

# NUMERICAL STUDY OF MICROPOLAR FLUID FLOW HEAT AND MASS TRANSFER OVER VERTICAL PLATE: EFFECTS OF THERMAL RADIATION AND MAGNETIC FIELD

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## Abstract

In this paper, we examine the thermal radiation effect on heat and mass transfer in steady laminar boundary layer flow of an incompressible viscous micropolar fluid over a vertical flat plate, with the presence of a magnetic field. Rosseland approximation is applied to describe the radiative heat flux in the energy equation. The resulting similarity equations are solved numerically. Many results are obtained and representative set is displayed graphically to illustrate the influence of the various parameters on different profiles. The conclusion is drawn that the flow field, temperature, concentration and microrotation as well as the skin friction coefficient and the both local Nusselt and Sherwood numbers are significantly influenced by Magnetic parameter, material parameter and thermal radiation parameter.

**Keywords:** Boundary layer, Micropolar fluid, MHD, Heat and Mass transfer, Thermal Radiation.

## Résumé

Dans cet article, nous examinons l'effet de rayonnement thermique sur le transfert de chaleur et de masse dans les flux de la couche limite laminaire stationnaire d'un fluide visqueux incompressible micropolaire sur une plaque plane verticale, avec la présence d'un champ magnétique. Rosseland approximation est appliqué pour décrire le flux de chaleur par rayonnement dans l'équation de l'énergie. Les équations de similitude résultant sont résolues numériquement. De nombreux résultats sont obtenus représentés graphiquement pour illustrer l'influence des différents paramètres sur des profils différents. La conclusion est tirée que le champ d'écoulement, la température, la concentration et micro-rotation ainsi que le coefficient de frottement de la peau et les nombres de Nusselt et deux Sherwood locales sont fortement influencés par le paramètre magnétique, le paramètre de matériau et le paramètre de rayonnement thermique.

**Mots clés:** La couche limite, Fluide micropolaire, MHD, Transfert de masse et de chaleur, Radiation thermique

## ملخص

في هذه المقالة، ندرس تأثير الإشعاع الحراري في نقل الحرارة والكتلة في تدفق الطبقة الحدودية لسائل لزج غير قابل للانضغاط ميكروبلر على لوحة مسطحة عمودية، مع وجود مجال مغناطيسي. يتم تطبيق تقريب روزلند لوصف تدفق الحرارة عن طريق الإشعاع في معادلة الطاقة. يتم حل المعادلات التشابه الناتج عددياً. ويتم الحصول على العديد من النتائج عن طريق منحنيات بيانية لتوضيح تأثير مختلف إعدادات على ملامح مختلفة. الاستنتاج أن حقل التدفق، ودرجة الحرارة، تركيز وسرعة دوران الجزيئات وهكذا معامل الاحتكاك والعديد المحليين لنسبت وشروط تتأثر بشدة بواسطة المعلمة المغناطيسية، المعلمة المادية والمعلمة الإشعاع الحراري.

**الكلمات المفتاحية:** الطبقة الحدودية، سائل ميكروبلر، منيتو هيدرو دينميك، نقل الكتلة والحرارة، الإشعاع الحراري.

## I. INTRODUCTION

The micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can affect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. It has many practical applications, for example, analyzing the behavior of exotic lubricants, colloidal suspensions, and solidification of liquid crystals, extrusion of polymer fluids, cooling of metallic plate in a bath, animal blood, body fluids and so forth. These kinds of the fluids have been proposed and modeled firstly by Erigen [1]. The theory of micropolar fluids takes into account the microscopic effects arising from the local microstructure and intrinsic motion of the fluid elements, and is expected to provide a mathematical model for non-Newtonian fluid behavior. Such fluids can be subjected to surface and body couples in which the material points in a volume element can undergo motions about the center of masses along with deformation. These fluids have been used in boundary layer flow and heat transfer by many authors for different geometries under diverse physical conditions [2-9]. Moreover, the study of magnetic field with presence of thermal radiation effects has important applications in physics and engineering such as plasma studies, MHD power generators cooling of nuclear reactor etc. A. Ishak et al [10] discuss Magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical surface demonstrate that, in the assisting flow case, solutions could be obtained for all positive values of  $\lambda$ , while in the opposing flow case the solution terminated with a saddle-node bifurcation at  $\lambda = \lambda_c$  ( $\lambda_c < 0$ ). The value of  $|\lambda_c|$  increases with an increase in  $K$ , thus micropolar fluid delays the boundary layer separation, which in turn increases the range of similarity solutions compared to Newtonian fluid. Influence of thermophoresis and chemical reaction on MHD micropolar fluid flow with variable fluid properties has been considered by K. Das [11] who found that in presence of different physical effects can be modified the flow, heat and mass transfer. K. Bhattacharyya et al [12] examined Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet, he found the microrotation effect on the flow executes when the material parameter changes. It is observed that due to increase in material parameter the steady flow needs more amount of mass suction. The skin friction coefficient, the couple stress coefficient and the heat transfer coefficient decrease with the material parameter for first solution and increase for second solution. Also, in both solutions the heat transfer coefficient increases with thermal radiation parameter. Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity has investigated numerically by M. A.A. Mahmoud [13] who shown that the thermal boundary thickness and Nusselt increase as the thermal conductivity parameter increases, while the thermal boundary

thickness decreases as the radiation parameter increases. M.M. Rahman, T. Sultana [14] studied numerically the radiative heat transfer flow of micropolar fluid with variable heat flux in a porous medium they demonstrate that the thermal radiation combined with the vortex viscosity can be change the flow and heat transfer transport phenomena in the porous medium. A numerical model is developed to examine the combined effects of Soret and Dufour on mixed convection magnetohydrodynamic heat and mass transfer in micropolar fluid-saturated Darcian porous medium in the presence of thermal radiation, non-uniform heat source/sink and Ohmic dissipation is presented by Dulal Pal, Sewli Chatterjee [15]. The analytical solution is presented for the effect of radiation on flow of a magneto-micropolar fluid past a continuously moving plate with suction and blowing has studied by M.A. Seddeek et al [16]. The steady laminar magnetohydrodynamic (MHD) boundary-layer flow past a wedge with constant surface heat flux immersed in an incompressible micropolar fluid in the presence of a variable magnetic field is investigated by Ioan Pop et al [17] they concluded that Numerical results show the micropolar fluids display drag reduction and consequently reduce the heat transfer rate at the surface, compared to the Newtonian fluids and the opposite trends are observed for the effects of the magnetic field on the fluid flow and heat transfer characteristics. An analysis has been performed for heat and mass transfer with radiation effect of a steady laminar boundary-layer flow of a micropolar flow past a nonlinearly stretching sheet, using the similarity transformation is presented by Kai-Long Hsiao [18]. M. A.A. Mahmoud and S. E. Waheed [19] considered MHD stagnation point flow of a micropolar fluid towards a moving surface with radiation, they were found that the Nusselt number increased as the magnetic parameter, radiation parameter and Prandtl number increase. In addition, an increase in the material parameter caused a decrease in the Nusselt number. M. Abd-El Aziz [20] studied Thermal radiation effects on magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences. The problem of the flow and heat transfer of a micropolar fluid in an axisymmetric stagnation flow on a cylinder with variable properties and suction is discussed by E. M. E. Elbarbary and N. S. Elgazery [21]. The effects of Joule-heating, chemical reaction and thermal radiation on unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid are analyzed by A. J. Chamkha [22]. In the present paper, we consider the effects of thermal radiation on heat and mass transfer hydro-magnetic flow over vertical flat pate in presence micropolar fluid. Based on the literature survey only the papers by N. A. Yacoba and A. Ishak [23]. The resulting similarity solutions of the governing equations are obtained. Many results are obtained and representative set is displayed graphically to illustrate the influence of the

various dimensionless parameters. It is in the main objective to determine the influence of the simultaneous effects on heat and mass transfer from the plate to the external medium.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider a convection–radiation of two-dimensional steady laminar flow, heat and mass transfer of an incompressible viscous micropolar fluid along a semi-infinite permeable plate under the influence of a transverse magnetic field of uniform strength  $B_0$  which is applied normally to the plate. It is assumed that the magnetic Reynolds number is very small and hence the induced magnetic field is negligible, as compared with the applied magnetic field. The wall is maintained at constant temperature and concentration,  $T_w$  and  $C_w$ , respectively, and these values are assumed to be greater than the ambient temperature and concentration,  $T_\infty$  and  $C_\infty$ , respectively. The flow is assumed to be in the x-direction which is taken along the plate and y-axis is normal to it (see Figure 1), under these assumptions and Boussinesq approximation, the governing equations for boundary layer flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$\rho j \left( u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - \kappa \left( 2N + \frac{\partial u}{\partial y} \right) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (5)$$

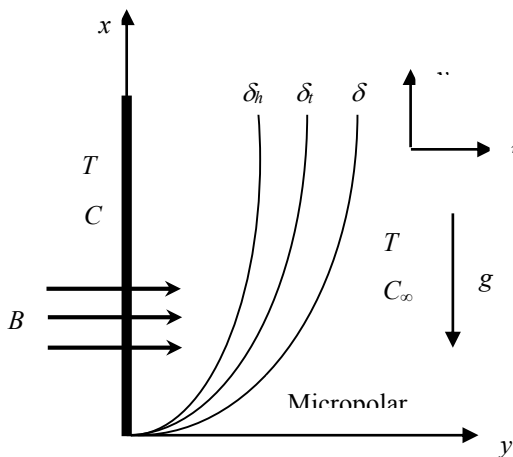


Fig. 1. Physical model and coordinate system of the problem.

Here  $T$  is the temperature,  $C$  is the species concentration in the boundary layer and  $N$  component of microrotation vector normal to x-y plane.  $\kappa$ ,  $\gamma$  are the vortex viscosity and spin gradient viscosity, we follow the work of many recent authors [18, 23] by assuming that  $\gamma = (\mu + \kappa/2)j$ .  $\rho$ ,  $\mu$ ,  $\nu$  and  $\beta$  are the density, dynamic viscosity, kinematic viscosity and volumetric thermal expansion coefficient of the fluid,  $\beta^*$  is the mass expansion coefficient while  $\alpha$  represent molecular thermal diffusivity of the medium and  $j$  represent microinertia density.  $\sigma$  is the electrical conductivity of the fluid,  $B_0$  is the strength of magnetic field, the gravitational acceleration is denoted by  $g$ . The coefficient that appear in Eqs. (5) is the mass diffusivity. Using the Rosseland approximation for radiation, the radiative heat flux is simplified as:

$$q_r = -\frac{4\sigma^*}{3k_1} \frac{\partial T^4}{\partial y} \quad (6)$$

Where  $\sigma^*$  is the Stefan-Boltzmann constant parameter,  $k_1$  is the Mean absorption coefficient. We presume that the temperature variation within the flow is such that  $T^4$  may be expanded in a Taylor's series. Expanding  $T^4$  about  $T_\infty$  and neglecting higher order terms we get,  $T^4 = 4T_\infty^3 T - 3T_\infty^4$ . Now Eq. (4) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k_1 \rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

The corresponding wall and for stream boundary conditions are defined as follows

$$\text{at } y = 0 : u = 0, v = 0, j = 0, T = T_w, C = C_w$$

$$\text{and } N = -\frac{1}{2} \frac{\partial u}{\partial y} \quad (8)$$

$$\text{at } y \rightarrow \infty : u = U, T = T_\infty, C = C_\infty, N = 0$$

We now introduce dimensionless quantities [see, 23] as

$$\eta = \left( \frac{U}{\nu x} \right)^{\frac{1}{2}} y, \psi = (\nu x U)^{\frac{1}{2}} f(\eta),$$

$$N = U \left( \frac{U}{x \nu} \right)^{\frac{1}{2}} h(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Where  $\eta$  is the independent similarity variable,  $f(\eta)$  is the dimensionless stream function,  $h(\eta)$  is the dimensionless microrotation,  $\theta(\eta)$  is the dimensionless temperature and  $\phi(\eta)$  is the dimensionless concentration. Further,  $\psi$  is the stream function which is defined in the usual way as

$$u = \partial \psi / \partial y, v = -\partial \psi / \partial x$$

The velocity components become

$$u = Uf'(\eta) \quad \text{and} \quad v = \frac{I}{2} \left( \frac{\nu U}{x} \right)^{1/2} (\eta f' - f)$$

Using the dimensionless quantities, the equations (1-7) with the boundary conditions (8) can be further reduced to a set of ordinary differential equations for which numerical solutions are more easily determined

$$(1+K)f''' + \frac{1}{2}ff'' - \frac{Ha^2}{Re}f' + Kh' + Gr_x\theta' + Gm_x\phi = 0 \quad (9)$$

$$\left(1 + \frac{K}{2}\right)h'' - K(2h + f'') + \frac{1}{2}(hf)' = 0 \quad (10)$$

$$\theta'' \left(1 + \frac{4}{3Rt}\right) + \frac{1}{2}Prf\theta' = 0 \quad (11)$$

$$\phi'' + \frac{1}{2}Scf\phi' = 0 \quad (12)$$

The boundary conditions become

$$y = 0, \eta = 0 : f(0) = 0, f'(0) = 0, h(0) = -\frac{1}{2}f''(0), \theta(0) = 1, \phi(0) = 1$$

$$y \rightarrow \infty, \eta \rightarrow \infty : f'(\infty) = 1, h(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0$$

(13)

Where the various parameters are defined by

$$Gr_x = \beta g(T_w - T_\infty)^2 x / U^2 \quad (14)$$

$$Gm_x = \beta^* g(C_w - C_\infty)^2 x / U^2 \quad (15)$$

$$K = \kappa / \mu \quad (16)$$

$$R = \frac{3Rt}{4 + 3Rt} \quad \text{With} \quad Rt = \frac{k_1 k_2}{4\sigma^* T_\infty^3} \quad (17)$$

$$Ha = B_0 x \sqrt{\sigma / \mu} \quad (18)$$

$$Re = \frac{\rho U x}{\mu} \quad (19)$$

$$Pr = \nu / \alpha \quad (20)$$

$$Sc = D / \nu \quad (21)$$

Here  $Gr_x$ ,  $Gm_x$ ,  $K$ ,  $R$ ,  $Ha$ ,  $Re$ ,  $Pr$  denote the local thermal Grashof number, local mass Grashof number, the material parameter, the radiation parameter, the Hartmann number, the magnetic Reynolds number and the Prandtl number respectively, while  $Sc$  is the Schmidt number. The physical quantities of interest are the skin friction coefficient  $C_f$ , the local Nusselt number  $Nu_x$  and the local Sherwood

number  $Sh_x$ , which are defined as

$$C_f = \frac{2\tau_w}{\rho U^2}, \quad Nu_x = \frac{xq_w}{k_2(T_w - T_\infty)},$$

$$Sh_x = \frac{xq'_w}{D(C_w - C_\infty)}$$

Where the surface shear stress  $\tau_w$ , the surface heat flux  $q_w$  and the surface mass flux  $q'_w$  are given by

$$\tau_w = \left[ (\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0},$$

$$q_w = -k_2 \left( \frac{\partial T}{\partial y} \right)_{y=0} - \frac{4\sigma^*}{3k_1} \left( \frac{\partial T^4}{\partial y} \right)_{y=0},$$

$$q'_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0}$$

Using the dimensionless variables, we obtain

$$C_f Re^{1/2} = (2 + K)f''(0),$$

$$\frac{Nu_x}{Re^{1/2}} = -\left(1 + \frac{4}{3Rt}\right)\theta'(0), \quad \frac{Sh_x}{Re^{1/2}} = -\phi'(0)$$

### III. RESULTS AND DISCUSSION

The set of the coupled ordinary differential Eqs (9) – (12) is highly nonlinear and cannot be solved analytically. Together with the boundary conditions (13), they form a two point boundary value problem (BVP) which can be solved using the routine `bvp4c` of the symbolic computer algebra software MATLAB, this routine is based on the finite differences method that implements the 3-stage Lobatto collocation formula and the collocation polynomial provides a continuous solution that is fourth-order accurate uniformly in the interval of integration. Mesh selection and error control are based on the residual of the continuous solution. The collocation technique uses a mesh of points to divide the interval of integration into subintervals. The flow regions is controlled by thermophysical parameters, namely  $Gr_x$ ,  $Gm_x$ ,  $K$ ,  $R$ ,  $Ha$ ,  $Re$ ,  $Pr$  and  $Sc$ . Numerical computations are carried out for different values of the parameters shown in all figures. Preliminary calculations are conducted to check the numerical results. We focus on the velocity, temperature, mass profiles and angular velocity, mainly for their relatively changes under the magnetic field and results are presented for a reasonable range of magnetic parameter and material parameter. It is interesting to show the influence of all the control parameters on the velocity, temperature, mass and angular velocity profiles respectively.

#### A. Effect of combined magnetic parameter $Ha$ and material parameter $K$

Figure 2 shown the influence of the magnetic field under Hartmann number  $Ha$  and report of vortex viscosity and dynamic viscosity which represent by material parameter  $K$ , on the behaviours of the dimensionless velocity along

the vertical plate with similarity variable  $\eta$ . From this figure we notes that the velocity decreases with increasing of the both of two parameters near the wall. Thus due to the increase of material parameter the hydrodynamic boundary layer thickness increase. Figure 3 illustrate the variation of the temperature fields versus the similarity variable  $\eta$ , we observes that the temperature increases with increasing of  $Ha$  and  $K$ . therefore the increasing of the thermal boundary layer thickness. The distributions of the microrotation of particles (angular velocity) with  $\eta$  are presented in figure 4, we remarks that there is an inflexion point in  $\eta = 3.0$  and  $\eta = 4.0$  for  $K = 1.0$  and  $4.0$  respectively. Before theses points the microrotation increases with increasing of  $Ha$  near the plate, after these points, the Hartemann number affect slightly on the angular velocity far from the wall. Figure 5 represent the variation of concentration with variable  $\eta$ , from this figure, we observe that the increasing of the both parameters leads to increasing of concentration profiles. We can conclude that the presence of magnetic field and micro constituents in rotation can be seen as strong tools to regulate the flow, heat and mass transfer.

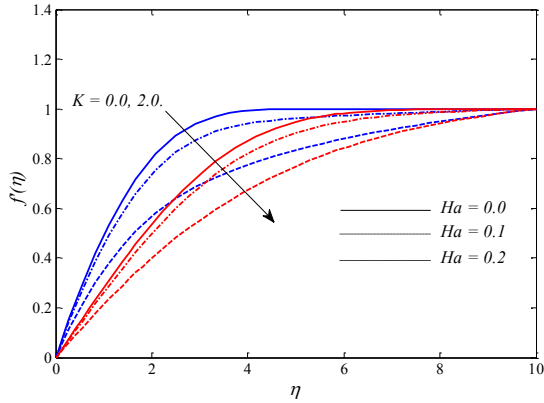


Fig. 2. Velocity profiles  $f'(\eta)$  for ( $Gr_x=0.1$ ,  $Gm_x=0.1$ ,  $R=0.5$ ,  $Pr=0.7$ ,  $R=2.0$  and  $Sc=0.6$ )

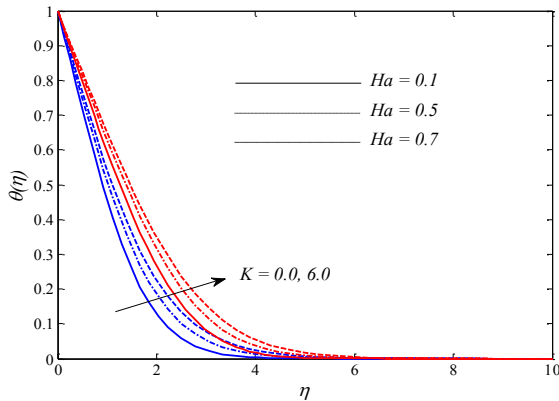


Fig. 3. Temperature distribution  $\theta(\eta)$  for ( $Gr_x = 3.0$ ,  $Gm_x = 3.0$ ,  $Re = 0.5$ ,  $Pr = 0.71$ ,  $R = 1.0$  and  $Sc = 0.6$ )

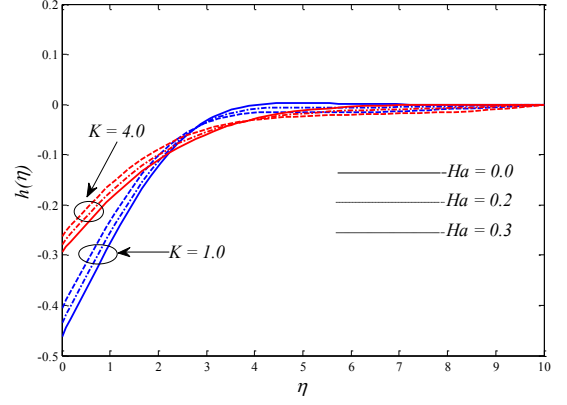


Fig. 4. Microrotation profiles  $h(\eta)$  for ( $Gr_x=0.5$ ,  $Gm_x=0.5$ ,  $Re=1.0$ ,  $Pr=5.0$ ,  $R=1.0$  and  $Sc=0.6$ )

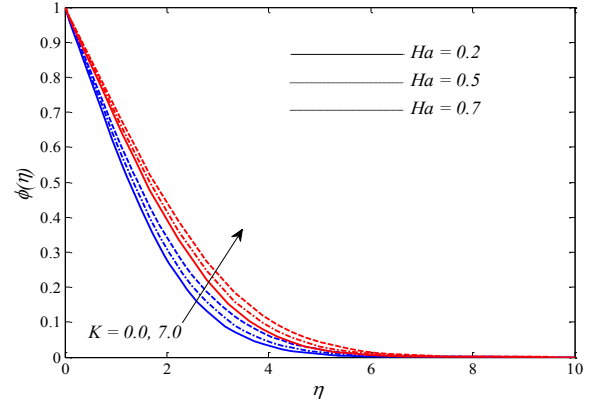
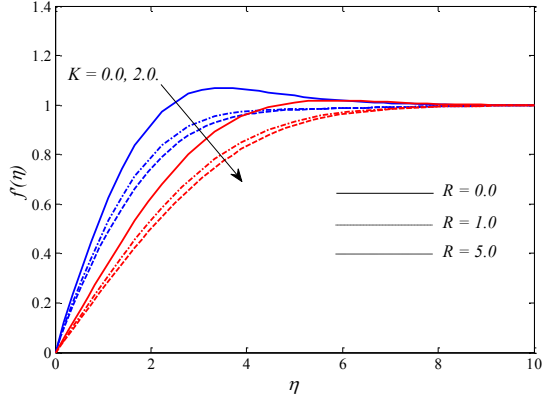
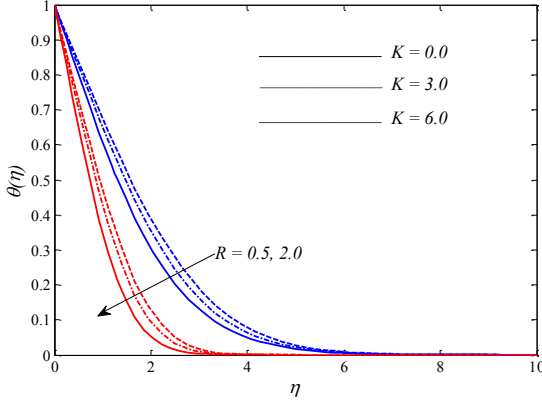
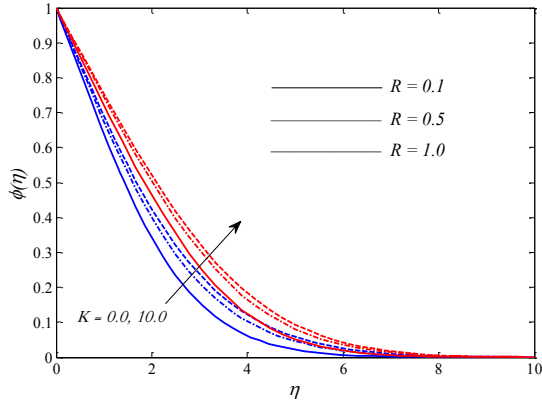
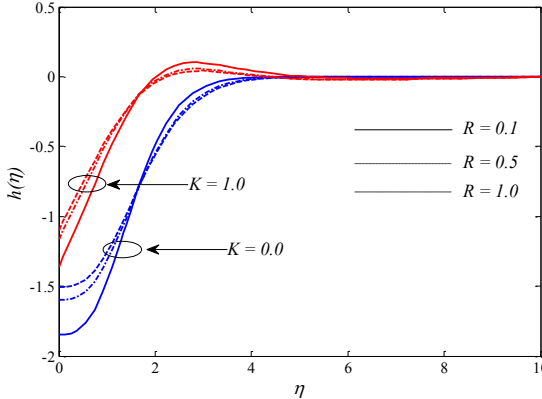


Fig. 5. Concentration profiles  $\phi(\eta)$  for ( $Gr_x = 2.0$ ,  $Gm_x = 2.0$ ,  $Re = 1.0$ ,  $Pr = 0.7$ ,  $R = 0.1$  and  $Sc = 0.3$ )

## B. Effect of coupled radiation parameter R and material parameter K

Figures (6-9) represent the variation of the dimensionless behaviours of fluid velocity, temperature, mass and microrotation velocity, respectively, in opposition to variable  $\eta$  with varying the thermal radiation parameter  $R$  and the material parameter  $K$ . From figure 6, we notes that if the both of radiation parameter and material parameter increase the fluid velocity profiles decrease. in addition, it is seen that the temperature increase with increasing of the material parameter and decrease if the radiation parameter increase, this leads to reduce the boundary layer thickness as illustrate in figure 7. Concerning the concentration fields, we remark that this last increase with increasing of the both of radiation and material parameters as shown in figure 8. The behaviours of the microrotation is demonstrate in figure 9, we remarks that there is an inflexion points in near off  $\eta = 2.0$  for  $K = 0.0$  and  $1.0$  respectively. Before theses points the microrotation increases with increasing of  $R$  and  $K$  near the plate, moreover, the radiation flux affect faintly on extreme from the wall. It is seen that all of this figures satisfy the boundary conditions (13), thus support the validity of the present results. We can conclude that the presence of radiation flux and micro constituents in rotation can be seen as physically powerful tools to regulate the flow, heat and mass transfer.


 Fig. 6. Velocity profiles  $f'(\eta)$  for ( $Gr_x = 0.1$ ,  $Gm_x = 0.1$ ,  $Re = 1.0$ ,  $Pr = 0.7$ ,  $Ha = 0.1$  and  $Sc = 0.6$ )

 Fig. 7. Temperature distribution  $\theta(\eta)$  for ( $Gr_x = 3.0$ ,  $Gm_x = 3.0$ ,  $Re = 0.5$ ,  $Pr = 0.71$ ,  $Ha = 0.5$  and  $Sc = 0.3$ )

 Fig. 8. Concentration profiles  $\phi(\eta)$  for ( $Gr_x = 2.0$ ,  $Gm_x = 2.0$ ,  $Re = 1.0$ ,  $Pr = 0.7$ ,  $Ha = 0.7$  and  $Sc = 0.3$ )

 Fig. 9. Microrotation profiles  $h(\eta)$  for

( $Gr_x = 2.0$ ,  $Gm_x = 2.0$ ,  $Re = 0.5$ ,  $Pr = 2.0$ ,  $Ha = 0.3$  and  $Sc = 0.6$ )

### C. Variations of Skin friction coefficient, Nusselt and Sherwood number

Now, we are concern to the variations of Nusselt and Sherwood number for different values of Hartmann parameter  $Ha$ , the material parameter  $K$ , the parameter of thermal radiation  $R$  and the both local Grashof number that are illustrated in the table I and II. It can be seen that the magnetic field and radiation flux can reduce the heat and mass transfer rate on the other hand the increasing in locale thermal Grashof number enhance the heat transfer in addition there is an enhancement of the local Sherwood number as the local mass Grashof number increase. However, the both of Nusselt and Sherwood number decrease when the material parameter increase. Moreover, we remark that the same results for the skin friction coefficient for changing the Hartmann number, radiation parameter and the both local Grashof. However compared to the variation of material parameter as illustrate in table III.

TABLE I: Values of  $Nu_x/Re^{1/2}$  for selected values of  $Ha$ ,  $Gr_x$ ,  $R$  and  $K$  with ( $Gm_x = 4.0$ ,  $Re = 1.0$ ,  $Pr = 0.71$ ,  $Sc = 0.3$ ).

$Ha$	$Gr_x$	$K = 0.0$			$K = 3.0$		
		$R = 0.1$	$R = 1.0$	$R = 2.0$	$R = 0.1$	$R = 1.0$	$R = 2.0$
0.2	0.1	1.8870	0.5672	0.3785	1.7640	0.4819	0.3160
	0.5	1.9470	0.5755	0.3830	1.8170	0.4891	0.3197
	1.0	2.0160	0.5854	0.3883	1.8780	0.4975	0.3241
	2.0	2.1340	0.6036	0.3982	1.9840	0.5131	0.3325
0.5	0.1	1.7700	0.5369	0.3604	1.6710	0.4590	0.3024
	0.5	1.8320	0.5459	0.3652	1.7250	0.4667	0.3064
	1.0	1.9040	0.5565	0.3709	1.7860	0.4757	0.3111
	2.0	2.0280	0.5760	0.3816	1.8950	0.4922	0.3200

TABLE II: Values of  $Sh_x/Re^{1/2}$  for selected values of  $Ha$ ,  $Gm_x$ ,  $R$  and  $K$  with ( $Gr_x = 01.0$ ,  $Re = 0.5$ ,  $Pr = 0.7$ ,  $Sc = 0.6$ ).

$Ha$	$Gm_x$	$K = 0.0$			$K = 2.0$		
		$R = 0.1$	$R = 1.0$	$R = 2.0$	$R = 0.1$	$R = 1.0$	$R = 2.0$
0.1	0.1	0.4411	0.3777	0.3593	0.3917	0.3376	0.3222
	0.5	0.4566	0.4002	0.3853	0.4055	0.3576	0.3450
	1.0	0.4735	0.4233	0.4110	0.4206	0.3781	0.3678
	2.0	0.5019	0.4598	0.4504	0.4461	0.4107	0.4028
0.2	0.1	0.4276	0.3616	0.3415	0.3808	0.3250	0.3084
	0.5	0.4440	0.3858	0.3696	0.3953	0.3462	0.3328
	1.0	0.4618	0.4102	0.3971	0.4111	0.3677	0.3568
	2.0	0.4914	0.4484	0.4385	0.4376	0.4016	0.3934

TABLE III: Values of  $C_f Re^{1/2}$  for selected values of  $Ha$ ,  $Gr_x$ ,  $R$  and  $K$  with ( $Gm_x = 4.0$ ,  $Re = 02.0$ ,  $Pr = 0.70$ ,  $Sc = 0.3$ ).

$Ha$	$Gr_x$	$K = 0.0$			$K = 2.0$		
		$R = 0.1$	$R = 1.0$	$R = 2.0$	$R = 0.1$	$R = 1.0$	$R = 2.0$
0.1	0.1	8.408	8.330	4.155	10.261	10.156	10.132
	0.5	9.186	8.801	8.707	11.221	10.705	10.586
	1.0	10.132	9.378	9.194	12.388	11.375	11.142
	2.0	11.948	10.497	10.142	14.624	12.673	12.224
0.2	0.1	8.344	8.266	8.247	10.184	10.080	10.056
	0.5	9.119	8.737	8.644	11.141	10.628	10.510
	1.0	10.062	9.314	9.131	12.304	11.299	11.067
	2.0	11.873	10.432	10.078	14.534	12.596	13.037

## CONCLUSION

The effects of magnetic field and thermal radiation on steady laminar boundary layer flow of an compressible electrically conducting micropolar fluid over a permeable vertical plate is studied. Using the similarity transformation, the governing system of non linear partial differential equations are transformed into non linear ordinary differential equations and are solved numerically using the routine bvp4c of the symbolic computer algebra software MATLAB. Numerical results are presented to illustrate the details of the flow, heat and mass transfer characteristics and their dependence physical parameters. We can conclude the following results from our investigation:

- 1) Increasing the magnetic parameter leads to acceleration of the angular velocity, increase the both of dimensionless temperature and concentration, but the effect is reverse for the flow velocity.
- 2) The species concentration and microrotation profiles increase near the plate for increasing the thermal radiation parameter, however the flow velocity and temperature profiles decrease.
- 3) Increasing the material parameter leads to improve the temperature, angular velocity and species mass profiles which then enhance the both of heat and mass boundary layer thickness, but the effect is reverse for the flow velocity.
- 4) It is seen that the skin friction coefficient decrease with increasing of magnetic, thermal radiation parameters, while it increase for increase the material parameter.
- 5) The heat and mass transfer rate decrease for increasing of magnetic parameter, thermal radiation and material parameter.

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