NOVEL ANALYTICAL EXPRESSIONS FOR THE ELECTROMAGNETIC PARAMETERS OF CYLINDRICAL STRIPLINES

Yamina BEKRI, Nadia BENABDALLAH[2], Djamal LACHACHI[1] and Nasreddine BENAHMED[1]
[1]Département de Génie Electrique et Electronique, Université de Tlemcen
N_Benahmed@yahoo.fr
[2]Département de Physique, Ecole Préparatoire des Sciences et Techniques de Tlemcen
N_Benabdallah@yahoo.fr
Reçu le : 25/01/2015 *Accepté le : 19/04/2015*

Abstract

In this paper, we present novel and rigorous analytical expressions for the electromagnetic (EM) parameters of cylindrical striplines.

The analytical expressions of the EM-parameters (characteristic impedance (Z_c), inductance per unit length (L) and capacitance per unit length(C)), deduced from rigorous analysis by the finite element method (FEM) and curves fitting techniques, can be easily implemented in CAD simulation tools, to design components for communication systems.

We compare some of our results with those obtained by other methods and we find them very close.

The relative errors between our numerical and our analytical results are less than 3% in a wide range, indicating the good accuracy of our closed-form expressions of cylindrical striplines.

Keywords : *Cylindrical stripline / characteristic impedance / inductance per unit length / capacitance per unit length / finite element method / analytical expressions.*

Résumé

Dans cet article, de nouvelles expressions pour les paramètres électromagnétiques (EM) de striplines cylindriques sont présentées.

Les expressions analytiques des paramètres EM (impédance caractéristique (Zc), inductance linéique (L) et capacité linéique (C)), déduites d'une part d'une analyse rigoureuse par la méthode des éléments finis (FEM) sous l'environnement FreeFEM et d'autre part des techniques de lissage de courbes sous l'environnement Origin50, peuvent être facilement utilisées pour concevoir des composants pour les systèmes de communications.

Une comparaison de nos résultats avec ceux de la littérature scientifique obtenus par d'autres méthodes, montre clairement le bon accord entre ces derniers.

Les erreurs relatives entre nos résultats numériques et analytiques sont inférieures à 3% dans un large intervalle, indiquant la bonne précision de nos expressions proposées pour les striplines cylindriques.

Mots clés : Striplines cylindriques / impédance caractéristique / inductance linéique / capacité linéique / méthode des éléments finis / expressions analytiques .

1. INTRODUCTION

The design of microstrip antennas and microstrip arrays on cylindrically shaped substrates necessitates the development of highly accurate computer-aided design tools for microstrip integrated circuits on curved substrates [1, 2].

By taking advantage of the cylindrical configuration, applications can be found which include using flexible dielectric material in the design of transition adaptors, baluns, filters, and impedance transformers.

Different solution methods have been reported in the literature for obtaining the parameters of the cylindrical stripline [3-7]. Wang [3] solved Laplace's equation by a dual series method and presented extensive results on the characteristic impedance of cylindrical stripline. Joshi et al. [4] determined the characteristic impedance of homogeneous filled cylindrical striplines by a residue calculus technique. Zeng et al. [5] used conformal transformation to find expressions for the characteristic impedance in a closed form for cylindrical and elliptical striplines with zero and finite thickness strip conductors. Chan et al. [6] analyzed a class of cylindrical transmission lines by using an iterative approach. Reddy et al. [7] obtained a closed-form expression for the characteristic impedance of a cylindrical stripline with multilayer dielectrics.

In this work, based on the FEM analyses done under FreeFEM environment and curves-fitting techniques under Origin50, new, simple and exact expressions for the characteristic impedance, the inductance per unit length and the capacitance per unit length of the cylindrical stripline are reported.

2. CYLINDRICAL STRIPLINES

The geometry of interest is shown in figure 1, which depicts an infinitely circular arc strip (r_2, 2α) placed between the two cylindrical ground planes with inner and outer radius of r_1 and r_3, respectively, and a cylindrical substrate with relative dielectric constant ε_r.

2.1 FEM results

In this section, we illustrate the design and the modeling of the cylindrical stripline focusing on the calculation of characteristic impedance, inductance per unit length and capacitance per unit length.

To validate our obtained numerical and analytical results, first we studied the structure presented in figure 1 with ratios $r_3/r_1=2$ and $r_2/r_1=\sqrt{2}$ already analyzed by others methods. Under freeFEM environment [8-10], we show in figure 2 the FEM meshes of the cylindrical stripline using triangular elements. The obtained potential distributions are presented in Fig. 3 for α = 40° and α = 140°.

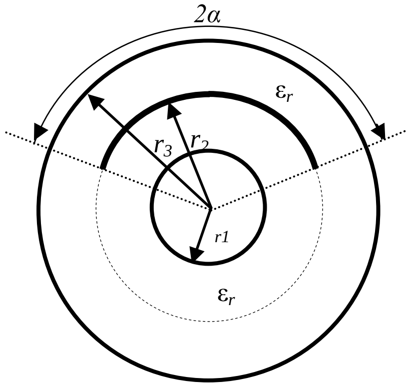


Figure 1 : The cross-sectional view of the cylindrical stripline

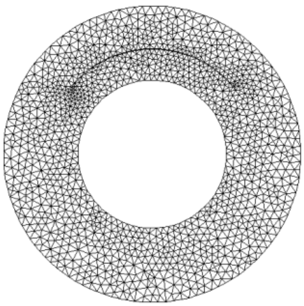
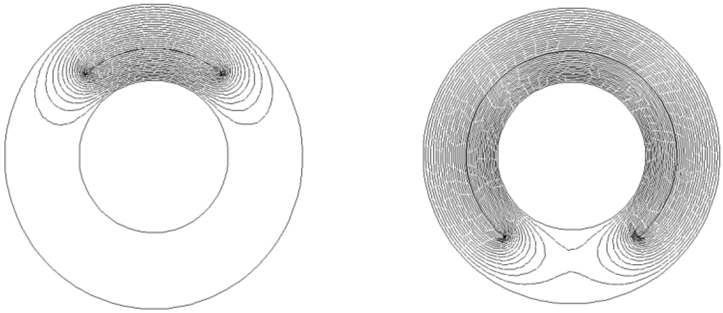


Figure 2 : FEM meshes of the cylindrical stripline



a) b)

Figure 3 : 2D potential destribution of the cylindrical stripline with α =40° on a) α =140° on b)

Table I shows the finite element results for the normalized characteristic impedance of the cylindrical stripline compared with the work of previous investigations [5, 7].

Table 1: Normalized characteristic impedance as a function of strip half-angle obtained from different methods

α (°)	$\sqrt{\varepsilon_r}\, Z_c\ (\Omega)$, $r_3/r_1 = 2$, $r_2/r_1 = \sqrt{2}$			
	FEM	**[5]**	**[6]**	**[7]**
10.20	98.0383	98.97	99.53	100.70
20.04	64.5378	64.93	65.21	65.83
29.88	48.1842	48.39	48.56	48.90
39.73	38.4312	38.49	38.68	38.90

This table shows clearly the good coherence between our FEM results and those of the scientific literature.

For the same ratio $r_3/r_1=2$ and for different values (r_2/r_1) varying between 1.1-1.9, the influence of the strip half-angle (α) varying between 5-179° on the EM-parameters of the cylindrical stripline can be seen in Figs. 4 through 6 for $\varepsilon_r = 1$.

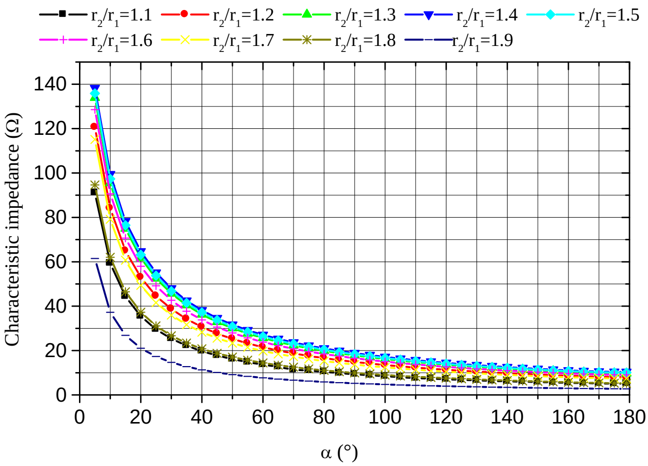


Figure 4 : Characteristc impedance of cylindrical stripline as a function of strip half-angle for different strip radius to inner conductor radius ratios

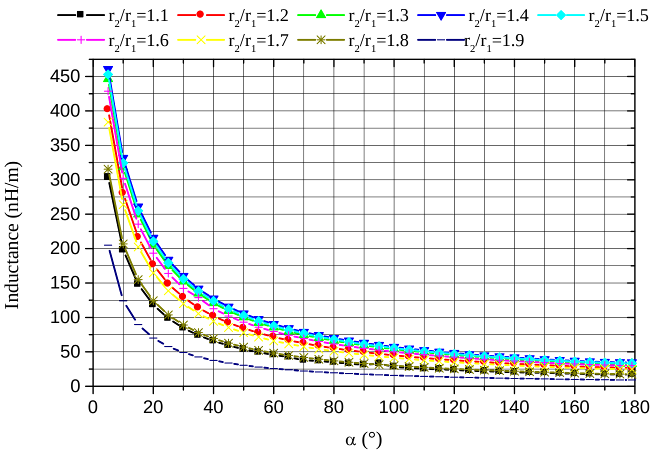


Figure 5 : Inductance per unit length of cylindrical stripline as a function of strip half-angle for different strip radius to inner conductor radius ratios

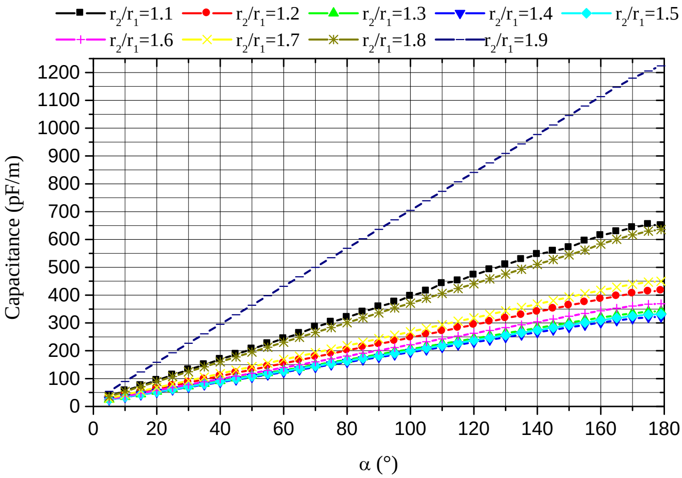


Figure 6 : Capacitance per unit length of cylindrical stripline as a function of strip half-angle for different strip radius to inner conductor radius ratios

2.2 Derivation of analytical expressions

2.2.1 Characteristic impedance

Using curve fitting techniques under Origin50 environment, it is found that the characteristic impedance of the cylindrical stripline can be expressed as:

$$Z_c = \frac{1}{\sqrt{\varepsilon_r}}(Z_{c0} + A_1 e^{-\alpha/t_1} + A_2 e^{-\alpha/t_2}) \quad (\Omega) \qquad (1)$$

Where:

For $r < 1.5$

Z_{c0} =-51.76+53.34r+17.95r^2-17.76r^3

A_1=-2602.35+6084.4r-4534.67r^2+1128.33r^3

t_1 =-291.36+683.13r-520.42r^2+132.36r^3

A_2 =-267.94+213.93r+218.85r^2-142.93r^3

t_2 =-1556.9+3687.99r-2830.98r^2+724.3r^3

$r = r_2 / r_1$

For $r \geq 1.5$

$Z_{c0} = \text{-}8887.77\text{+}21587.02r\text{-}19615.8r^2\text{+}7915.26r^3\text{-}1197.46r^4$

$A_1 = \text{-}33247.47 + 81131.57\mathrm{r} - 73965.47\mathrm{r}^2 + 29982.5\mathrm{r}^3 - 4562.06\mathrm{r}^4$

$t_1 = 7213.71\text{-}17477.45r\text{+}15869.92r^2\text{-}6392.61r^3\text{+}963.61r^4$

$A_2 = =\text{-}32807.31\text{+}79533.0r\text{-}72090.48r^2\text{+}29038.79r^3\text{-}4390.60r^4$

$t_2 = 139309.0\text{-}339004.11r\text{+}308973.2r^2\text{-}124958.05r^3\text{+}18920.61r^4$

$r = r_2 / r_1$

2.2.2 Inductance per unit length

The inductance per unit length of the cylindrical stripline can be expressed by relation (2):

$$L = \frac{1}{\varepsilon_r}(L_0 + A_1 e^{-\alpha / t_1} + A_2 e^{-\alpha / t_2}) \quad \left(\frac{nH}{m}\right) \tag{2}$$

Where:

For $r < 1.5$

$L_0 = \text{-}291.33\text{+}461.19r\text{-}164.9r^2$

$A_1 = 415.025\text{-}63.3e^{\text{-}(r\text{-}1.1)/0.0838}$

$t_1 = \text{-}264.15\text{+}620.8r\text{-}472.9r^2\text{+}120.3r^3$

$A_2 = \text{-}1921.92\text{+}3104.9r\text{-}1121r^2$

$t_2 = \text{-}1321.3\text{+}3145.98r\text{-}2415.95r^2\text{+}618.53r^3$

$r = r_2 / r_1$

For $r \geq 1.5$

$L_0 = -33587.1\text{+}81204.34r\text{-}73460.1r^2\text{+}29509.35r^3\text{-}4444.02r^4$

$A_1 = \text{-}93684.33\text{+}230658.64r\text{-}212025.76r^2\text{+}86657.61r^3\text{-}13294.8r^4$

$t_1 = 9850.37\text{-}23640.22r\text{+}21257.1r^2\text{-}8480.23r^3\text{+}1266.24r^4$

$A_2 = =\text{-}176747.03\text{+}422471.25r\text{-}377727.47r^2\text{+}150000.13r^3\text{-}22340.71r^4$

$t_2 = 150148.01\text{-}364310.08r\text{+}331069.54r^2\text{-}133511.18r^3\text{+}20159.12r^4$

$r = r_2 / r_1$

2.2.3 Capacitance per unit length

Finally, the capacitance of the cylindrical stripline is given by relation (3):

$$C = \varepsilon_r (a + b\alpha) \qquad \left(\frac{pF}{m}\right) \tag{3}$$

Where:

For $r < 1.5$

a=-7184.74+18012.65r-14910.6r^2+4084.93r^3

b=276.078-610.35r+453.39r^2-112.41r^3

$r = r_2 / r_1$

For $r \geq 1.5$

a=10506.26-25206.12r+22672.4r^2-9049.37r^2+1352.92r^4

b=4014.99-9871.22r+9102.14r^2-3730.27r^3+573.53r^4

$r = r_2 / r_1$

2.3 Comparison between analytical and numerical results

In figures 7 to 9 we illustrate comparisons between our analytical and numerical results for the cylindrical stripline.

The relative errors are less than 3% in a wide range of geometrical parameters, indicating the good accuracy of our closed-form expressions of cylindrical striplines.

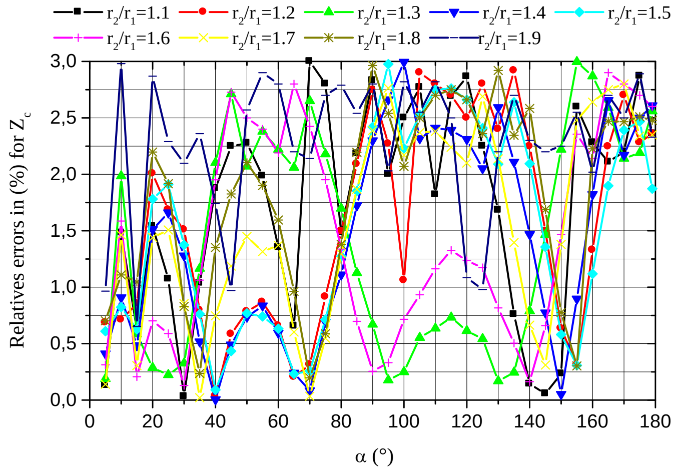


Figure 7 : Relatives errors between our analytical and numerical results for the characteristic impedance

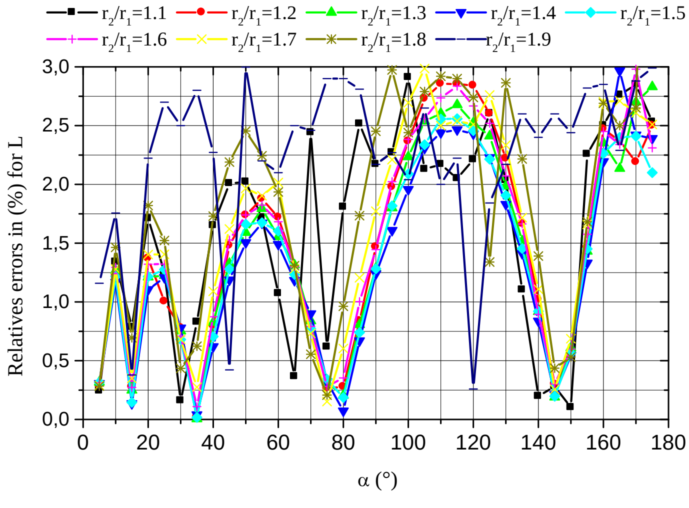


Figure 8 : Relatives errors between our analytical and numerical results for the inductance

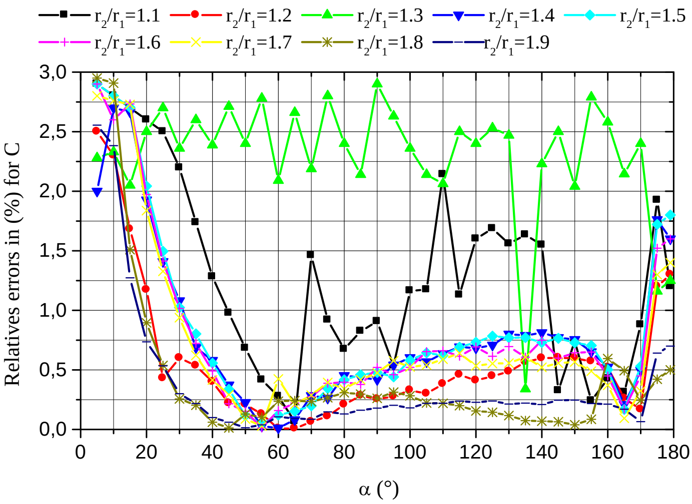


Figure 9 : Relatives errors between our analytical and numerical results for the capacitance

3. CONCLUSION

Closed-form expressions for the characteristic impedance, the inductance per unit length and the capacitance per unit length are proposed for the cylindrical stripline.

The study presents exact extensive expressions for all cylindrical striplines in a wide range of (r_2/r_1) ratios and strip half-angle values (α). The expressions are simple and accurate and thus of practical interest.

Agreement between our numerical and analytical results and previous methods reported in the scientific literature confirmed the validity of the proposed approach.

REFERENCES

[1] A. Nakatani and N. G. Alexopoulos, "Modeling microstrip circuits and microstrip antennas on cylindrical substrates," in Antennas and Propagation Society International Symposium, vol. 24, pp. 439-442, June 1986.

[2] A. Nakatani, N. G. Alexopoulos, N. K. Uzunoglu, and P. L. E. Uslenghi, "Accurate Green's function computation for printed circuit antennas on cylindrical substrates," Electromagnerics, vol. 6, no. 3, July–Sept., 1986.

[3] Y. C. Wang, "Cylindrical and cylindrically warped strip and microstrip line," IEEE Transactions on Microwave Theory and Techniques, vol. MTT-26, pp. 20-23, January 1978.
[4] K. K. Joshi, and B. N. Das, "Analysis of elliptic and cylindrical striplines using Laplace's equation,'' IEEE Transactions on Microwave Theory and Techniques, vol. MTT-28, pp. 381-386, April 1980.
[5] L. R. Zeng and Y. X. Wang, "Accurate solutions of elliptical and cylindrical striplines and microstrip lines," IEEE Transactions on Microwave Theory and Techniques, vol. MTT-34, pp. 259-265, February 1986.
[6] C. H. Chan, and R. Mittra, "Analysis of a class of cylindrical multiconductor transmission lines using- an iterative approach," IEEE Transactions on Microwave Theory and Techniques, vol. MTT-35, pp. 415-424, April 1987.
[7] C. J. Reddy and M.D. Deshpande, "Analysis of cylindrical stripline with multilayered dielectrics," Transactions on Microwave Theory and Techniques, vol. MTT-34, June 1986, pp. 701-706.
[8] www.FreeFEM.org
[9] D. Lachachi, N. Benabdallah and N. Benahmed, "Coupleurs branchline TEM et quasi-TEM à lignes coaxiales classiques», Communication Science & technology (COST), vol. no. 15, July 2014, pp. 27-46.
[10] Y. Bekri, N. Benabdallah, D. Lachachi and N. Benahmed, "Analyze Quasi-TEM Rectangular Coax Couplers," Microwaves & RF, vol. 54, no 1, January 2015, pp. 68-72.