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ALGORITHM FOR THE MIN-MAX PROBLEM OF A OPTIMAL CONTROL

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Abstract

In this paper we consider an optimal control non linear problem with bounded constraints on reachability set. The interest of this problem is the min-max model who has several practical applications and the method of resolution .The first generalizations of this method to the min-max problem in linear programming and in optimal control are presented in the results [6-10]. The method proposed [1-3] in this paper is a generalisation to the problem with a bounded goal set.

***Keywords**: Optimal control, support-control, ε-Optimal.*

Résumé

Un problème non linéaire de contrôle optimal avec des contraintes bornées sur l'ensemble d'accessibilité a été considéré. L'intérêt du problème réside en premier lieu du modèle du min-max qui a plusieurs applications pratiques et de la méthode de résolution adaptée de la méthode du simplexe. Celle-ci permet le démarrage de l'itération à partir d'un point intérieur et permet l'obtention d'une solution approchée. La méthode proposée est basée sur la méthode adaptée [1-3] appliquée à plusieurs problèmes de programmation linéaire et de contrôle optimal [4-7]. Les premières généralisations de cette méthode au problème du min-max sont dans les résultats [8-10]. Ici, on a fait une généralisation au problème avec un ensemble d'accessibilité borné et le maintien de la spécificité du min-max, car autrement on aurait un nombre élevé de contraintes.

***Mots clés**: Contrôle optimal, contrôle-support, ε-Optimal.*

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ملخص

إشكالية غير خطية للتجكم الآمثل ذات متطلبات مستمرة على مجموعة الحلول الممكنة أخذت بعين الاعتبار.

أهمية الإشكالية تمكن أولا في الطابع الأدنى – الأقصى الذي له عدة تطبيقات عملية وطريقة الحل المطبقة متبنية من طريق Simplexe، هذه الأخيرة تسمح ببداية طريقة التحليل ابتداء من نقطة داخلية وتسمح بالحصول على حل مقرب الطريقة المقترحة تعتمد على الطريقة المتبنية (1-3) المطبقة على عدة إشكاليات البرامج الخطية والتحكم الأمثال، التعميمات الأولى لهذه الطريقة على إشكاليات أقصى – أدنى موجودة في النتائج (6-10) هنا قمنا بتعميم إلى إشكالية ذات مجموعة Accessibilité محدودة و إبقاء خصوصية أدنى- أقصى وألا سيكون لدينا عدة مرتفع من المعادلات.

***الكلمات المفتاحية:** تحكم الآمثل، الطابع الأدنى – الأقصى، ε-optimal.*

1. INTRODUCTION

A mini-max problem of optimal control with bounded constraint is considered. After having given the essential definitions, we constructed the criteria of optimality and the iterration of algorithm which is based on the concept of so-called support control.

2. PROBLEM FORMULATION

In the class of piecewise continues functions $U = \{u(t), t\in T=[0,t^*]\}$, we consider the problem

$$J(u) = \min_{k\in K}(c_k'.x(t_*) + \alpha_k) \to \max_{u\in U} \quad (1)$$

$$\dot{x} = Ax + bu \ , x(0) = x_0 \ , \quad (2)$$

$$g_1 \le H.x(t_*) \le g_2 \ , U = \{u(t): d_1 \le u(t) \le d_2\}, \ t \in T=[0,t^*] \ . \quad (3)$$

where $x(t) = (x_j, j\in J)$ is a n-vector representing the state of the system at the moment t, x_0 the initial position, $u(t)$ the control at the moment t (input signal) limited by the scalars d_1 , d_2 ; $Hx(t^*)$ is the out put signal limited by the scalars m-vectors $g_1 = g_1(I)$, $g_2 = g_2(I)$, A , H are $n\times n$, $m\times n$-constant matrix, *ban*-vector, c_k' , $k\in K$, constant vector representing the cost; α_k , $k\in K$, are scalars; $I = \{1,2,...,m\}$, $J = \{1,2,...,n\}$ the set of indices; $K = \{1,2,.....\rho\}$ the set of indices of the quality criterion components $J(u)$. C' is the transpose of C.

Let's solve the equation (2) by the method of Cauchy, we get:

$$x(t) = F(t).x_0 + \int_0^t F(s)F^{-1}(t_*)bu(s)ds \ , \ t \in T; \quad (4)$$

where $F(t) = \exp(A.t)$, $t \in [0, t^*]$ is the fundamental matrix, solution of the homogeneous system: $dF(t)/dt = AF(t)$, $F(0)=Id$. Replacing the solution $x(t)$ given by (4) in the problem (1)-(3), let's get the following equivalent problem :

$$J(u) = \min_{k\in K}(c'_k F(t_*)x_0 + \int_0^{t_*} c_k(t)u(t)dt + \alpha_k)dt \to \max_{u\in U}, \quad (5)$$

$$g_1 - HF(t_*)x_0 \le \int_0^{t_*} P(t)u(t)dt \le g_2 - HF(t_*)x_0, \quad (6)$$

$$d_1 \le u(t) \le d_2,\ t\in T = [0, t^*] \quad (7)$$

where $c'_k(t) = c'_k F(t_*)F^{-1}(t)b, k\in K$;$P(t)=HF(t_*)F^{-1}(t)b$, $t \in T=[0, t_*]$.

Definition 2.1

The control u will be admissible if u and $x(t)$, $t\in T$ satisfy constraints (2) -(3).

The admissible control u will be an optimal control if $J(u^0) = \max J(u)$, and u_ε is ε-optimal control if $J(u_\varepsilon)-J(u^0) \le \varepsilon$.

In the set I, we choose a subset $I_c = \{i_1, i_2, ..., i_r\}$, with $|I_c| = r$; and in the set T, we choose the subset $T_c = \{t_j, j = 1,...,r\}$ formed of its isolated moments.

Let the matrix $P_c = P(I_c, T_c) = (P_i(t), i\in I_c, t\in T_c)$, where $P_i(t)$ is the i^{th} element of the vector function $P(t) = Hq(t)$, here $q(t)$, $t\in T$ is the solution of the system : $\dot{q} = -Aq$, $q(t_*)=b$.

Definition 2.2

The set $Q_c = \{I_c, T_c\}$ is called the constraint support if $\det P_c \ne 0$.

Suppose Q_c is the constraint support. The vector of potential and the vector of estimation functions will be constructed as follows:

$$y'_k(I_c) = (c'_k q(t), t\in T_c)P_c^{-1}, y'_k(I\ ) = 0; I_h = I - I_c; k\in K; \quad (8)$$

$$\Delta_k(t) = y'_k(t)P(t) - c'_k q(t), t\in T, k\in K. \quad (9)$$

Let us form the following sets

$$T_f = \{t_j \in T - T_c\}, j = r+1,...,s \text{ and } K_f \subset K,$$

with $|K_f| = |T_f| + 1$.

By using the following sets T_f and K_f, we formed the matrix $\Delta_f = (\Delta(K_f, T_f); e(K_f))$; where

$(\Delta(K_f, T_f) = (\Delta_k(t);\quad k\in K_f, t\in T_f)$;

$e(K_f) = (e_k = 1, k\in K_f)$

Definition 2.3

The set $Q_f = \{T_f, K_f\}$ is called the functional support if $\det \Delta_f \neq 0$.

By using the last row of matrix Δ_f^{-1}, we build the vector $\lambda' = (\lambda'[K_f], \lambda'[K_H])$

$$\lambda'[K_f] = (0'[T_f], 1)\Delta_f^{-1},\ \lambda'[K_H] = 0'[K_H], \quad (10)$$

where $0'[T_f]$ and $0'[K_H]$ are null row vectors, $K_H = K/K_H$.

The functional support Q_f is said to be regular if $\lambda[K_f] \ge 0$ [2].

From now on only regular support is considered.

Definition 2.4

The set $Q_p = \{Q_c, Q_f\}$ formed by the constraints support and by the functional support is called the problem support.

Definition 2.5

The pair $\{u, Q_p\}$ formed by an admissible command and the problem support is called the command support.

3. IMPROVEMENT OF THE QUALITY CRITERION

Suppose $\{u, Q_p\}$ the command support of problem (1)-(3), and consider another admissible command $\bar{u}(t) = u(t) + \Delta u(t)$, $t\in T$ and the corresponding trajectory $\bar{x}(t) = x(t) + \Delta x(t)$, $t \in T$.

Let us build the following vectors

$W[K] = (\omega_k, k\in K)$ and $\overline{W}[K] = (\bar{\omega}_k, k\in K)$,

where: $\omega_k = c'_k x(t^*) + \alpha_k - J(u)$, $k\in K$; $\bar{\omega}_k = c'_k \bar{x}(t^*) + \alpha_k - J(\bar{u})$, $k\in K$. (11)

$\Delta W[K] = (\Delta\omega_k, k\in K)$

with $\Delta\bar{\omega}_k = \bar{\omega}_k - \omega_k = C'_k\Delta x(t^*) - J(u)$, $k\in K$.

By construction $\Delta\omega_k \ge -\omega_k$, $k\in K$. By using the equation

$$\Delta x(t^*) = \int_0^{t^*} F(t^*)F^{-1}(t)b\Delta u(t)dt \quad \text{given by [4],}$$

we obtain:

$$\Delta J(u) = J(\bar{u}) - J(u) = -\int_0^{t^*}\Delta(t)\Delta u(t)dt + \sum_{i\in I_c}\nu_i\upsilon_i - \sum\lambda_k\Delta\omega_k, \quad (12)$$

where $\nu(I) = (\nu_i, i\in I) = \sum_{k\in K}\lambda_k y_k(I)$;

$\upsilon(I) = (\upsilon_i, i\in I) = H\Delta x(t^*)$;

$\Delta(t) = \lambda_k\Delta_k(t), t\in T$.

The maximum of this increase (12) under the constraints:

$$d_1 - u(t) \le \Delta u(t) \le d_2 - u(t), t\in T,$$
$$g_{1i} - H(i,J)x(t^*) \le \upsilon_i \le g_{2i} - H(i,J)x(t^*), i\in I_c, \quad (13)$$
$$\Delta\omega_k \ge -\omega_k, k\in K,$$

is equal to:

$\beta(u, Q_p) =$

$$\int_{T^+}\Delta(t)(u(t) - d_1)dt + \int_{T^-}\Delta(t)(u(t) - d_2)dt$$

$$+\sum_{k\in K_f}\lambda_k\omega_k+\sum_{\nu_i\prec}\nu_i\upsilon_{1i}+\sum_{\succ}\nu_i\upsilon_{2i}$$

$$=\int_0^{t^*}\varepsilon(t)\,dt+\varepsilon_1+\varepsilon_2. \qquad (14)$$

called suboptimum value of the command-support $\{u\ ,\ Q_p\}$,where

$T^{+}=\{t\in T/\Delta(t)\succ$ $\prec$

$\upsilon_1(I)=(\upsilon_{1i},i\in I)=g_1(I)-Hx(t^*);$

$\upsilon_2(I)=(\upsilon_{2i},i\in I)=g_2(I)-Hx(t^*).$

4. ε-MAXIMUM PRINCIPLE

Consider the trajectory $x(t),t\in T$ of the direct system (2) and the function $\psi\ (t)$ solution of the conjugate system:

$$\begin{cases}\dot{\psi} \\ \psi(t^*)=\sum_{k\in K_f}\lambda_k\left(H'y_k-c_k\right).\end{cases}$$

Let $H(x,\psi,u)=\psi\,'(Ax+bu)$ be the Hamiltonian function.

Theorem 1

The command support $\{u,Q_p\}$ is ε-optimal if the following relations are satisfied:

$$H(x(t),\psi(t),u(t))=\max_{d_1\le u\le d_2}H(x(t),\psi(t),u(t))-\varepsilon(t),t\in T,$$

$$\nu'Hx(t^*)=\max_{g_1\le Z\le g_2}\nu'Z-\varepsilon_1,$$

$$\lambda'\omega=-\max_{\overline{\omega}\ge 0}\left(-\lambda\overline{\omega}\right)+\varepsilon_2, \qquad (15)$$

$$\int_0^{t^*}\varepsilon(t)\,dt+\varepsilon_1+\varepsilon_2\le\varepsilon.$$

Sufficient condition

We assume that the condition (15) is verified. By using the function $\Delta(t)$, $t\in T$, written under another form $\Delta(t)=-\psi'(t)b$, $t\in T$, the suboptimality-value (14) of support control $\{u,Q_p\}$ becomes:

$$\beta(u,Q_p)=$$

$$-\int_{T^+}\Delta(t)(d_1-u(t))\,dt-\int_{T^-}\Delta(t)(d_2-u(t))dt \quad +$$

$$\sum_{\nu_i\prec}\nu_i\upsilon_{1i}+\sum_{\succ}\nu_i\upsilon_{2i}+\sum_{k\in K_f}\lambda_k\omega_k$$

$$=\int_{T^+}\psi'(t)b(d_1-u(t))\,dt-\int_{T^-}\psi'(t)b(d_2-u(t))dt$$

$$+\sum_{\nu_i\prec}\nu_i\left(g_{1i}-H(i,I)x(t^*)\right)$$

$$+\sum_{\nu_i\succ}\nu_i\left(g_{2i}-H(i,I)x(t^*)\right)+\sum_{k\in K_f}\left(\lambda_k\omega_k-\lambda_k.0\right)$$

$$=\int_0^{t^*}\left[\max_{d_1\le u\le d_2}H(x(t),\psi(t),u)-H(x(t),\psi(t),u(t))\right]dt$$

$$+\max_{g_1\le Z\le g_2}\nu'Z-\nu'Hx(t^*)+\max_{\overline{\omega}\ge 0}\left(-\lambda'\overline{\omega}\right)$$

From this we obtain:

$$\beta(u,Q_p)=\int_0^{t^*}\varepsilon(t)\,dt+\varepsilon_1+\varepsilon_2\le\varepsilon; \qquad (16)$$

This implies that the command $u(t)$, $t\in T$ is ε-optimal.

Necessary condition

Let $u(t)$, $t\in T$, be the ε-optimal command, and we calculate the value of suboptimality

$$\beta(u,Q_p)=-\int_{T^+}\Delta(t)(d_1-u(t))\,dt-\int_{T^-}\Delta(t)(d_2-u(t))dt$$

$$+\sum_{\nu_i\prec}\nu_i\upsilon_{1i}+\sum_{\succ}\nu_i\upsilon_{2i}+\sum_{k\in K_f}\lambda_k\omega_k \qquad (17)$$

We introduce the adjoin problem of the problem (1)-(3):

$$J^*(\xi)=J^*(\lambda,\nu_1,\nu_2,f_1,f_2)=$$

$$\lambda'(K)\alpha(K)-\nu_1'(I)g_1(I)+\nu_2'(I)g_2(I)+\psi'(0)x_0$$

$$-\int_0^{t^*}f_1(t)\,d_1dt+\int_0^{t^*}f_2(t)\,d_2dt\to\min$$

$$\begin{aligned}&\dot{\psi} \qquad \psi(t^*)=\lambda'(K)C(K)-\nu'H;\\ &-\psi'(t)b-f_1(t)+f_2(t)=0,t\in T;\\ &\nu+\nu_1-\nu_2=0;e'(K)\lambda(K)=1;\\ &f_1(t),f_2(t)\ge 0,t\in T;\lambda(K)\ge 0,\nu_1,\nu_2\ge 0\end{aligned} \qquad (18)$$

Let $\xi=(\lambda,\nu_1,\nu_2,f_{1,},f_2)$ the set where ν, ν_1, ν_2 are m-vectors, $\lambda(K)$, and the functions $f_1(t)$, $f_2(t)$, $t\in T$, defined as follows:

$$\lambda(K)=\left(\lambda(K_f),\lambda(K/K_f)=0\right),\nu(I)=\sum_{k\in K}\lambda_k y_k;$$

$$\begin{aligned}&f_1(t)=\Delta(t),f_2(t)=0,\ \text{if}\ \Delta(t)\ge 0;\\ &f_1(t)=0,f_2(t)=-\Delta(t),\ \text{if}\ \Delta(t)\prec\end{aligned} \qquad (19)$$

$$\nu_{1i}=0,\nu_{2i}=\nu_i\ \ \text{if}\ \nu_i\ge 0;\nu_{1i}=-\nu_i,\nu_{2i}=0\ \ \text{if}\ \nu_i\prec .$$

Is the admissible solution of the adjoin problem (18). Let $\xi^\circ=\left(\lambda^\circ,\nu^\circ{}_1,\nu^\circ{}_2,f^\circ{}_{1,},f^\circ{}_2\right)$ the optimal solution of the problem (18). From the value of the suboptimality (17) and by due to the relations (9), (10) and (19) we obtain:

$$\beta(u,Q_p)=\nu'\int_0^{t^*}P(t)u(t)\,dt-\sum_{k\in K}\lambda_k\int_0^{t^*}c_k'q(t)u(t)\,dt$$

$$-\int_0^{t^*}f_1(t)\,d_1dt+\int_0^{t^*}f_2(t)\,d_2dt-\nu'Hx(t^*)+\nu_2'(I)g_2(I)$$

$$-\nu_1'(I)g_1(I)-J(u)+\sum_{k\in K}\lambda_k\left(c_k'x(t^*)+\alpha_k\right)$$

$$=J(u^\circ)-J(u)+\lambda'(K)\alpha(K)+\nu_2'(I)g_2(I)$$

$$-\nu_1'(I)g_1(I)+\psi'(0)x_0-\int_0^{t^*} f_1(t)d_1dt+\int_0^{t^*} f_2(t)d_2dt$$
$$-\lambda'^{\circ}(K)\alpha(K)-\nu_2'^{\circ}(I)g_2(I)+\nu_1'^{\circ}(I)g_1(I)-\psi'^{\circ}(0)x_0$$
$$+\int_0^{t^*} f_1^{\circ}(t)d_1dt-\int_0^{t^*} f_2^{\circ}(t)d_2dt.$$

Assume $\beta=\beta(u,Q_p)=\beta_u+\beta_c$, where

$$\beta_u=J(u^{\circ})-J(u)$$

is the measure of the non-optimality of the command $u(t)$, $t \in T$; and

$$\beta_c=J^*(\xi)-J^*(\xi^{\circ})=\lambda'(K)\alpha(K)+\nu_2'(I)g_2(I)$$
$$-\nu_1'(I)g_1(I)+\psi'(0)x_0-\int_0^{t^*} f_1(t)d_1dt+\int_0^{t^*} f_2(t)d_2dt$$
$$-\lambda'^{\circ}(K)\alpha(K)-\nu_2'^{\circ}(I)g_2(I)+\nu_1'^{\circ}(I)g_1(I)-\psi'^{\circ}(0)x_0$$
$$+\int_0^{t^*} f_1^{\circ}(t)d_1dt-\int_0^{t^*} f_2^{\circ}(t)d_2dt$$

is the measure of the non-optimality of the support Q_p.

If in the command $u(t)$, $t\in T$, we associate an optimal support Q_p° to $\beta_c=0$.

From this we deduce

$$\beta(u,Q_p^{\circ})=\beta_u\leq\varepsilon. \qquad (20)$$

Let's put:

$$\varepsilon(t)=\Delta(t)(u(t)-d_1),t\in T^+;$$
$$\varepsilon(t)=\Delta(t)(u(t)-d_2),t\in T^-;$$
$$\varepsilon(t)=0, if\, \Delta(t)=0,t\in T;$$
$$\varepsilon_1=\sum_{\nu_i\prec} \nu_i\upsilon_{1i}+\sum_{\succ} \nu_i\upsilon_{2i};$$
$$\varepsilon_2=\sum_{k\in K}\lambda_k\omega_k.$$

From this, by using the definition of the co-command $\Delta(t)$, $t\in T$, we obtain:

$$\varepsilon(t)=\psi'(t)(Ax(t)+bd_1)-\psi'(t)(Ax(t)+bu(t)), if\, \psi'(t)b\prec$$
$$\varepsilon(t)=\psi'(t)(Ax(t)+bd_2)-\psi'(t)(Ax(t)+bu(t)), if\, \psi'(t)b\succ$$
$$\varepsilon(t)=0, if\, \psi'(t)b=0,t\in T.$$

By introducing the "Hamiltonian" function, $\varepsilon(t)$ will be equal to:

$$\varepsilon(t)=\max_{d_1\leq u\leq d_2} H(x(t),\psi(t),u)-H(x(t),\psi(t),u(t)),t\in T. \qquad (21)$$

In addition we have:

$$\varepsilon_1=\sum_{\nu_i\prec} \nu_i g_{1i}+\sum_{\succ} \nu_i g_{2i}-\nu'(I)H(I,J)x(t^*).$$

That is to say

$$\varepsilon_1=\max \nu'Z-\nu'Hx(t^*);$$
$$\varepsilon_2=\max_{\varpi\geq 0}\left(-\lambda'(K)\varpi(K)\right)+\lambda'(K)\omega(K). \qquad (22)$$

By using the conditions (20), (21) and (22), we obtain the ε-maximum condition (16).

5. ALGORITHM

Let $\varepsilon\succ$ be a given positive number .Suppose that the starting command-support $\{u,Q_p\}$ does not satisfy the ε-maximum principle. An iteration of this algorithm consists in improving the command support $\{u,Q_p\}\rightarrow\{\bar{u},\bar{Q}_p\}$ with $J(\bar{u})\geq J(u)$.

Consider $\bar{u}(t)=u(t)+\theta\,\Delta u(t), t\in T$, another admissible command of the problem (1)-(3), where $\Delta u(t)$ is the admissible direction, $\theta\geq 0$ being the maximum admissible step a long this direction.

Let's choose the numbers $\alpha\succ$ and $h\succ$, (α,h are the parameters of this algorithm), where h is a very small value.

Let us form the following sets:

$$T_0=\{t\in T:|\psi'(t)b|\leq\alpha\},$$
$$T^*=\{t\in T:|\psi'(t)b|\succ$$
$$T^*=T/T_0;$$

and subdivide T_0 into intervals $[\underline{\tau}_j,\bar{\tau}_j[, j=\overline{1,N};$ so as:

$$\mathrm{T}_0=\bigcup_{j=1}^{N} \quad ,\bar{\tau}_j[\cap[\underline{\tau}_i,\bar{\tau}_i[=\varnothing,\ i\neq j \text{ and}$$
$$\bar{\tau}_j-\underline{\tau}_j\leq h, j=\overline{1,N},$$
$$T_p\subset\{\tau_j, j=\overline{1,N}\}, u(t)=u_j= \text{Constant},$$
$$t\in[\underline{\tau}_j,\bar{\tau}_j[, j=\overline{1,N}.$$

Assume that $\Delta u(t)=\begin{cases} d_1-u(t), if\, \Delta(t)\prec \\ d_2-u(t), if\, \Delta(t)\succ \end{cases}$;

and ℓ $\begin{cases}\theta\Delta u(t), t\in[\underline{\tau}_j,\bar{\tau}_j[, j=\overline{1,N}, \\ \theta, j=N+1; \end{cases}$

where N+1 is the index corresponding to the set $T^*=T\setminus T_0$.

Let us calculate the following quantities:

$$q_j=\int_{\underline{\tau}_j}^{\bar{\tau}_j} P(t)dt; g_k(j)=\int_{\underline{\tau}_j}^{\bar{\tau}_j} c_k' q(t)dt, k\in K;$$
$$q_{N+1}=\int_{T^*} P(t)\Delta u(t)dt;$$
$$g_k(N+1)=\int_{T^*} c_k' q(t)\Delta u(t)dt, k\in K;$$
$$d_{1j}=d_1-u_j, d_{2j}=d_2-u_j, j=\overline{1,N}; d_{1N+1}=0; d_{2N+1}=1;$$

$\theta\Delta u(t) = \ell \quad , \bar{\tau}_j[, j=\overline{1,N}; 0 \le \theta \le 1$;

$V_1 = g_1 - Hx(t^*), V_2 = g_2 - Hx(t^*)$;

$g_k = \left(g_k(j), j=\overline{1,N+1}\right), G = \left(q_j, j=\overline{1,N+1}\right)$

$\bar{d}_1 = \left(d_{1j}, j=\overline{1,N+1}\right), \bar{d}_2 = \left(d_{2j}, j=\overline{1,N+1}\right)$,

$\ell \quad \ell \quad \overline{N+1})$.

By using these quantities, the maximum of functional (13) under the constraints (14), becomes the following support problem:

$$\min_k \left(g_k^{'}\ell \right) \to \max_{\ell};$$
$$V_1 \le G\ell \tag{23}$$
$$\bar{d}_1 \le \ell$$

While solving this problem by the adapted method [2], we obtain the ε-optimal solution $\{\ell \}$, where S_p is going to be the optimal support of the problem (23).

The new command will be equal to:

$\bar{u}(t) = u(t) + \ell \quad T^*$;

$\bar{u}(t) = u(t) + \ell \quad , \bar{\tau}_j\left[, j=\overline{1,N}\right.$;

(24)

and satisfy $J(\bar{u}) \ge J(u)$.

By using the support S_p, we construct the support $\bar{Q}_p$ [1] of the problem (1)-(3). If the command support $\{\bar{u}, \bar{Q}_p\}$ does not satisfy theorem 1, we change the support $\bar{Q}_p$ [1].

By using $\bar{Q}_p = \{\bar{I}_c, \bar{T}_c, \bar{T}_f, \bar{K}_f\}$ we calculate $\bar{\Delta}_k(t)$ (9), $\bar{\lambda}_k$ (10), and $\Delta(t) = \sum_{k \in \bar{K}_f} \bar{\lambda}_k \bar{\Delta}_k(t)$. By following, we construct the quasi-command $\omega = (\omega(t), t \in T)$:

$$\omega(t) = d_1, if\, \bar{\Delta}(t) \succ$$
$$\omega(t) = d_2, if\, \bar{\Delta}(t) \prec \tag{25}$$
$$\omega(t) \in [d_1, d_2], if\, \bar{\Delta}(t) = 0, t \in T;$$

and its corresponding quasi-trajectory $\chi = (\chi(t),\ t \in \mathrm{T})$, solution of the equation $\quad \omega, \chi(0) = x_0$.

If the following relations are satisfied

$$\begin{cases} g_1 \le H\chi(t^*) \le g_2 \\ J(\omega) = c_k^{'}\chi(t^*) + \alpha_k, k \in K_f. \end{cases}$$

Then the quasi-command $\omega(t)$ is optimal for the problem (1)-(3).

If not, then we determine the following vector:

$$\begin{pmatrix} \gamma(\bar{T}_p) \\ \gamma(s+1) \end{pmatrix} = R^{-1} \begin{pmatrix} g_{12}(\bar{I}_c) - H(\bar{I}, J)\chi(t^*) \\ C'(\bar{K}_f)\chi(t^*) + \alpha(\bar{K}_f) - e(\bar{K}_f)J(\omega) \end{pmatrix} \tag{26}$$

where $R = \begin{pmatrix} P(\bar{I}_c, \bar{T}_c) & P(\bar{I}_c, \bar{T}_f) & 0 \\ -C'(\bar{K}_f)q(\bar{T}_c) & -C'(\bar{K}_f)q(\bar{T}_f) & e(\bar{K}_f) \end{pmatrix}$,

$g_{12} = \begin{cases} g_{1i}, if\, \bar{v}_i \prec \\ g_{2i}, if\, \bar{v}_i \succ \end{cases}$.

Here $\det R \ne 0$, because $\det P(\bar{I}_c, \bar{T}_c) \ne 0$ and $\det \bar{\Delta}_f \ne 0$.

Let us calculate the following quantities:

$$\beta_k = \gamma(s+1) + J(\omega) - c_k^{'}\chi(t^*) - \alpha_k - \sum_{j=1}^{s} c_k^{'}q(t_j)\gamma(t_j), k \in \bar{K}_H;$$

$$\gamma_1(\bar{I}_H) = \sum_{j=1}^{s} P(\bar{I}_H, t_j) + H(\bar{I}_H, J)\chi(t^*) - g_1(\bar{I}_H);$$

$$\gamma_2(\bar{I}_H) = \sum_{j=1}^{s} P(\bar{I}_H, t_j) + H(\bar{I}_H, J)\chi(t^*) - g_2(\bar{I}_H).$$

Two cases are possible:

- *First case*:

If the following relations:

i) $\left\|\gamma(\bar{T}_p)\right\| \le \mu$ (μ is the method parameter).

ii) $\beta_k \le 0, k \in \bar{K}_H; \gamma_1(\bar{I}_H) \ge 0; \gamma_2(\bar{I}_H) \le 0;$ (27)

are satisfied, then go to the final procedure.

- *Second case*:

Otherwise the support $\bar{Q}_p$ will be changed by using the dual method [2]:

$\bar{Q}_p \to \tilde{Q}_p$ And a new iteration begins with $\{\bar{u}, \tilde{Q}_p\}$, where $\bar{\alpha} \prec$.

6. FINAL PROCEDURE

Suppose that the quasi-command $\omega(t)$, $t \in T$ (25) and its corresponding quasi-trajectory $(\chi(t), t \in T)$, constructed by the support $\bar{Q}_p$, the relation (27) are satisfied.

Suppose that $T_0 = \{t \in T : \bar{\Delta}(t) = 0\}$ is formed by the isolated moments $t_j, j = \overline{1,s}$; and

$$\dot{\Delta}(t_j) \ne 0, j = \overline{1,s}, s = |\bar{T}_p|.$$

The final procedure consists in finding the solution $\tau^{\circ}{}_p = \left(\tau_j, j=\overline{1,s}\right)$ and $\tau^{\circ}(s+1)$ of the following system by using the Newton method of the system:

$$(d_2 - d_1)\sum_{j=1}^{s} sign\,\dot{\Delta}(t_j)\int_{t_j}^{\tau_i} \left(\bar{I}_c, t\right)dt = g_{12}(\bar{I}_c) - H(\bar{I}_c, J)\chi(t^*),$$

$$-(d_2 - d_1)\sum_{j=1}^{s} sign\,\dot{\Delta}(t_j)\int_{t_j}^{\tau_i} c_k q(t)dt + \tau(s+1) = c_k^{'}\chi(t^*) + \alpha_k, k \in \bar{K}_f. \tag{28}$$

To solve the system (28), we apply the Newton method. We take as initial the approximation:

$$\tau_p^{(0)} = \left(\tau_j^{(0)}, j = \overline{1,s}\right), \tau_{(s+1)}^{(0)}, \tau_p^{(0)} = \left\{t_j, j = \overline{1,s}\right\}, \tau_{(s+1)}^{(0)} = \gamma(s+1).$$

where $Q_p^{(0)} = \left\{\overline{I}, \tau_p^{(0)}, \overline{K}_f\right\}$.

Let $\tau_p^{(k)} = \left\{\tau_j^{(k)}, j = \overline{1,s}\right\}, \tau_{(s+1)}^{(k)}$ be the k^{th} approximation.

Suppose $\det P\left(\overline{I}_c, \tau_c^{(k)}\right) \neq 0$ and $\det \overline{\Delta}_f^{(k)} \neq 0$; where

$$P\left(\overline{I}_c, \tau_c^{(k)}\right) = \left(H\left(\overline{I}_c, J\right) q\left(\tau_j^{(k)}\right), j = \overline{1,s}\right) \text{ and}$$

$$\overline{\Delta}_f^{(k)} = \left(\overline{\Delta}\left(\overline{K}_f, \tau_f^{(k)}\right), e\left(\overline{K}_f\right)\right),$$

$$\overline{\Delta}\left(\overline{K}_f, \tau_f^{(k)}\right) = \left(\overline{\Delta}_k(t), t \in \tau_f^{(k)}, k \in \overline{K}_f\right);$$

$$\overline{\Delta}_k(t) = \left(c_k' q\left(\tau_j^{(k)}\right), j = \overline{1,s}\right)' P\left(\overline{I}_c, \tau_c^{(k)}\right)^{-1} P\left(\overline{I}_c, t\right) - c_k' q(t),$$

$t \in T, k \in K$

The $(k+1)^{th}$ approximations $\tau_p^{(k+1)}$ and $\tau_{(s+1)}^{(k+1)}$ will be equal to:

$$\tau_p^{(k+1)} = \tau_p^{(k)} + \frac{1}{d_2 - d_1}\left\{sign\dot{\Delta}(t_j)\gamma\left(\tau_j^{(k)}\right), j = \overline{1,s}\right\}$$
$$\tau_{(s+1)}^{(k+1)} = \tau_{(s+1)}^{(k)} + \gamma(s+1) \quad (29)$$

where $\left(\gamma\left(\tau_p^{(k)}\right), \gamma_{(s+1)}^{(k)}\right)$, the vector (26), calculated with the support $Q_p^{(k)} = \left\{\overline{I}_c, \tau_p^{(k)}, \overline{K}_f\right\}$.

Let $\tau_p^{\circ}, \tau_{(s+1)}^{\circ}$, be the solution of the system (28). Then the quasi-command $\omega^{\circ}(t), t \in T$ (25) calculated with the support Q_p° is the optimal command for the problem (1) - (3) and Q_p° is the optimal support.

Remark 1. The system (28) is obtained by using the relations (26) and (27).

CONCLUSION

The criteria of optimality was obtained by the pontriaguin maximum principle. The constructed of an a optimal control $u°(t)$, $t \in T$, requires to solve $s+\left|K_f\right|$ equations by the Newton method. Its interesting to solve the problem (1)-(3) with the multidimentionnal functions $u(t)$, $t \in T$.

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