(B)} = (\mathbf{x}_1, \mathbf{x}_4, \mathbf{x}_1 \dots \mathbf{x}_n)$$
(12)

with n samples At the end we obtain:

$$\mathbf{X}^* = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \dots, \mathbf{x}^B)$$
(13)

With each replicate contains n samples.

The probability that a particular value xi is left out is :

$$P = (1 - \frac{1}{n})^n \tag{14}$$

#### 2.4. The proposed Algorithm:

Our method is based, first, on increasing the number of snapshots received on array network using bootstrap technique.

Determination the number of source first is essential for high resolution method. It should use AIC or MDL algorithm to determine the order of the model. But, in this work, we followed the same spirit given in [3, 5]. The idea is making an estimation of source number using beamforming or Capon method applied to the global array output. If q peaks appear, we re-apply MUSIC algorithm by restricting our research in intervals around each q peaks.

Then, applying spatial downsampling to the array network, if L is a downsampling factor that means we obtain L different DOAs estimates. Among these L sets, we keep only the highest number of peaks in each interval.

The new algorithm can be summarized s fellow:

- Step1: Applying bootstrap technique to generate new samples by sampling with replacement from original data.
- Step2: First estimation the number of source on global array network using beamforming or Capon method.
- Step3: Defining the set of intervals where research re refined.
- Step4: Divide the global antenna array into L interleaving subarrays,
- Step 5: On each subarray, we apply MUSIC Algorithm. The number of MUSIC spectrum peaks equals to the number of sources.
- Step 6: The number of sources is selected from the q intervals for the L subarrays that present the maximum number peaks,
- Step7: computing the final DOA, after sorting and calculate the average from each interval and selected subarray presenting the maximum peaks

$$\widehat{\theta}_{j} = \frac{1}{p} \sum_{l=1}^{p} \widehat{\theta}_{j}^{l}$$
(15)

Where  $\hat{\theta}_{l}^{l}$ , l=1..p represents p estimates DOA from different sub-arrays.

#### 3. SIMULATIONS AND RESULTS

To illustrate the performance of our method, some numerical results are presented to analyze and compare the estimation of behavior of the new proposed algorithm. A Uniform Linear Array (ULA) is constituted of N=10 sensor spacing of half-length wave length is employed. Assume that there are two closely spaced uncorrelated narrowband signal sources with the same wavelength  $\lambda$ ,  $\theta_1 = 32^{\circ}$ 

and  $\theta_2 = \theta_1 + \delta \theta$ , where  $\delta \theta$  is a very small angle difference. Simulation results were obtained based on 100 Monte Carlo simulation runs. The method proposed in this paper is denoted 'SSBoot'.

In fig. 4, as presented in [3] and [6], we compare the performance of the two methods: using MUSIC alone and MUSIC combined just with spatial sampling, called SS-MUSIC, to estimate the DOA's of two sources very closed in the space for low SNR=5dB and different angles of separation. the histograms of the number of sources detected by the two standard SS-MUSIC and MUSIC methods are shown, for different distances between the sources, and for different signal-to-noise ratios (SNR). This figure demonstrates the interest of the SS-MUSIC algorithm, which manages to detect closed sources in space and for low signal-to-noise ratios. For example, when the angle of separation is 2<sup>0</sup>, MUSIC alone (at left) detects just one source. However, SS-MUSIC at right side, detects one sources 85% and two sources 15%. We also note that the two algorithms never detect an additional source that corresponds to the noise.

Fig. 5, for angle of arrival  $-40^{\circ} - 20^{\circ} 60^{\circ}$  and  $80^{\circ}$  respectively, shows the performance of bootstrap for varying snapshots. When a few samples (20 snapshots) are received the response MUSIC spectrum is almost flat and the DOA are difficult to extract, but when these samples are bootstrapped at 200, 1000 then 2000 samples, the response increases and the peaks becomes noticeable. However, it demonstrates the effectiveness of the bootstrap method to improve the detection and estimation of DOA.

Fig. 6 illustrates the performance achieved by our method for a few snapshots and low SNR. In fact, for received low samples the detection is weak, it increases slowly when SNR increases. But when these samples are bootstrapped at 1000 snapshots the estimation rate improves and reaches the max rate for low SNR. However, our algorithm SSBoot bootstraps the received samples and uses the spatial sampling exhibits its estimation performance for the same number of snapshots, indeed the very close space source are detected for low SNR.

Figure 7 shows that for few samples the detection nearly breaks down. With bootstrap at 1000 snapshots, the detection starts to work although it performs unsatisfactorily because it suffers at low SNR. The proposed method SSBoot performs better, it has highest detection rate for low SNR and very close separation source. As we can see in the fig. 8, the DOAs MSE (Mean Square Error) against SNR for L=2, and angle difference  $\delta\theta$ =5<sup>0</sup>, it can be observed that the MSE for Only bootstrapped MUSIC method and our technique SSBoot that use Bootstrap, downsampling and MUSIC have practically the same estimation accuracy. It means that SSBoot improves the resolution but it doesn't enhance the estimation accuracy.





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#### 4. CONCLUSION

We introduced a new technique based on the combination of bootstrap technique, spatial downsampling and MUSIC method to improve the estimation of closed source number. It was shown that for the case of small sample size, the bootstrap technique is used to estimate and evaluate the resample data. The spatial sampling was also presented as downsampling method, which provides different subarrays and widens the angle separation of closed source when MUSIC Algorithm is applied.

Through different simulations, the performance and the effectiveness of the proposed approach are demonstrated. First, it's presented a comparison performance between MUSIC alone and SS-MUSIC, which combine the high resolution method MUSIC and spatial sampling technique. It's seen that is obvious that SS-MUSIC presents best results than MUSIC algorithm alone. Then it's illustrated different simulations that asses the performance of our new algorithm called "SSBoot"

which combine high resolution method MUSIC, spatial sampling and non-parametric bootstrap technique.

The results introduced in this paper prove that our methods are very attractive when few samples are available and outperform the ordinary technique at difficult scenarios especially for very close source and low SNR. Simulations have shown that spatial sampling and bootstrap techniques outperforms DOA estimation, when MUSIC method is applied for small sample size and very close sources. But it's demonstrated that our technique can't improve the estimation accuracy.

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