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# Evaluation of the Dynamic Reliability by Differential Equations

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***Abstract:*** **Dynamic reliability is defined as the part of the probabilistic analysis in dependability, which studies in an integrated manner the behavior of human-machine systems-software affected by a change in the underlying dynamics. In this article, we study the evaluation of the dynamic reliability of a test case, the heated tank. Our approach consists of four steps: the first one is to study the system in a functional analysis using the FAST method, the second step allows studying the tank through a dysfunctional analysis to obtain dangerous events which can lead to the failure system; the third step is used to model the system by a tree of events in the context of dependability and the last step is to model the different paths leading to failure and the final result is a mathematical model in the form of differential equations.**

***Keywords:*Dependability, reliability dynamic, functional analysis, FAST method, event tree, differential equations.**

## I. INTRODUCTION

Dependability [1] allows maintaining the proper functioning of a system or a product throughout its life cycle. Factors of dependability, reliability plays an important role because it measures the ability of a system to remain without failure. On the other hand, the dynamic reliability is defined as the part of the probabilistic analysis in dependability, which studies in an integrated manner the behavior of systems affected by changing underlying dynamics. Thus, it is necessary to use applied mathematics, such as probability theory and the field of differential equations for the modeling of complex systems.

This article is organized as follows. The second section presents our approach to assessing the reliability dynamics following four steps. The application of the approach to the test case heated tank is presented in the third section and the end result is the mathematical model in the form of differential equations.

## II. DESCRIPTION OF THE APPROACH

There are several methods for evaluating the dynamic reliability however, at present the problems of dynamic reliability have not been solved in the general case.

A study of different methods developed to assess the dynamic reliability, we found that the method of PDMP (piecewise Deterministic Markov Processes) [2], [3] is the most widely used because it can naturally take into account events the deterministic evolution of the physical parameters of the stochastic processes and events corresponding to the random demand or failures of system components.
To do this, to help meet the challenges posed by the dynamic reliability, we propose an approach integrating PDMP to define the different trajectories of a system and then calculate the probability of failure for each event using the dreaded differential equations.

### *A. Flowchart of the proposed approach:*

We give here a flowchart (Figure 1) shows the different steps of the methodology, followed by a description of each step:

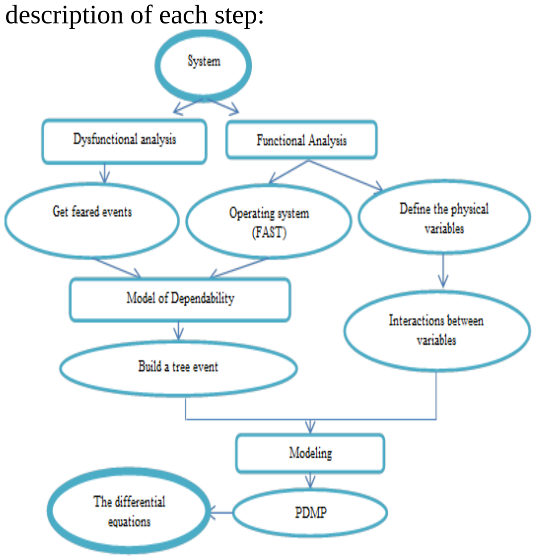

Figure 1: Flowchart of the proposed approach

*B. Description of the different steps:*

**1) Step 1: Functional Analysis**

The Functional Analysis [4] applies to the creation or improvement of a system.
We use Functional decomposition following FAST(Function Analysis System Technique) [5].
This technique allows to highlight the design process by showing the relationship between needs and solutions [3]. To use this method you must answer the following questions (Figure 2): why, when and how?

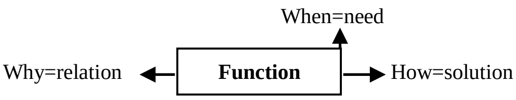


Figure 2: Implementation of FAST

At the level of functional analysis, we define:

- The physical variables:This step identifies the physical variables that describe the system from the functional decomposition.
-The interactions between variables: As we are in the context of dynamic reliability, one must study the interactions between these different physical variables as dynamic reliability takes into account these interactions.

**2) Step2: Dysfunctional Analysis**

Dysfunctional Analysis [6] is to imagine all the failures that can occur anywhere in the system. It is realized through three phases can be represented by the following figure:

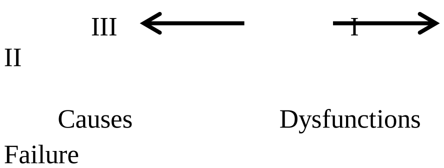


Figure3: The phases of the Dysfunctional Analysis

From Dysfunctional Analysis, we define the set of undesirable events that can lead to failure.

**3) Step 3: Model of Dependability**

In the context of dependability, we adopt the method of event trees which embodies a simple and natural [7] and reveals the consequences caused by different events, from an initiating event.

The approach generally [7] chosen to perform an analysis by event tree is as follows: define the initiating event to consider, identify the security functions provided to cope, build and operate tree and describe sequences events identified..

**4) Step 4: Modelling of the system by PDMP**

The PDMP [2], [3] are stochastic hybrid dynamical models, defining deterministic trajectories punctuated by random jumps.

A PDMP is determined by a hybrid process noted: X(t) = ($x_t$,$m_t$) :

- A discrete variable $m_t$present mode or process state at time t.
- A state variable Euclidean $x_t \epsilon R^d$.

The PDMP is determined by its local characteristics ($E_m$, $\Phi_m$, $\lambda_m$,$Q_m$) m ϵ M

**a. $E_m$a subspace open in $R^n$ :**

Let M be a countable set whose elements are called profiles, for all m ϵ M, let $E_m$is a subset opened in$R^n$. Let E = $\bigcup_{m\epsilon M} E_m \times \{m\}$

**b. The flowϕ :**defines the deterministic trajectory between two jumps.

**c. The intensity of jumps$\lambda_m$ :**

$\bar{E} \to R^+$is a measurable function characterizing the frequency jumps, and which satisfies:

∀ (x,m) ϵ E, ∍ e > 0 tel que $\int_0^e \lambda_m(\Phi_m(x,s)\, ds < \infty$.

**d. The measurement of transition of the process $Q_m$ :**

$Q_m$: $\bar{E} \times ß(\bar{E}) \to [0,1]$ checks for any couple (x,m) ϵ$\bar{E}$ : $Q_m(x, E - \{(x,m)\})$= 1

That is to say that the process should jump to a new mode and / or a new position

## III. CASE STUDY

We will apply ourapproach on a test case of heated tank which is an example of literature is representative of the gas and petroleum industry.

*A. Principle of System Operation[1] :*

The tank(Figure 4) contains the liquid whose level is measured by three sensors each connected to a unit. Unit 1 and 2 are used to add liquid in the vessel and unit 3of obtaining. It has 4 positions for each unit: O (Open), F (Closed), Ob (Open blocked) and Fb (Closed blocked).

The height of the liquid height h varies between 4 m and 10 m and the temperature θ is between 0 ° C and 100 ° C.

The tank is characterized by two jumps:Jumping random caused to drive failure and Jumping deterministic caused to control laws:

• The first control law is L1 when the liquid level is less than 6 meters. Units if they are not locked, position themselves in the mode m = (O, O, F) to fill the tank.

• The second control law L2 occurs when the level exceeds 8 m: units are placed in the mode m = (F, F, O) to remove excess liquid.

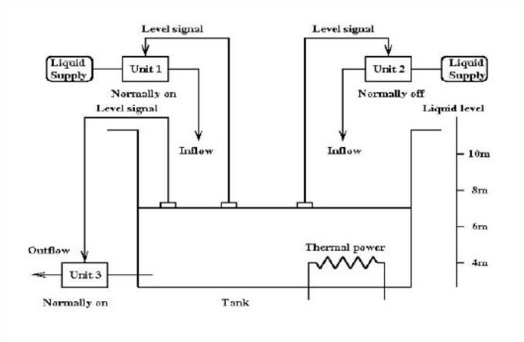


Figure4:Schema of thermal tank

*B. Application on the heated tank:*

The proposed approach follows four sequential steps described above:

**1) Step 1: Functional Analysis**

To study the functioning of the system and to identify its physical variables we need a functional decomposition.

➢ Functionaldecompositionfollowing FAST:

Following the methodology of the FAST approach, we obtained the following diagram (Figure 5) representing the functional decomposition of the tank.

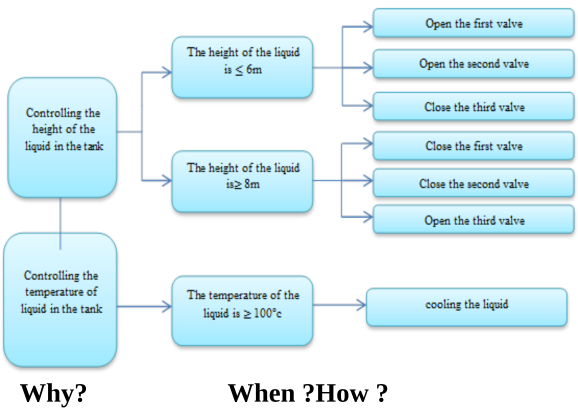


Figure5: Functional decomposition following FAST

➢ The physical variables:

According to the functional decomposition there we are three physical variables: height, temperature and mode of the system is the state of the component.

➢ The interactions between variables:

There are three types of interactions between these variables.

-***Interaction temperature-mode:*** The transition from one mode to another in the event of a drive failure depends on the evolution of the temperature.

- ***Interaction height-mode:*** The control laws L1 and L2, by a change in system mode, can change the trajectory of the pitch when it goes beyond 6m or 8m. So there is interaction between the height and the mode.

- ***Interaction height-temperature:*** The height and temperature are continuous variables that evolve over time. More precisely, the height is independent, but the temperature depends on the height.

**2) Step 2: Analysis Dysfunctional:**

To apply the Dysfunctional Analysis, we identify three phases we have seen previously:

Height of the liquid≤4m ▶ Dryness ▶ Failure of the tank

Height of the liquid≥10m ▶ Overflow ▶ Failure of the tank

Liquid temperature≥100º ▶ Overheating ▶ Failure of the tank

| III | I | II |
|---|---|---|
| Causes | Dysfunctions | Failure |

From the analysis we get 3 Dysfunctional feared events are: Dryness, Overflow and Overheating.

**3) Step 3: Model of Dependability:** To build the tree of events, it is necessary to identify the set of all combinations generated from the possible states.

➢ The different modes of tank:

As we saw previously each unit of the tank has 4 possible states: open, open blocked, closed and closedblocked. The tank has 64 possible modes [2].

- Different jumps of 4 states:

Each hop of the four states (Figure 6) are defined by [2]:

- An open state can only go to an open state blocked or closed blocked.
- A closed state can only go to an open state blocked or closedblocked.

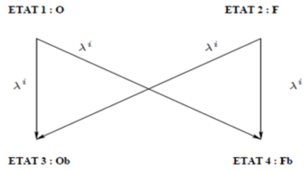


Figure6 : Different jumps of 4 states

- Construction of treeevents:

As in [2],we sought before constructing the tree event to simplify the system by reducing, if possible modes of tank starting from the initial condition m (0) = (O, F, O) by eliminating all states not derived from m (0). We obtain 37 modes [2] instead of 64 modes.

The following figure (Figure 7) describes the event tree constructed for the 37 modes obtained.

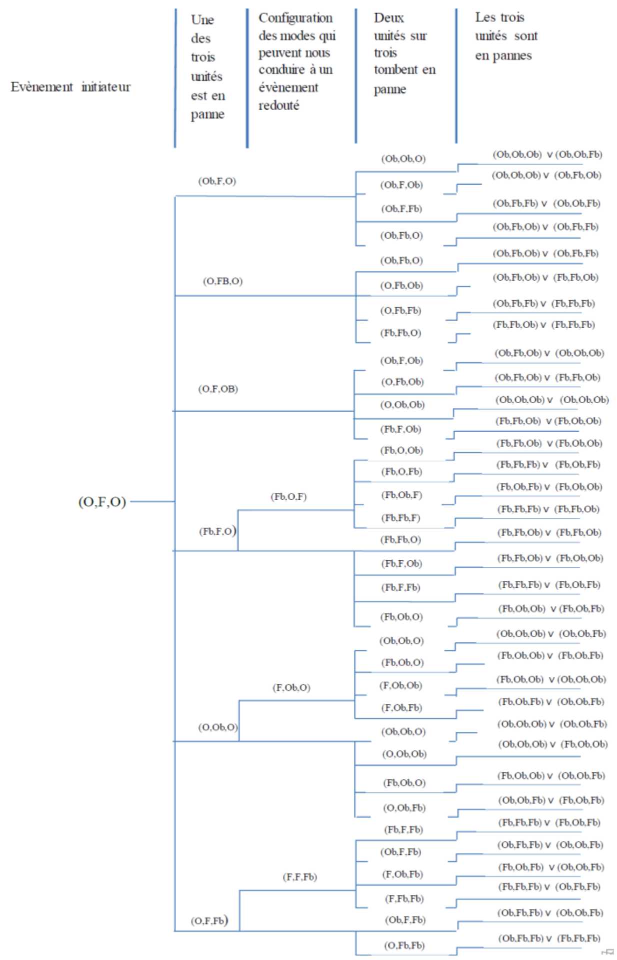


Figure7 :the tree of events obtained for 37 modes

**4) Step 4: Modelling of the system by PDMP [2], [3]**

- Identification of differential equations for the system:

Before identifying the local characteristics of the PDMP we present the system of differential equations [2], [3] following which describes the overall behavior of the tank:

$$(S) = \begin{cases} \dfrac{dh}{dt} = \gamma_1(\alpha) \\ \dfrac{d\theta}{dt} = \dfrac{\gamma_2(\alpha) - \gamma_3(\alpha)\theta}{h} \end{cases}$$

- Identification of the characteristics of PDMP:

According to the definition of the PDMP, the following are identified:

***a. The state spaceE :***

The height of the liquid must evolve in the interval [6, 8]. Beyond this range, the system reacts, through control laws. It is therefore considered that the height of 6 m and 8 m are the borders of the state space. The temperature of the liquid must evolve between 0 ° C and 100 ° C. So we can define the state space by the following formula:

E = ( [4,6] $\times$ [$\theta$,100] )$\cup$ ([6,8] $\times$ [$\theta$,100] )$\cup$( [8,10] $\times$ [$\theta$,100] )

### b. *The flow $\phi$ :*

Height and temperature represent the flow of $\phi$ PDMP describing the behavior of the tank. Flow associated with mode m is defined by :$\phi$(h, $\theta, t$)= (h(t), $\theta(t)$)where h(t) and $\theta(t)$are the solutions of the differential system (S)

### c. *The intensity $\lambda_m$ :*

The intensity of jump $\lambda_m$ for a mode m is the sum of the failure rates $2\lambda^i$ of units, for example from the transition graph obtained for the mode $m_1$ you can go to $m_{15}$ et $m_{27}$ with intensity $\lambda^1$ [as unit 1 fails], to $m_4$ et $m_5$ with intensity $\lambda^2$ [as unit 2 fails], to $m_2$ et $m_3$ with intensity $\lambda^3$ [as unit 3 fails].

So: $\lambda_1(\theta) = 2(\lambda^1(\theta) + \lambda^2(\theta) + \lambda^3(\theta))$

### d. *The measurement of transition Q :*

The kernel Q summarizes the probabilities of transition from mode m to another mode $m'$.

- Jumping with a control law $\Leftrightarrow$Transition probability = 1
- Jumping to a dreaded event $\Leftrightarrow$Transition probability =1/2
- Jumping to another mode after a failure of the unit $\Leftrightarrow$Transition probability = failure rate / intensityof jump.

➢ Constants of the differential system:

We note $\alpha$ the coefficient $\alpha = (\alpha_1, \alpha_2, \alpha_3)$defined for every i=1, 2, 3 by :

$$\alpha_i = \begin{cases} 0 & \text{if unit } i \text{ is closed or closed blocked} \\ 1 & \text{if unit } i \text{ is open or open blocked} \end{cases}$$

The following table (table1) shows the constants related to the differential system (S).

With $(\gamma_1(\alpha), \gamma_2(\alpha), \gamma_3(\alpha))$ in m.$h^{-1}$, q in m.$h^{-1}$it presents the flow measuring units $\theta_{in}$ in °c It presents the temperature of the liquid supplied by the units 1 and 2. K inm.°c.$h^{-1}$is a parameter related to the physical variables of the tank.

TABLE I
CONSTANTS RELATED TO THE DIFFERENTIAL SYSTEM (S)

| $\gamma_1(\alpha)$ | $\gamma_2(\alpha)$ | $\gamma_3(\alpha)$ | q | $\theta_{in}$ | K |
|---|---|---|---|---|---|
| $(\alpha_1 + \alpha_2 -$ | $(\alpha_1 + \alpha_2)q\theta_{in}$ | $+ K$ $(\alpha_1 + \alpha_2)q$ | 1.5 | 15 | 23.88915 |

We set also as constants:

$\theta_1 = \frac{q\theta_{in}+K}{q}$ = 30,9261°c , $\theta_2 = \frac{2q\theta_{in}+K}{2q}$ = 22,96305°c

So that we have: $\frac{\gamma_2(\alpha)}{\gamma_3(\alpha)} = \begin{cases} \theta_1 si (\alpha_1 + \alpha_2) = 1 \\ \theta_2 \, si \, (\alpha_1 + \alpha_2) = 2 \end{cases}$

The following table (TABLE II) shows the different configurations depending on the coefficient $\alpha$:

TABLE III
CONSTANTS RELATED TO THE DIFFERENTIAL SYSTEM (S)

| $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ | $\gamma_1(\alpha)$ | $\gamma_2(\alpha)$ | $\gamma_3(\alpha)$ | $\frac{\gamma_2}{\gamma_3}$ | $\frac{\gamma_3}{\gamma_1}$ |
|---|---|---|---|---|---|
| $\alpha = (0,1,0)$et $(1,0,0)$ | q | $q\theta_{in}$ + K | Q | $\theta_1$ | 1 |
| $\alpha = (1,1,0)$ | 2q | $2q\theta_{in}$ + K | 2q | $\theta_2$ | 1 |
| $\alpha = (1,1,1)$ | q | $2q\theta_{in}$ + K | 2q | $\theta_2$ | 2 |
| $\alpha = (0,0,0)$ | 0 | K | 0 | | |
| $\alpha = (0,1,1)$et $(1,0,1)$ | 0 | $q\theta_{in}$ + K | Q | $\theta_1$ | |
| $\alpha = (0,0,1)$ | -q | K | 0 | | 0 |

➢ Résolution des équations différentielles :

Whether (h(0), $\theta$(0), m(0)) the initial conditions of the tank, then the set of solutions of (S) are:

- If m(0) correspondsto the coefficient $\alpha$ = (0,1,0) or (1,0,0) So$\forall$t$\in R^+$ :

$$\begin{cases} h(t) = qt + h(t) \\ \theta(t) = \dfrac{h(0)(\theta(0) - \theta_1)}{h(t)} + \theta_1 \end{cases}$$

- If m(0) corresponds to the coefficient $\alpha$ = (1,1,0) So $\forall$t$\in R^+$ :

$$\begin{cases} h(t) = 2qt + h(t) \\ \theta(t) = \dfrac{h(0)(\theta(0) - \theta_2)}{h(t)} + \theta_2 \end{cases}$$

- If m(0) corresponds to the coefficient $\alpha =$ (1,1,1) So ∀t∈ $R^+$ :

$$\begin{cases} h(t) = qt + h(t) \\ \theta(t) = \dfrac{h^2(0)(\theta(0) - \theta_2)}{h^2(t)} + \theta_2 \end{cases}$$

- If m(0) corresponds to the coefficient $\alpha =$ (0,0,0) So ∀t∈ $R^+$ :

$$\begin{cases} h(t) = h(0) \\ \theta(t) = \dfrac{K}{h(0)} t + \theta(0) \end{cases}$$

- If m(0) correspondsto the coefficient $\alpha =$ (0,1,1) or (1,0,1) So ∀t∈ $R^+$ :

$$\begin{cases} h(t) = h(0) \\ \theta(t) = (\theta(0) - \theta_1) e^{-\frac{qt}{h(0)}} + \theta_1 \end{cases}$$

- If m(0) correspondsto the coefficient $\alpha =$ (0,0,1) So∀t∈ $R^+$ :

$$\begin{cases} h(t) = -qt + h(0) \\ \theta(t) = \theta(0) - \dfrac{K}{q} In\left(\dfrac{h(t)}{h(0)}\right) h(0) \end{cases}$$

We obtain the following formula:

$$h(t) = \gamma_1(\alpha)t + h(t) \quad \forall t \in R^+$$

## IV. CONCLUSIONS

In this article, we presented an approach to evaluate the dynamic reliability of a system.

The approach allows us to study the system considered by Dysfunctional and Functional Analysis in the context of dependability, then modeled by a tree of events to identify the different paths leading to the functioning and to the dysfunctioning set. Given the complexity and limitations of mathematics to evaluate analytically the dynamic reliability of a system, currently dynamic reliability is often accessible only by the PDMP method and in this context, we integrate the PDMP in our approach in order to define the different trajectories of the system and then calculate the probability of failure for each event using the differential equations.

We applied our approach to the test case, the heated tank, known in the literature. On the method of PDMP applied to this example [2], [3] we introduced upstream the steps of functional and dysfunctional analysis of the tank. We obtained respectively the functional diagram of the tank and the tree of events. The final result is a mathematical model in the form of differential equations, which express the trajectories leading to undesirable events.

The application of our approach on this example has allowed to validating the first two steps and to make the learning of the PDMP method. The continuity of this work is to test the approach on a real system; the final result will be the calculation of the failure probability for each undesirable event, by the model exploitation of the obtained differential equations.

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