FREE VIBRATION OF SPHERICAL SHELL USING FINITE ELEMENT APPROACH

M. BOUAZZA[1, 2], A. LAIREDJ [3], D. OUINAS[4], and A. HAMOUINE[1]

[1] *Department of Civil Engineering, University of Bechar, Bechar 08000, Algeria.*
[2] *Laboratory of Materials and Hydrology (LMH), University of Sidi Bel Abbes, Sidi Bel Abbes 2200, Algeria.*
[3] *Department of Mechanical Engineering, University of of Bechar, Bechar 08000, Algeria.*
[4] *Department of Mechanical Engineering, University of Mostaganem 27000, Algeria.*

Abstract: *Free vibration analysis of doubly curved composite spherical shell panel is investigated using FEM. The frequency–amplitude relations for the free vibrated spherical shell are computed using eigenvalue formulation and are solved employing a direct iterative procedure using Ansys 12; two types of modeling are proposed: Shell99 and Shell281. Effects of moisture and temperature on the vibration characteristics of laminated composite shells are examined.*

Keywords: stacking sequences**,** free vibration, spherical shell, finite element approach.

Introduction

Laminated composite shell panels are being widely used in a great variety of engineering applications, i.e., aeronautical, mechanical, chemical and other industries over the past three decades. The main reasons for this trend are outstanding mechanical properties of composite, such as high strength to weight ratio, excellent corrosion resistance and very good fatigue characteristics. Its ability to allow the structural properties to be tailored according to requirements adds to the versatility of composite for sensitivity application. The structures are very often subjected to large amplitude vibration when deformed and undeformed shapes are substantially different. It is no longer possible to define the state variables with the help of linear strain–displacement relations. The strains are significantly larger than the linear strains, and hence there is a need to investigate the nonlinear free vibration of the shell panels in the framework of the higher order shear deformation theory (HSDT) in the Green–Lagrange sense for more accurate prediction of the nonlinear response of the structures.

Dumir and Bhaskar [1] employed an assumed time-mode approach and orthogonal point collocation method to deduce the steady-state nonlinear response of rectilinearly orthotropic thin rectangular plate exposed to a uniformly distributed harmonic force based on von-Karman type dynamics. Sathyamoorthy [2] analyzed the effects of large amplitude on the flexural vibration extending the classical shallow shell theory for geometrically nonlinear analysis of moderately thick orthotropic shallow shells incorporating the rotary inertia effects and the nonlinearity in von-Karman sense. The equations of motion are derived using the Galerkin method and solved by Runge–Kutta integration procedure.

In the present study, the free vibrations of laminated spherical shells analysed using the finite element method. Two types of modeling are proposed: Shell99 and Shell281. Reduced lamina material properties at elevated moisture concentration and temperature are used in the analysis. Non-dimensional fundamental frequencies are evaluated for cross-ply laminates spherical shells at different moisture concentrations and temperatures for simply-supported boundary conditions. The results obtained are compared with their counterparts in the literature.

3. Formulation

A typical composite laminated spherical shell of radius (R_1=R_2), thickness h is shown in Fig. 1. Coordinates used for the study are ξ_1 axial, ξ_2 circumferential and ξ radial surface metrics.

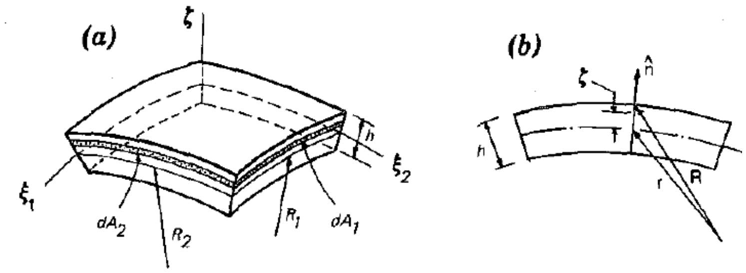

Fig.1. Geometry of laminated composite cylindrical shell, R_1=R_2.

A full shell with different mesh sizes is considered. The shells are discretized into quadrilateral elements. Details of a typical mesh, comprising 16 (4x4) elements, are shown in Fig. 2.

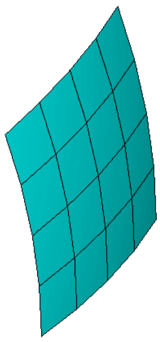

Fig. 2. Discretization with a quadrilateral element of Shell99 and Shell281.

The fig. 3 represents an example of layer stacking of the stratified Shell simulated on the software of calculation of the ANSYS.

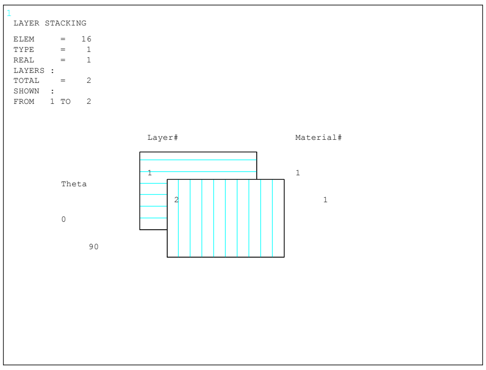

Fig. 3. Laminated spherical shells with two layers

4. Numerical studies

4.1. Validation problems

The un-symmetric simply supported laminated spherical (R_x= R_y= R) shells are considered with different R/a ratios. The following material properties are used in the analysis:

E_1= 25E_2, G_{23}=0.2E_2, G_{13}= G_{12} = 0.5E_2, ν_{12}= 0.25, a/h=100.

Table1. Non-dimensional fundamental frequency $\varpi = \omega a^2 \sqrt{\rho / E_2} / h$ versus radius to side length ratio of cylindrical shell (a/b=1, a/h=100)

R/a	0°/90°			
	a/h=100			
	Reddy [3]	Raja [4]	Shell99	Shell 281
1	125.930	125.956	128.2360	128.8160
2	67.361	67.376	67.3344	67.7623
3	46.002	46.011	45.9621	46.1990
4	35.228	35.235	35.2034	35.3624
5	28.825	28.831	28.8071	28.9290
10	16.706	16.708	22.7420	23.3012

The free vibration frequencies of laminated shells are presented in Table 1. The predicted natural frequencies by the present elements are in good agreement with those reported by Reddy [3] and Raja et al [4]. Simply supported cross-ply laminated spherical shell (a/h=100).

Figures 4-5 display the first five mode shapes of a 0°/90° cylindrical laminated shell (R/a=3) for simply supported of elements Shell 99 and Shell281 respectively, for the 4x4 grid .These graphic displays can be used as a checking for free vibration.

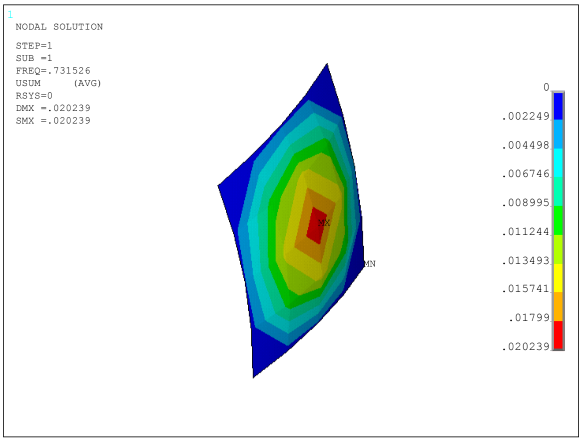

Fig.4. Clean frequencies of first mode, of laminated spherical shell, Shell99 element, (R/a=3).

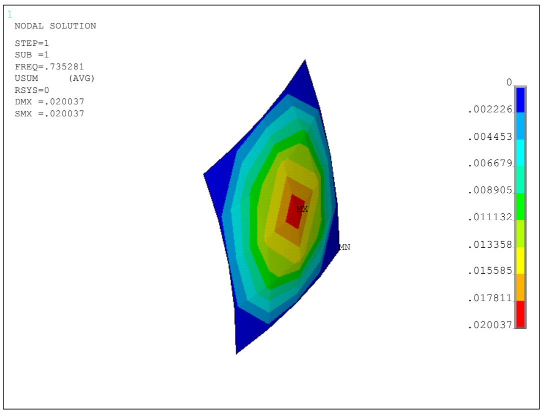

Fig.5. Clean frequencies of first mode, of laminated spherical shell, Shell281 element, (R/a=3).

4.2. Vibration response of multilayered shells taking into account the changing of material properties due to moisture variations

This example studies the effects of moisture concentrations on the material properties and the static response of the simply-supported laminated spherical shells for a uniform moisture distribution across plate thickness. Relationships between moisture concentrations and material properties [5] can be found in Table 2. Fig. 6 shows that the moisture has a small effect on the fundamental frequency**.**

Table 2. Elastic moduli of graphite/epoxy lamina at different moisture concentrations, $G_{xz}=G_{xy}$, $G_{yz}=0.5\ G_{xy}$, $\nu_{xy}=0.3$, $\beta1=0$, $\beta=0.44$

Elastic Moduli (GPa)	Moisture concentration C (%)						
	0	0.25	0.5	0.75	1.0	1.25	1.5
E_x	130	130	130	130	130	130	130
E_y	9.5	9.25	9.0	8.75	8.5	8.5	8.5
G_{xy}	6.0	6.0	6.0	6.0	6.0	6.0	6.0

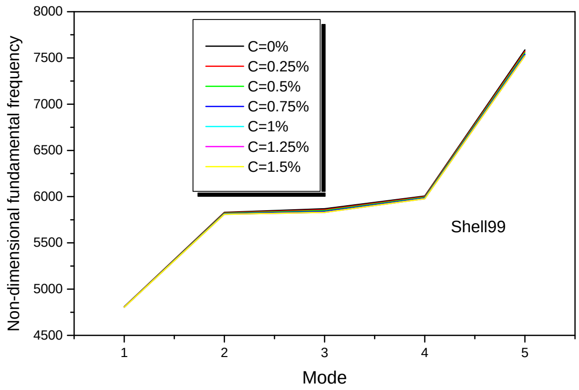

Fig. 6. Effect of moisture on the non-dimensional fundamental frequency, Shell99 element, (R/a=3).

4.3. Vibration response of multilayered shells taking into account the change of material properties due to temperature variations

In many investigations, material properties of composite laminates are assumed to be independent of temperature. However, the elastic moduli of laminates in general degenerate with the elevation of temperature. The change in material properties [5] due to a rise of temperature is given in Table 3. This example will illustrate the effects of temperature on material properties, and hence how internal stresses are affected. As material constants depend on temperature, relationship between the vibration response of laminates.

Table3. Elastic moduli of graphite/epoxy lamina at different temperatures, $G_{xz}=G_{xy}$, $G_{yz}=0.5\ G_{xy}$, $\nu_{xy}=0.3$, $\alpha_1=-0.3\text{x}10^{-6}$ /K, $\alpha_2=-28.1\text{x}10^{-6}$ /K

Elastic moduli (GPa)	Temperature T (K)					
	300	325	350	375	400	425
E_x	130	130	130	130	130	130
E_y	9.5	8. 5	8.0	7.5	7	6.75
G_{xy}	6.0	6.0	5.5	5.0	4.75	4.5

The variation of the Non-dimensional fundamental frequency change of laminated spherical shells according to mode for different values of temperature is presented in Figure 7. In this figure, the temperature is increased, the non-dimensional fundamental frequency decreases. The temperature influences an important manner on the clean frequencies of shells stratified.

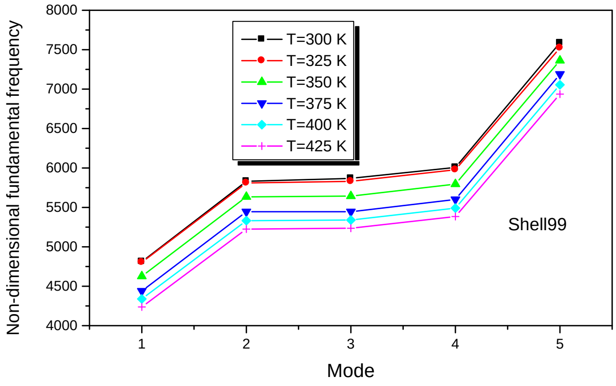

Fig.7. Effect of temperature on the non-dimensional fundamental frequency, Shell99 element, (R/a=3).

5. Conclusions

The free vibration analysis of doubly curved composite spherical shell is investigated using FEM. The frequency–amplitude relations for the free vibrated spherical shell are computed using eigenvalue formulation and are solved employing Ansys. The effects of the moisture and temperature on the non-dimensional fundamental frequency are examined.

Based on the numerical results the following conclusions are drawn.

(a) The finite element method results are in good agreement with those obtained with the theory of Reddy [3] and Raja et al [4].

(b) The frequency ratio is more pronounced when amplitude ratio increases.

(c) The temperature affects the non-dimensional fundamental frequencies.

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