Prime Number Function

Fonction nombre premier

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ABSTRACT

The objective of this research is to study prime numbers in relation to their distributions and some of the conjectures to which they are subject. To achieve our goal, we have developed a function having a defined starting set and as an ending set, the set of prime numbers. From the formulas of the domain of definition, we were able to prove the Goldbach conjecture and generalization. Another more detailed explanation of the distribution of prime numbers allowed us to prove the infinity of twin primes. In conclusion, this research exposes a detailed explanation of prime numbers as well as a little glimpse into their distribution and from that, a proof of many of these conjectures such as the Goldbach conjecture, the twin prime conjecture and others.

Mots-Clés:
Nombre Premier, Conjecture de Goldbach, Théorème des écarts, Conjecture des Nombres Premiers Jumeaux

RÉSUMÉ

L'objectif de cette recherche est d'étudier les nombres premiers en rapport avec leurs distributions et certaines des conjectures dont ils sont sujets. Pour atteindre notre objectif, nous avons élaboré une fonction ayant un ensemble de départ définie et comme ensemble d'arrivée l'ensemble des nombres premiers. À partir des formules du domaine de définition, nous avons pu démontrer la conjecture de Goldbach et une autre qui la généralise. Une autre explication de manière plus détaillée sur la répartition des nombres premiers nous a permis de prouver l'infinité des nombres premiers jumeaux. En conclusion, cette recherche expose une explication détaillée des nombres premiers ainsi qu'une petite aperçue sur leur distribution et à partir de cela, une démonstration de bons nombres de ces conjectures telles que la conjecture de Goldbach, la conjecture des nombres premiers jumeaux et d'autres.

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1. INTRODUCTION

A prime number is a natural number that admits exactly two distinct and positive divisors which are 1 and itself. According to this definition, the numbers 0 and 1 are neither prime nor composite because 0 is divisible by all integers and 1 is only divisible by itself [1]. Prime numbers are today the most important field of mathematics because of its importance on cryptography and its relation with the experiences of everyday life [1-2]. However, they contain mysteries that many seek to uncover. As a result, it becomes one of the most fascinating area in the world of mathematics. Among its problems, we have the Goldbach conjecture [4], the twin prime conjecture [6], the Riemann hypothesis and many others.
In what follows, we will, from the sieve of eratosthenes [3], first create a function with a defined starting set and an ending set which is nothing other than the set prime numbers. Then, the conditions of this starting set to prove the Goldbach conjecture [5], a conjecture which will be a special case for a conjecture that we will state and prove and finally prove the conjecture of twin prime numbers [7].

2. RESEARCH METHOD

2.1. Function of Prime Numbers :

2.1.1. Presentation of 3p - 2 + 2pn:

According to the erasthostein screen, the corollary can be that the sequence of compound numbers is: $\mathbf{p + pn}$, with $\mathbf{p}$ a prime number and $\mathbf{n}$ a natural integer:

If $\mathbf{p = 3, p + pn = U_n = 3 + 3n = 3; 6; 9; 12; \dots}$

If $\mathbf{p = 5, p + pn = V_n = 5 + 5n = 5; 10; 15; 20; 25; \dots}$

If $\mathbf{p = 7, p + pn = C_n = 7 + 7n = 7; 14; 21; 28; 35; \dots}$

and so on…

Let the sequence $\mathbf{Ni = In = 1 + 2n}$, with $\mathbf{n}$ a natural integer, be the set of odd numbers.

$\mathbf{In = 1; 3; 5; 7; 9; 13; 15; 17; 19; 21; 23; \dots}$

For **Un, Vn, Cn, ...** the first terms are prime numbers. So we can deduce that the composite numbers resulting from **Un, Vn, Cn, ...** and which are in **In** are respectively the terms of the sequences: $\mathbf{9 + 6n; 15 + 10n; 21 + 14n; \dots}$ The general formula that derives is :

$$\mathbf{3p + 2pn} \tag{1}$$

Then to find the prime numbers, we write all the terms of **In** and we find those which are not terms of $\mathbf{3p + 2pn}$ with $\mathbf{p}$ a prime number and $\mathbf{n}$ a natural number except **1**.
In conclusion, if an integer $\mathbf{l}$ is different from

$$\mathbf{3p - 2 + 2pn} \tag{2}$$

the sum

$$\mathbf{2 + l} \tag{3}$$

is a prime number.

2.1.2. The General function:

Let us find a function having as a starting set, a set included in **IN** and having as set of arrival the set of prime numbers.This function has as a starting set a set **E** included in **IN** and has for set of arrival the set of prime numbers noted **P**.

Let **M** be the function:

$$\begin{aligned} \mathbf{M:}\ & \mathbf{E \rightarrow P} \\ & \mathbf{n \rightarrow 2+n} \end{aligned} \tag{4}$$

$$\mathbf{M(n)=2+n}$$
$$\mathbf{Dm=E}$$

E: the set of odd numbers other than **3p - 2 + 2pa** with **p** an **odd prime number** and **a ϵ IN.**

$$\mathbf{E= Ni \setminus \{3p\text{-}2\text{+}2pa\} \cup \{0\}} \tag{5}$$

Ni is the set of odd numbers that is to say any term of the sequence

$$\mathbf{1 + 2m} \tag{6}$$

with **m ϵ IN.**

For prime numbers from **1** to **100**, we can take as sources of numbers of prime numbers, odd prime numbers which are from **1** to **10**. For these odd primes, we have **3**; **5** and **7.**

If **p = 3** then **3p-2 + 2pa = 7 + 6a,** if **p = 5** then **3p – 2 + 2pa = 13 + 10a**, if **p = 7** then **3p – 2 + 2pa = 19 + 14a.**

Finally, to find the prime numbers which are from **1** to **100**, we take for the terms of **3p – 2 + 2pa**, **7 + 6a**, **13 + 10a**, **19 + 14a** that we must extract from Ni to find the numbers of **E.**

As a result, we write all the odd numbers from **1** to **100**, we underline the terms of **7 + 6a**, **13 + 10a**, **19 + 14a**, we note the numbers not underlined and we add **0** in the set.

To verify directly whether a number m of **Ni** is respectively a term of **7 + 6a**, **13 + 10a**, **19 + 14a**, we make

$$\boldsymbol{a} = \frac{\mathbf{m-7}}{\mathbf{6}}, \boldsymbol{a} = \frac{\mathbf{m-13}}{\mathbf{10}}, \boldsymbol{a} = \frac{\mathbf{m-19}}{\mathbf{14}}, \tag{7}$$

If there is **a** a belonging to **IN,** then **m** belongs to the sequence where **a** is.

Example :

Ni = 1 ;3 ;5 ;7 ;9 ;11 ;13 ;15 ;17 ;19 ;21 ;23 ;25 ;27 ;29 ;31 ;33 ;35 ;37 ;39 ;41 ;43 ;45 ;47 ;49 ;51 ;53 ;55 ;57 ;
59 ; 61 ;63 ;65 ;67 ;69 ;71 ;73 ;75 ;77 ;79 ;81 ;83 ;85 ;89 ;91 ;93 ;95 ;97 ;99

So E = { 0 ;1 ;3 ;9 ;11 ;15 ;17 ;21 ;27 ;29 ;35 ;39 ;41 ;45 ;51 ;57 ;59 ;65 ;69 ;71 ;77 ;81 ;95 ;99 }

For all **n ϵ E**, **M(n) = 2+n** is a **prime number**. **M** is the sequence that makes it possible to find the numbers of **P (set of prime numbers)**.

M(n) = 2 ;3 ;5 ;7 ;11 ;13 ;17 ;19 ;23 ;27 ;31 ;37 ;41 ;43 ;47 ;53 ;59 ;61 ;67 ;71 ;73 ;79 ;83 ;89 ;97

Application :

Find the prime numbers from **100** to **300**.

For this purpose, it is possible to use as a source of prime numbers 11; 13; 17 and 19 to widen the numbers of the set **E** obtained previously. If we replace 11; 13; 17 and 19 respectively in **3p – 2 + 2pa**, we will have **31 + 22a**; **37 + 26a**; **49 + 34a; 55 + 38a** therefore:

$$\mathbf{E = Ni \setminus \{7 + 6a; 13 + 10a; 19 + 14a; 31 + 22a; 37 + 26a; 49 + 34a; 55 + 38a\} \cup \{0\}} \tag{8}$$

for **prime numbers** from **1** to **300**.

Ni = 99; 101; 103; 105; 107; 109; 111; 113; 115; 117; 119; 121; 123; 125; 127; 129; 131; 133; 135; 137; 139; 141; 143; 145; 147; 149; 151; 153; 155; 157; 159; 161; 163; 165; 167; 169; 171; 173; 175; 177; 179; 181; 183; 185; 187; 189; 191; 193; 195; 197; 199; 201; 203; 205; 207; 211; 213; 215; 217; 219; 221; 223; 225; 227; 229; 231; 233; 235; 237; 239; 241; 243; 245; 247; 249; 251; 253; 255; 257; 259; 261; 263; 265; 267; 269; 271; 273; 275; 277; 279; 281; 283; 285; 287; 289; 291; 293; 295; 297; 299

E= {99 ;101 ;105 ;107 ;111 ;125 ;129 ;135 ;137 ;149 ;155 ;161 ;171 ;177 ;179 ;189 ; 191 ;195 ;197 ;195 ;197 ;227 ;231 ;237 ;245 ;255 ;257 ;261 ;275 ;279 ;285 ;287 ;291 ;297 ;299

M(n) = 101; 103; 107; 109; 113; 127; 131; 137; 139; 151; 157; 163; 167; 173; 179; 181; 191; 193; 197; 199; 211; 223; 227; 229; 233; 239; 241; 251; 257; 263; 269; 271; 277; 281; 283; 293

2.2. Demonstration of the Goldbach Conjecture:

Goldbach's conjecture says that all even numbers from **4** can be written as the sum of **2 prime numbers**. The Goldbach Conjecture does not mean that any even number greater than or equal to **4** can be written only by the sum of **2 prime numbers**, they can be written as the sum of 2 compound numbers and also, you must know that the sum of 2 prime numbers still does not give an even number. The Goldbach Conjecture means that for every even number, there are at least **2 prime numbers** such that their sums give this even number knowing that the **2 primes** can be equal or different.

Let **Md = 4 + 2d** with $d \in$ IN, the sequence being the even numbers beginning with **4**. If the Goldbach Conjecture is true then there exists at least one **P1** and **P2** such that **P1** + **P**2 = Md or **2 + a + 2 + b = 4 + 2d** with **a ϵ E** and **b ϵ E**.

$$\mathbf{a+b = 2d} \tag{9}$$

Check if the hypothesis is true:

$$\mathbf{a \in IN}$$
$$\mathbf{b \in IN}$$
$$\text{If } \mathbf{a \in \mathit{E}} \Rightarrow \mathbf{a \neq 3p - 2 + 2pn} \text{ with } \mathbf{n \in IN}$$
$$\text{If } \mathbf{b} \in E \Rightarrow \mathbf{b \neq 3p - 2 + 2pn} \text{ with } \mathbf{n \in IN}$$

Let **f (x)** be the function defined on **IR** and

$$\mathbf{f\ (x) = 3p\text{-}2 + 2px} \tag{10}$$

f is a polynomial function that can be derived on **IR** from the derived function **f '(x)** given by:

$$\mathbf{f\ '(x) = 2p} \tag{11}$$

$\forall$ **x ϵ IR**, **f'(x) > 0** so **f** is continuous and strictly increasing on **IR**

The smallest value of **f** is **4** with **p = 2** for all **x** ϵ IN then **f > 0** $\forall$ ***x ϵ* IN**,

$$\mathbf{b > 0 \Rightarrow -b < 0} \text{ then } \mathbf{-b \neq 3p\text{-}2 + 2pn}$$

$$\mathbf{2d\text{-}b \neq 3p\text{-}2 + 2pn + 2d} \tag{12}$$

Let **Un = 2pn + 2d** and **Vn = 2p'n'**; One is even for all **n ϵ IN**.

$$\forall \mathbf{\ d \in IN}, \exists \mathbf{\ p' \in P,\ n' \in IN \setminus 2pn + 2d = 2p'n'} \tag{13}$$

The terms of **Vn** are included in **Un**, for all **n ϵ IN**, there exists at least one **d** allowing to say that **Un = Vn** with

d ϵ IN.

So, one can transform the writing **2pn + 2d** into **2pn** where

$$\mathbf{2d\text{-}b \neq 3p - 2 + 2pn} \tag{14}$$

then there exists at least one

$$\mathbf{a = 2d - b} \tag{15}$$

and that **a** is a number belonging to **E**.

$$\mathbf{a > 0 \Rightarrow -a < 0}$$

$$-a \neq 3p - 2 + 2pn$$

$$2d - a \neq 3p - 2 + 2pn + 2d,$$

from what is shown above, we can transform **2pn + 2d** into **2pn** where:

$$2d - a \neq 3p - 2 + 2pn \tag{16}$$

Then there exists at least one **b = 2d - a** such that **b** is a number belonging to **E**.

Finally, there exists at least

$$a + b = 2d$$

such that **a** and **b** are numbers belonging to **E**.

$$2 + a + 2 + b = 4 + 2d$$

$$M(a) + M(b) = Md \tag{17}$$

then for every even number greater than or equal to **4** there are at least **2 prime numbers** such that their sum is equal to this even number.

2.2.1. Partial conclusion 1:

Goldbach's Conjecture is true.

$$\forall\, n \in IN,\ \exists\, a, b \in E \,/\, 2 + a + 2 + b = 4 + 2n \tag{18}$$

Note :

By demonstrating this **Goldbach's Conjecture**, it allowed me to discover that for any positive integer number other than **0** and **2**, that is **m** its **half**, there exists at least **m prime number** such as for all even numbers greater than or equal to this even **number**, it is possible to add the **m prime numbers** to have the even numbers. For example, for **4**, **2** is its **half**, all even numbers greater than or equal to **4** can be written as the sum of **2 prime numbers**. For **6**, **3** is its **half**, all even numbers greater than or equal to **6** can be written as the sum of **3 prime numbers**. For **8**, **4** is its **half**, all the even numbers greater than or equal to **8** can be written as the sum of **4 prime numbers.**

2.3. Demonstration of my Conjecture:

I remind you that my **Conjecture** says that if there is a positive even number other than **0** and **2**, **m** is its **half**, all **prime numbers** greater than or equal to **d** can be written as the sum of **m prime numbers.**

For example, for **4**, **2** is its **half**, all even numbers greater than or equal to **4** can be written as the sum of **2 prime numbers.** For **6**, **3** is its **half**, all even numbers greater than or equal to **6** can be written as the sum of **3 prime numbers.** For **8**, **4** is its **half**, all the even numbers greater than or equal to **8** can be written as the sum of **4 prime numbers.**

First of all, we know that if **n** $\in$ **E, M(n) = 2 + n** is a **prime number.**

Let **d** be an even number, **m** is its **half**:

The even numbers greater than **d** are the terms of the sequence

$$d + 2a$$

with $\mathbf{a} \in \mathbf{IN}$.

$$\text{If } \mathbf{n} \in \mathbf{E}, \mathbf{n} \neq \mathbf{3p - 2 + 2\,pl} \text{ with } \mathbf{l} \in \mathbf{IN}.$$
$$\mathbf{3p - 2 + 2\,pl} > 0 \text{ for all } \mathbf{l} \in \mathbf{IN}.$$

$$\text{Let } \underbrace{\mathbf{n_1, n_2, n_3, \ldots, n_m}}_{\mathbf{m\ times}}$$

$$\mathbf{-n_1 - n_2 - n_3 - \ldots - n_m} < \mathbf{0} \text{ bacause } \mathbf{n_1, n_2, n_3 \ldots, n_m} > \mathbf{0}$$
$$\text{So } \mathbf{-n_1 - n_2 - n_3 - \ldots - n_m} \neq \mathbf{3p - 2 + 2pl}$$
$$\mathbf{2a - n_2 - n_3 - \ldots - n_m = n_1}$$
$$\mathbf{2a - n_2 - n_3 - \ldots - n_m} \neq \mathbf{3p - 2 + 2pl + 2a}$$

Let $\mathbf{A = 2p_1l_1}$ et $\mathbf{B = 2p_2l_2 + 2a}$, any element of **A** is in **B** so we can transform $\mathbf{2pl + 2a}$ in $\mathbf{2pl}$ from where

$$\mathbf{2a - n_2 - n_3 - \ldots - n_m} \neq \mathbf{3p - 2 + 2pl} \tag{19}$$

So, there is at least one

$$\mathbf{n_1} \neq \mathbf{3p - 2 + 2pl} \text{ such as } \mathbf{n_1 = 2a - n_2 - n_3 - \ldots - n_m}$$

Valid for

$$\mathbf{n_2 = 2a - n_1 - n_3 - \ldots - n_m}$$
$$\mathbf{n_3 = 2a - n_1 - n_2 - \ldots - n_m}$$
$$\mathbf{\ldots}$$
$$\mathbf{n_m = 2a - n_1 - n_2 - n_3 - \ldots}$$

So there are at least $\mathbf{n1, n2, n3, \ldots, nm} \in \mathbf{E}$ such as

$$\mathbf{2a = n_1 + n_2 + n_3 + \ldots + n_m}$$
$$\mathbf{2m + 2a = 2 + n_1 + 2 + n_2 + 2 + n_3 + \ldots + 2 + n_m}$$
$$\text{Or } \mathbf{2m = d} \text{ so}$$
$$\mathbf{d + 2a = 2 + n_1 + 2 + n_2 + 2 + n_3 + \ldots + 2 + n_m}$$

$\mathbf{2 + n_1}$;$\mathbf{2 + n_2}$;$\mathbf{2 + n_3}$;… ;$\mathbf{2 + n_m}$ are respectively prime numbers equal to $\mathbf{M(n_1); M(n_2); M(n_3); \ldots ; M(n_m)}$.

2.3.1. Partial conclusion 2:

Any integer even number **d** different from **2** and **0**, let **m** its **half**, all the numbers greater than or equal to **d** can be written as the sum of **m prime numbers.**

2.4. Variance Theorem:

We will elaborate a new **theorem** called **variance theorem** which is stated as follows: every even number is the **difference** of **2 prime numbers.** For example, **2 = 5 - 3; 4 = 7 - 3; 6 = 17-11; 8 = 13-5; 10 = 17 - 7** etc.

Let's go to the **proof** of the **theorem:**

The theorem can be formulated as follows:

$$\mathbf{2 + a - (2 + b) = 2d} \tag{20}$$

with **a** and **b** numbers of the set **E** and of a number of **IN.**

$$\mathbf{2 + a - (2 + b) = 2d}$$

$$\mathbf{2 + a - 2 - b = 2d}$$

$$\mathbf{a - b = 2d \Rightarrow a = 2d + b} \tag{21}$$

Let $\mathbf{b} \in \mathbf{E}$,

$$\mathbf{b \neq 3p - 2 + 2pn}$$

$$\mathbf{b + 2d \neq 3p - 2 + 2pn + 2d}$$

But the elements of **2pn** are included in **2pn + 2d** then

$$\mathbf{b + 2d \neq 3p - 2 + 2pn + 2d \Rightarrow b + 2d \neq 3p - 2 + 2pn}$$

from where

$$\mathbf{a \neq 3p - 2 + 2pn.} \tag{22}$$

Then every even number can at least be written as the distance of **2 prime numbers.**

2.5. TWIN PRIME NUMBER CONJECTURE:

In this part, let us first look at the distribution of prime numbers. The distribution of prime numbers is not random, they follow a determined course but which however knows conditions. Each prime number found is a kind of process that eliminates the appearance of the following **prime numbers.** These are **independent.** But the sequence of **prime numbers** sometimes loses its terms through the distribution of divisible numbers to the terms they give. In addition to exceeding the square of a **prime number**, the latter increases the conditions of existence of the sequence from which **prime numbers** become increasingly rare.

The **prime number 2** will allow us to find all primes less than $\mathbf{3^2}$ according to the formula

$$\mathbf{2 + 2n + 1} \text{ or } \mathbf{2 + 2n - 1} \tag{23}$$

with $\mathbf{n \in IN}$. The prime numbers from this formula are **3; 5; 7.**

Now, all the following prime numbers are found by the formula

$$\mathbf{3 + 3n + 1} \text{ or } \mathbf{3 + 3n - 1} \tag{24}$$

with **n** an **odd number**, but these sequences experience conditions that increase after having exceeded the square of a **prime number.**

Up to the number $\mathbf{5^2}$, the suite knows no conditions. Thus, prime numbers from the formula are: **11; 13; 17; 19; 23.**

After having exceeded the square of **5**, the continuation begins to have conditions:

so **3 + 3n + 1** is a **prime number** and

$$\text{if } \frac{\mathbf{(3+3n+1)-5}}{5} \in \mathbf{IR \setminus Z}$$

$$\text{if } \frac{\mathbf{(3+3n-1)-5}}{5} \in \mathbf{IR \setminus Z},$$

Let $\mathbf{p_1; p_2; p_3; p_4; p_5; \ldots; p_n}$ **prime numbers** different from **2** and **3** with $\mathbf{p_1 < p_2 < p_3 < p_4 < p_5 < \ldots < p_n}$, **the prime numbers** greater than $\mathbf{pn^2}$ and lower than **pn + 12** are the **terms** of the sequence **3 + 3n + 1** or **3 + 3n - 1** satisfying the following **conditions:**

$$\frac{\mathbf{(3+3n-1)-p}}{\mathbf{p}} \in \mathbf{IR \setminus Z} \tag{25}$$

$$\text{with } \mathbf{p} \in \{\mathbf{p_1; p_2; p_3; p_4; p_5; \ldots; p_n}\}.$$

The results of operations:

$$\mathbf{3 + 3n - 1} \text{ or } \mathbf{3 + 3n + 1},$$

If they are **prime**, are the **consequences** of **the twin prime numbers.**

The probability of finding a term of **3 + 3n - 1** or **3 + 3n + 1** thanks to the **5 + 5n** and **7 + 7n** term is higher than the others because **5** and **7** are close to **6**.

With the multiplication of prime numbers, it is often rare to fall on a **prime number**, but the prime numbers that add to the terms always become **larger.**

The terms **3 + 3n + 1** or **3 + 3n - 1** are more present in a series of numbers than that of **p + pn** even though they increase more and more because they are very large. If the terms of **3 + 3n - 1** or **3 + 3n + 1** fall on a term of **p + pn**, then there will be **3 * p** difference for it to fall on a **term** of **p + pn** and the **terms** of **3 + 3n - 1** or **3 + 3n + 1** occur several times in this gap.

The presence of **twin prime numbers** is becoming increasingly rare but they will always come in a **moment.**

2.5.1. Partial conclusion 3:

There is an **infinity** of **twin prime numbers.**

3. RESULTS AND ANALYSIS

3.1. Sub section 1

Goldbach's Conjecture is true.

3.2. Sub section 2

Any integer even number **d** different from **2** and **0,** let **m** ist half, all the numbers greater than or equal to **d** can be written as the sum of **m Prime Numbers.**

3.1. Sub Section 3

Then every even number can at least be written as the distance of **2 primes numbers.**

3.2. Sub Section 4

There is an **infinity** of **twin prime numbers.**

4. CONCLUSION

To overcome this study, we can conclude that the prime numbers show more and more fascinating aspects. The elaboration of a function giving as an end set the set of prime numbers allowed us to prove the veracity of Goldbach's conjecture as well as another which generalizes it and in addition to show a variance of prime numbers.

On the other hand, with a more detailed explanation of the distribution of prime numbers, we have explained the infinity of twin prime numbers.

Thus, from the results obtained, we can conclude that the appearance of prime numbers does not emanate from chance but follows a well-determined logic.

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