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Rectangular Antenna Array Optimization using Wind Driven Optimization

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Abstract: Antenna arrays are considered as important type used today for long distance communication with a very high gain. The design of such antenna depends on parameters and desired behavior performing the task, this project handle the application of a new type of nature-inspired global optimization methodology in the design of an optimized planar antenna array which ensures minimum side lobes and high directivity, this new optimization method is based on the atmospheric motion and it is known as Wind Driven Optimization (WDO) a population based iterative heuristic global optimization algorithm technique for multi-dimensional and multi-modal problems with the potential to implement constraints on the search domain. The optimal values obtained results in a good suppression of the side lobe level for the different antenna configurations with several sorts of excitation: Amplitude only, phase only, both amplitude and phase. Besides, the directivity is not worse than that of the uniform one.

Keywords: Rectangular antenna arrays, wind driven optimization, sidelobe level, directivity.

1. INTRODUCTION

In long distance communication, there is great need for very directive antennas with very high gain due to the radiation pattern limitations of a single antenna; several single antenna elements can be combined to form an array [1]. Arrays of antennas are used to direct radiated power towards a desired angular sector. The number, geometrical arrangement, and relative amplitudes and phases of the array elements depend on the angular pattern that must be achieved. Once an array has been designed to focus towards a particular direction, it becomes a simple matter to steer it towards some other direction by changing the relative phase of the array elements.

The designed array should allow signals from a desired direction to add constructively while simultaneously adding destructively in the undesired directions, hence an array may be regarded as a spatial filter with high gain in the desired signal direction and low gain elsewhere. Theoretically, the array should be designed with a maximum directivity and minimum side lobe level so as to achieve maximum signal to noise plus interference ratio at the output of the array antenna. However, this is only true if the interferences are evenly distributed, or assume certain distribution patterns over the whole spatial domain, the maximum directivity design may not be the best design. Therefore, if the designer of the array does not know the distribution of the directions of the interferences, an alternative design such as side-lobe level reduction may be preferred. In this project we concentrated on designing directivity and side-lobe levels of uniformly and non-uniformly distributed antenna elements along a planar, circular and a concentric ring by varying the amplitude only, the phase-only, amplitude and phase simultaneously using specific optimization techniques [2].

Over the decades, several researchers have generated different solutions to linear and non-liner optimization problems. For that purpose there is no known single optimization method available for solving all optimization problems. A lot of optimization methods have been developed for solving different types of optimization problems in recent years. The modern optimization methods (sometimes called nontraditional optimization methods) are very powerful and popular methods for solving complex engineering problems , some of these methods are nature inspired techniques such as Simulated Annealing (SA) [8,9], Genetic Algorithm (GA) [10], Particle Swarm Optimization (PSO) [11] and Ant Colony System (ANS) [12] – as systems based on animal behavior, and Invasive Weed Optimization (IWO) [13] – from emulation for the vegetal growing patterns. More recently, a new optimization algorithm was invented called Galaxy Based Search Algorithm [14].

Synthesis and optimization problems in electromagnetics have long utilized these nature-inspired techniques to varying degrees of success. Application areas within the field of electromagnetics are very wide, ranging from antenna design to metamaterial synthesis. In the search for new methods, a novel nature-inspired optimization algorithm called the Wind Driven Optimization (WDO) technique is introduced in this chapter. In essence, the WDO is a population based iterative heuristic global numerical optimization technique for multi-dimensional and multi-modal problems with the ability to implement constraints on the search domain. The inspiration for WDO comes from atmospheric motion in which the trajectory of an infinitesimally small air parcel can be described via Newton's second law of motion. In the next sections we structure to give the description of WDO technique in detail along with the underlying physical equations of atmospheric motion, and a parameter study will be conducted to aid in tuning the WDO algorithm [15].
This work addresses the problem of finding the optimum, that is the minimum SLL and maximum Directivity using Wind Driven Optimization (WDO), In essence, WDO is a population based iterative heuristic global optimization technique for multi-dimensional and multimodal problems with the potential to implement constraints on the search domain. The inspiration for WDO comes from atmospheric motion in which the trajectory of an infinitesimally small air parcel can be described via Newton's second law of motion.

2. PROBLEM FORMULATION

For an N×M retangular two dimensional arrays, the array factor is expressed as:

$$AF(\theta,\phi) = \sum_{n=1}^{N} I_{1n}\left[\sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi+\beta_x)}\right] e^{j(n-1)(kd_y \sin\theta\sin\phi+\beta_y)} \quad (1)$$

The antenna array is seen as the product of two linear array factors. The idea of the optimization task is similar to the one dimensional (linear) array described in the previous equations with the exception that the overall performance parameters are taken to be the worst of the two dimensions.
Each element contribute to the overall radiation pattern by three controls that can be acted upon to shape the pattern. This suggests many strategies of pattern synthesis that are dealt with in this work.
The work starts by finding the element excitation amplitudes to have a radiation pattern with the least sidelobe level. The adoption of the amplitude-only control of sidelobe level has been due to the fact that it produced the best performance in terms of sidelobe level than the two other alternatives (position-only and phase-only). The fitness function used to achieve this is:

$$f_1 = \sum_{i=1}^{N} \frac{1}{\Delta\theta_i} \int_{\theta_{li}}^{\theta_{ui}} \left|AF(\theta)^2\right| d\theta \quad (2)$$

Where θ_{li} and θ_{ui} are the boundaries of the spatial regions where the radiation pattern maxima are intended to be suppressed, $\Delta\theta_i$ represents the bandwidth ($\theta_{ui} - \theta_{ui}$).
It should be noted that the amplitudes are confined in the interval [0,1] which might lead to values that are very far from each other. This is problem as the feeding network becomes complex and difficult to implement. An attempt to overcome this problem is to include the dynamic range (DR) of these amplitudes into the optimization process. The dynamic range is defined simply to be:

$$DR = \frac{a_{\max}}{a_{\min}} \quad (3)$$

with its contribution to the fitness function being

$$f_3 = w_d DR \quad (4)$$

Where w_d is a weighting factor to adjust the dynamic range and can be defined as:

$$w_d = \begin{cases} 0 & if\ DR < some\ value \\ 1 & elsewhere \end{cases} \quad (5)$$

Once the excitation amplitudes that produce the lowest possible sidelobe level have been found, the next level of optimization is to incorporate a null steering capability to the array. The forcing on null placement leads automatically to a loss in performance in terms of sidelobe level. This needs to be taken into account in the optimization process. The idea is to perform perturbations on the three controls (amplitude, phase and position) and their combinations to place nulls in some desired directions while preserving the sidelobe level at some acceptable level. For null placement, the fitness function is defined as:

$$f_2 = \sum_{\theta=0}^{\theta=180} w(\theta)\left|F_0(\theta) - F_d(\theta)\right| + SLL \quad (6)$$

Where $w(\theta)$ is a weighting vector to force the array to place the null(s) in their desired directions and is defined as:

$$w(\theta) = \begin{cases} 100 & \text{if } \theta = \text{desired direction} \\ 1 & \text{Elswhere} \end{cases} \quad (7)$$

$F_0(\theta)$ is the initial pattern and $F_d(\theta)$ is the desired pattern which is merely the initial pattern except at the desired nulls where it is forced to be zero. This is summarized as:

$$F_d(\theta) = \begin{cases} 0 & \text{for null directions} \\ \text{initial pattern} & \text{elsewhere} \end{cases} \quad (8)$$

The term *SLL* accounts for the maximum allowable increase in sidelobe level due to null placement. This is done by introducing a penalty on the increase in sidelobe level as:

$$SLL = \begin{cases} 10 & \text{if } SLL \geq \text{some value} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

3. THE WIND DRIVEN OPTIMIZATION TECHNIQUE

In the Earth's atmosphere, the wind blows in an attempt to equalize horizontal imbalances in air pressure [16], it blows in the direction from a region of high pressure to low pressure at a velocity which is proportional to the pressure gradient [55, 81] that from where the WDO technique is inspired. The term "wind" actually refers to the large-scale horizontal air motion particularly in the lowest layer of the Earth's atmosphere called the troposphere. The troposphere is the lowest layer which extends from the surface of the earth's crust up to 18 km, where the layer thickness may vary based on the location's latitude [17]. Due to differences in the solar energy reaching to the different locations on earth surface, the temperature can vary significantly among regions. Areas with higher temperature would have rising warm air and regions with lower temperatures would have sinking cold air causing the air density to decrease in high temperature areas and to increase in low temperature areas. Due to earth's gravitational field, g, the mass of atmosphere applies a force on the earth's crust, where the air pressure can simply be defined by the force exerted per unit area [18]. Since temperature differences lead to variations in air pressure at different locations, horizontal differences in air pressure causes the air to move from high pressure regions to low pressure regions [19]. This movement is due to the pressure gradient ∇P which can be calculated as the pressure change over a distance [20].

There are two distinct descriptions of atmospheric behavior, which are Lagrangian and Euler descriptions [20]. In the Euler description, the air is treated as a fluid system and a continuum which is described by the fluid motion governed by the equations of continuum mechanics [20], [21]. On the other hand, the Lagrangian description represents the atmosphere as the collection of many infinitesimal fluid parcels [21], [22]. In the derivation of the equations for WDO, the Lagrangian description is preferred due to fact that it would simplify the numerical algorithm and reduce computational overhead that the WDO would need during the optimization of a real world problem.

The starting point for calculating an air parcel’s trajectory is Newton’s second law ofmotion, which provides accurate results when applied to the analysis of atmospheric motion in the Lagrangian description [18], [19], [21]. It states that the total force applied on an air parcel causes it to accelerate with an acceleration a in the same direction as the applied total force according to:

$$\rho a = \sum F_i \tag{10}$$

Where ρ is the air density for an infinitesimally small air parcel and F_i represents all the individual forces acting on the air parcel. To relate the air pressure to the air parcel's density and temperature, the ideal gas law can be utilized and is given by:

$$P = \rho RT \tag{11}$$

Where P is the pressure, R is the universal gas constant and T is the temperature in Kelvin.
In the physical three-dimensional atmosphere, the gravitational force F_G is a vertical force directed toward the earth's surface. However, if the center of the earth is considered to be the center of the rectangular coordinate system, then we can claim that the gravitational force simply pulls the air parcelstowards the origin of the coordinate system in all three dimensions, which is also more easily mapped to N–dimensional space. For this reason, the gravitational force is included in the algorithm as a force on all N-dimensions and is directed towards the center of the coordinate system. In its simplest form, the gravitational force F_G can be defined as [22]:

$$F_G = \rho \delta V g \tag{12}$$

Utilizing the ideal gas law, the density can be written in terms of the pressure, the temperature and the universal gas constant and inserted into yielding:
A population of air parcels would start at random positions in the search space their velocity and position would be adjusted at each iteration and they would move towards an optimum pressure location at the end of the last iteration. Considering that actually the optimum pressure point on the N-dimensional search space corresponds to the optimum solution for the optimization problems, WDO offers a simple but efficient way to tackle them.
For each dimension, WDO allows the air parcels to travel only in the bounds of [-1, 1] and if an air parcel tries to travel outside of these bounds at any dimension, then its position at that particular dimensions is set to the boundary value. For example, if the air parcel tries to go from 0.82 to beyond 1 at the x-dimension, the position for the next iteration is forced to be 1 on the x-dimension. In literature, there has been many other boundary conditions proposed [22], yet for the WDO, the gravitational pull would pull any air parcels that are stuck at the boundaries back in to the feasible search space.
It should be also pointed out that the updated velocity of the air parcels should be limited to a maximum value per iteration. The main reason for this is to simply prevent air parcels from taking large steps and overlook certain regions in the search space. To limit the magnitude of the velocity following simple rule is implemented:

$$\vec{u}^{\bullet}_{new} = \frac{u_{\max} \vec{u}_{new}}{\left| \vec{u}_{new} \right|} \tag{13}$$

Where the direction of the motion is preserved but the magnitude is limited to be no more than $\vec{u}_{\max}$ at any dimension and represents the velocity after limited to maximum speed $\vec{u}^{\bullet}_{new}$. So far the theory behind the WDO, the operators of the algorithm and implemented constraints are discussed. The algorithm starts with the initialization stage where all parameters related to the WDO as well as the other parameters related to the optimization problem have to be defined. Also, one has to define a pressure function (fitness function) and define parameter boundaries. Once the optimization problem is set up, then the population of air parcels are randomly distributed over the N-Dimensional search space with a random velocity assigned to them. Next step is to evaluate the pressure (fitness) values of each air parcel at their current positions. Once, the pressure values are evaluated, the population is ranked based on their pressure and velocity update is applied along with the restrictions as given in (13). The positions for the next iteration are updated and the boundaries are checked to prevent any air parcel from running out of feasible search space. One all the updates are carried out, the pressure at new locations is to be evaluated once again and this iterative procedure continues until the maximum number of iterations are reached. Finally, the best pressure location at the end of the last iteration is the optimization result and the best candidate solution to the problem.

The flowchart in Fig. 1 illustrates the mechanism of work of the wind driven optimization algorithm.

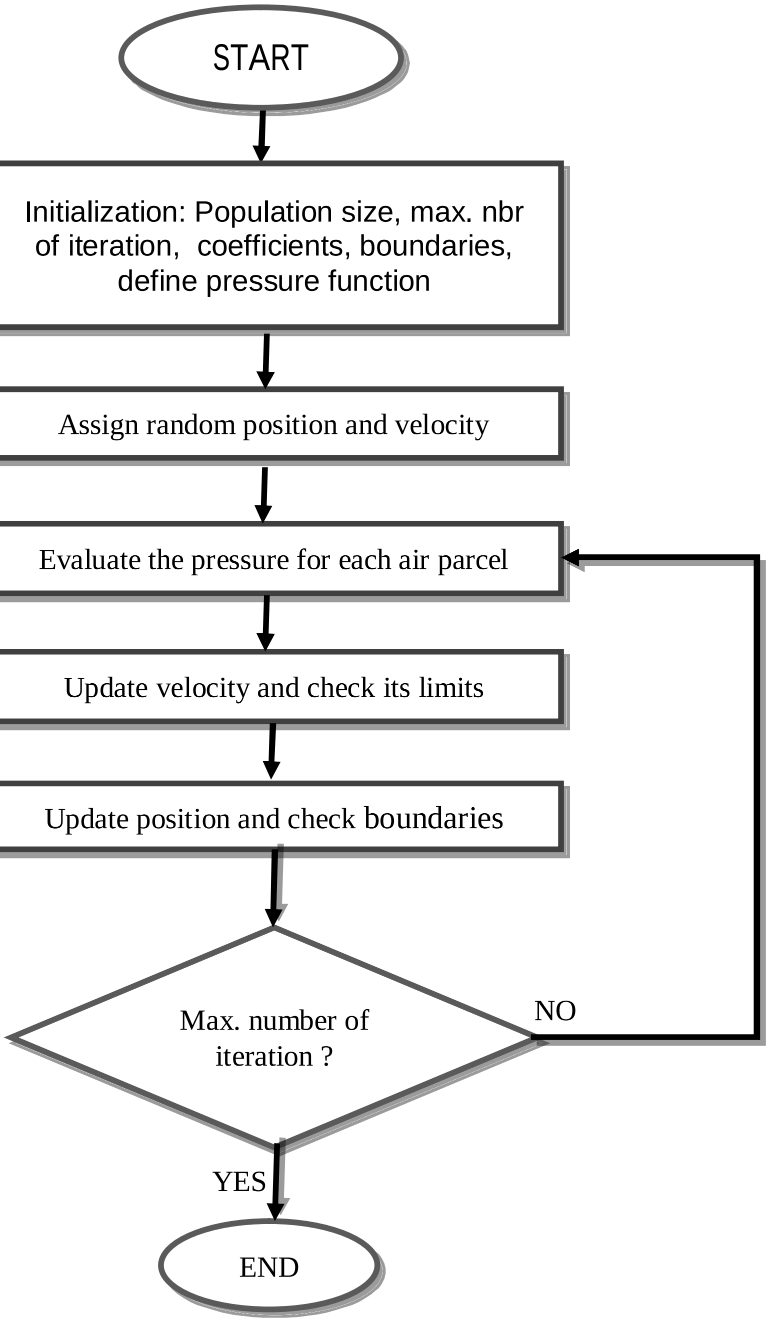

Fig. 1 Flowchart of the wind driven optimization (WDO)

4. RESULTS AND DISCUSSIONS

the variations of the excitation amplitude and the phase in the following way:

- Amplitude (I): It varies in the interval [0, 1].
- Phase: the phase excitation varies in [0, 2π].

An array of 100 elements laying along the x-y plane (10X10 array) is considered, with: ($\beta_x = \beta_y = 0$ and $d_x = d_y = \lambda / 2$ where $\lambda = 1$ and $phi = 0$) . Since the number of elements is the same over each dimension, the optimization task is performed only on one array and the overall 2D array factor can be found as the multiplication of the two array factors. The overall radiation characteristics (SLL and DIR) can be found as the linear array characteristics themselves.

For the uniform case (I=1 and α= 0), the array is characterized by : SLL = –12.97dB and a Directivity of 21.7 dB.

For non-uniform case, different stratagies are adopted. First, only with the variation of the excitation Amplitudes to optimize the SLL then Directivity separately and then both. Second, we vary only the excitation phase angles with the same objectives and finally the two paremeters are varied together (excitation amplitudie and phase) with the same objectives.

A. Excitation amplitude-only variation

Setting the parameters presented above and optimizing for the SLL only, we see from fig. 2 that the sidelobe level is reduced from -12.7dB to -25.8385dB. However, one notices a reduction in the Directivity to 19.72dB.

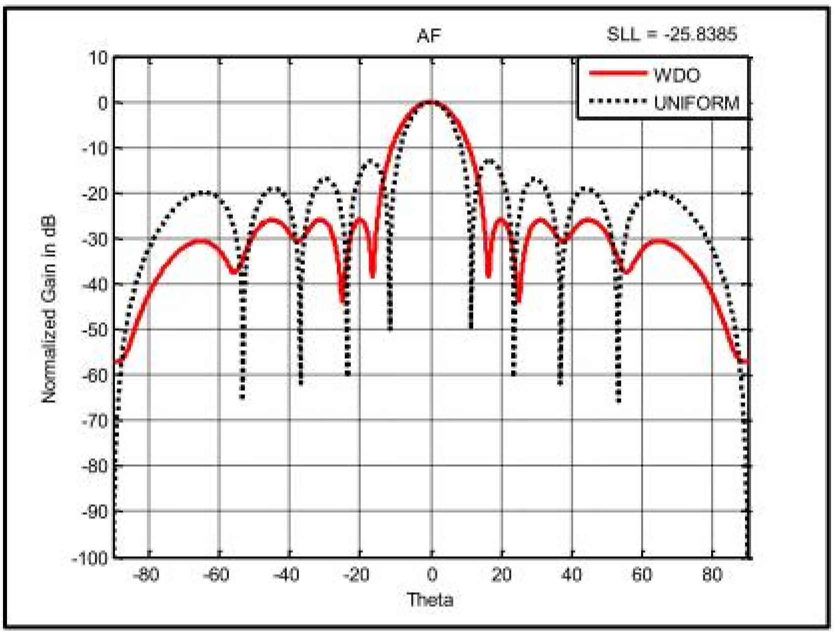

Fig. 2 Array factor when optimizing for SLL in Amplitude-only variation case

Optimization of only Directivity: It is clear from fig. 3 that the directivity is almost the same. Indeed the optimized directivity is at 21.2214dB. However, there is a little improvement in the sidelobe level and the value achieved is SLL = -13.7613dB.

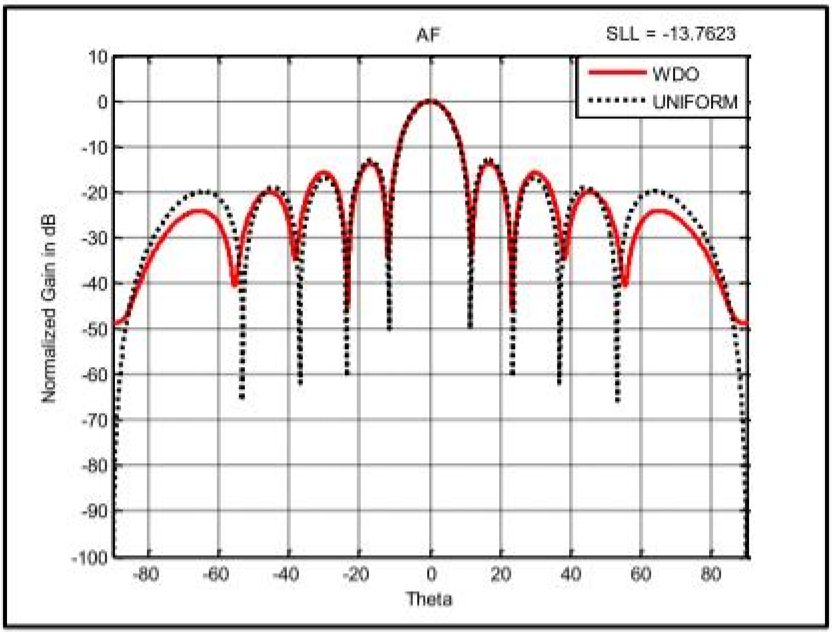

Fig. 3 Array factor when optimizing for Directivity in Amplitude-only variation case

Optimization of both SLL and Directivity: fig. 4 shows the array factor when optimizing for both SLL and Directivity. The uniform one has a ratio SLL/DIR of 0.6. In this step, this ratio has been increasd to 1.1909 as we have obtained a better SLL but a decreasing directivity (Directivity = 19.6328dB and SLL = -23.3810dB).

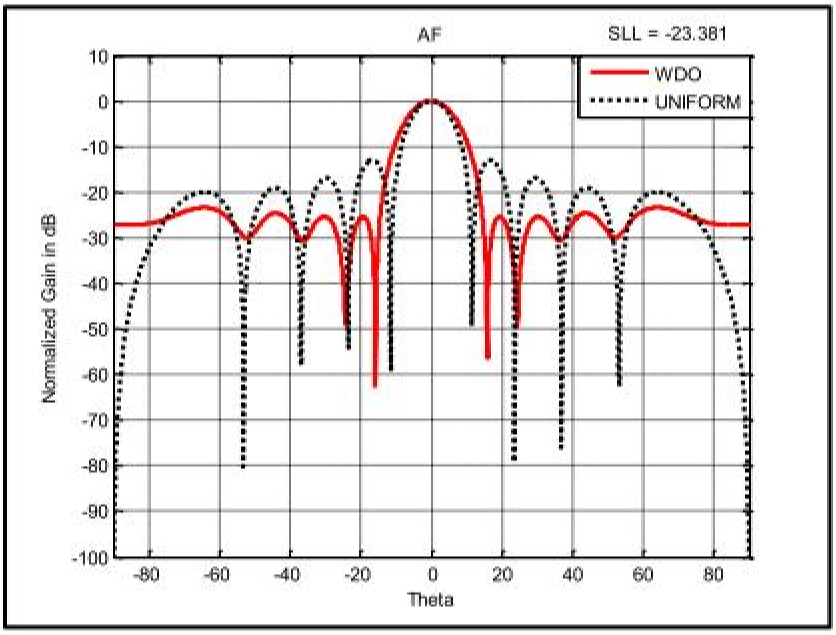

Fig. 4 Array factor when optimizing for both SLL and Directivity in Amplitude-only variation case

B. Excitation phase-only variation

In this part and from fig. 5, we see that the optimized array factor is almost like the uniform one with a small decrease and increase in Directivity and SLL, respectively.

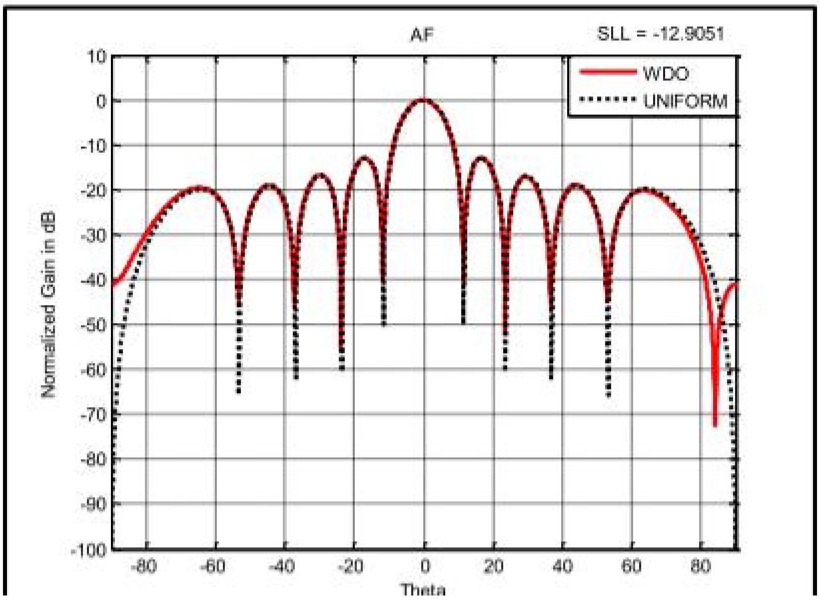

Fig. 5 Array factor when optimizing for Directivity in phase-only variation case

Optimization of SLL only: when the excitation phases are varied by the WDO, the array factor in fig. 6 is obtained and a better SLL is achieved. However, the directivity goes in the wrong direction and decreases instead of increasing. The obtained SLL is -18.5981dB and Directivity 17.0681dB.

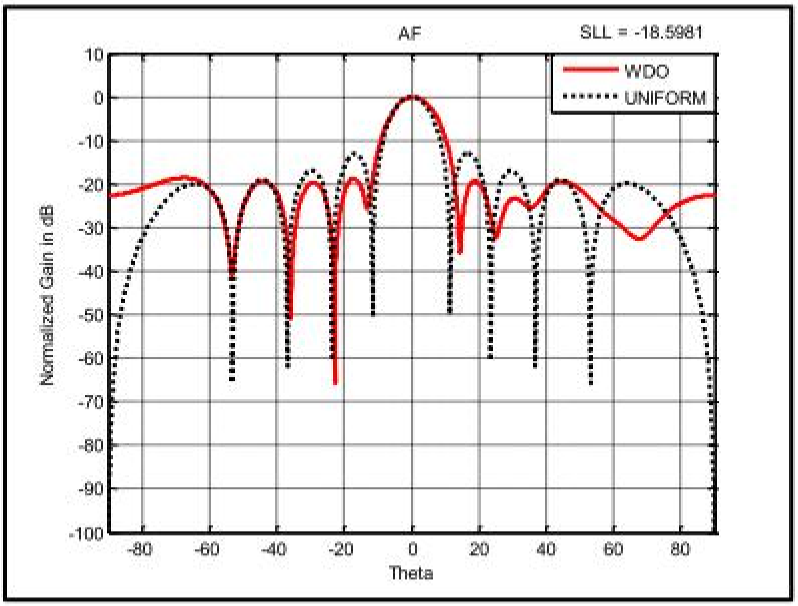

Fig. 6 Array factor when optimizing for SLL in phase-only variation case

Optimization of both SLL and Directivity, fig. 7 show the pattern of the array factor when optimizing for both SLL and Directivity. The ratio SLL/DIR is optimized from 0.6 to 0.98 so a better SLL is produced but with a decreasing Directivity. The obtained results are: (SLL = -17.0559dB, Directivity = 17.3801dB).

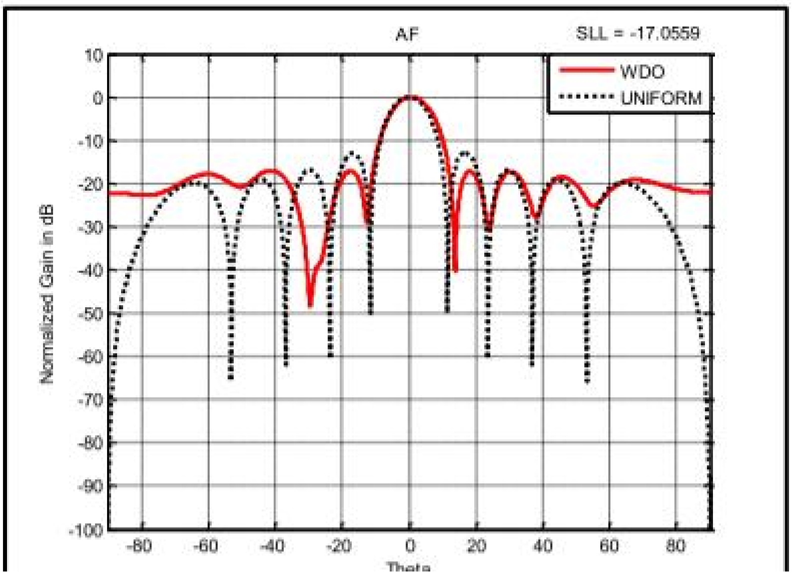

Fig. 7 Array factor when optimizing for both SLL and Directivity in phase-only variation case

C. Excitation Amplitude and Phase variation

In this part, the variation of both Amplitude and Phase to optimize SLL only is done. From fig.8, one can see that the sidelobe level has been improved compared to the one of the uniform case. However, the directivity get worst a little bit and decreased also.

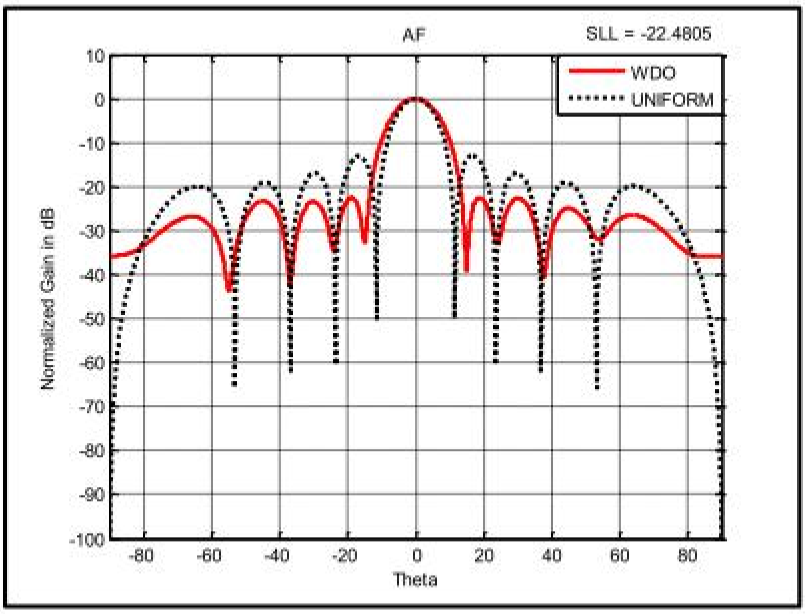

Fig. 8 Array factor when optimizing for SLL in both amplitude and phase variation case

With both Amplitude and Phase variation and optimizing the Directivity only, we see that the obtained directivity of 21.4078 dB is not a better value than that of the uniform one. However, the

SLL is changed from -12.96628dB to -13.3491dB.Fig.9 shows the array factor of the optimized array.

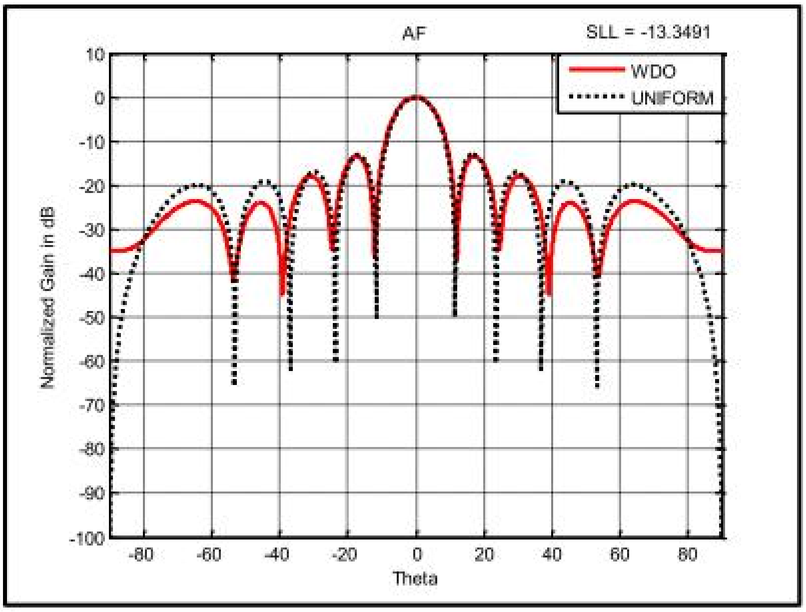

Fig. 9 Array factor when optimizing for Directivity in both amplitude and phase variation case

Now, varying the Phase and the Amplitude produces a better SLL and Directivity at the same time. It is clearly seen that a much better SLL than the uniform case is obtained on one hand with a deterioration in the directivity in the other hand. Fig. 10 shows the array factor of the optimized array.

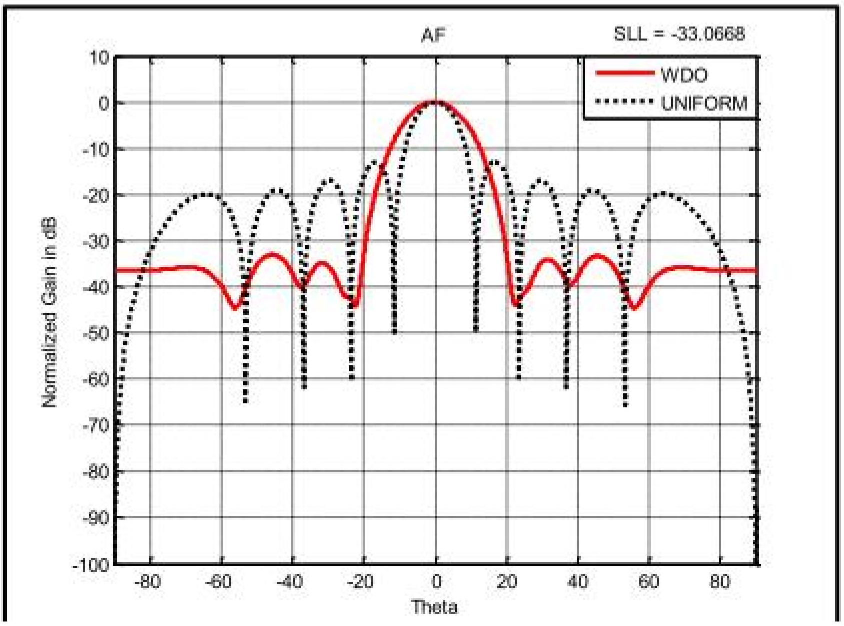

Fig. 10 Array factor when optimizing for both SLL and Directivity in amplitude-phase variation case

D. Results discussion

From the previous optimization results, we conclude that the best SLLs obtained are –33. 0668 dB – 25.8385 dB with the corresponding directivities of 18.1059 dB and 19.7209 dB, respectively. These were generated by the cases of Amplitude and Phase and Amplitude-only variations. However, the best directivity is of the uniform one followed by 21.6727dB generated from Phase-only variation. On the other hand, the worst SLL obtained after that of the uniform is -12.9052dB generated from Phase-only when the objective was to maximize the directivity.

5. CONCLUSIONS

In this paper, a new nature-inspired global optimization technique, called Wind driven optimization is used in the design of phased antenna arrays. This method is inspired from the motion of air parcels in wind, and it is a population-based iterative methods aiming to improve the best candidate solution over time. In terms of having the position and velocity update rules, WDO could be compared against the Particle Swarm Optimization (PSO), where PSO is based on a swarm of particles those share information about the search space to achieve better results. The WDO algorithm generates the non-uniform excitation amplitude & phase for the Rectangular planar arrays in question with a set of dimension, minimum and maximum boundaries, the performance of the antennas arrays was observed and studied in terms of side lobe level and directivity. Also, their subsequent array patterns were generated for observation. From these results, it was clearly shown that the optimal design is done by finding optimal excitation currents and phasing of the elements of the array. The simulated results reveal that the optimal design offers a considerable SLL reduction along with reduction of Directivity compared to the corresponding uniform arrays.

The WDO algorithm has successfully obtained the minimum SLL and an comparable value of directivity to the uniform case. Furthermore, WDO can be focused upon exploration of other parameters like gain, beamwidth and first null to null width by varying more parameters like spacing and phase shift.

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