FAST AND EFFICIENT FEM ANALYSIS OF E AND H PLANE WAVEGUIDE BENDS DISCONTINUITIES

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ABSTRACT

Rectangular waveguide discontinuities are frequently used in many applications in communication systems. Fast and efficient technique, based on finite element method FEM is applied in this paper to the analysis of bends in lossless rectangular waveguide for both H and E plane. These junctions, excited by fundamental TE_{10} mode, are analysed with taking into account the effects of higher order modes due to the presence of neighbouring discontinuities.
The scattering matrix of many complex microwave devices is obtained by chaining S matrices of simple discontinuities.
All these numerical results have been validated by comparison with results available in the scientific literature.

Index terms**:** Bends, Rectangular waveguides, Finite Element Method, Discontinuities, S matrices.

RESUME

Les discontinuités en guides d'ondes rectangulaires sont fréquemment employées dans plusieurs applications des systèmes de communication. Une technique rapide et efficace, basée sur la méthode des éléments finis FEM est appliquée dans cet article pour l'analyse des coudes en guides d'ondes rectangulaires sans pertes dans les deux plans H et E. Ces jonctions, excitées par le mode fondamental TE_{10}, sont analysées en tenant compte de l'influence des modes supérieurs due à la présence des discontinuités proches.
La matrice de dispersion de plusieurs dispositifs micro-ondes complexes est obtenue par le chaînage des matrices S des discontinuités simples.
Tous ces résultats numériques ont été validés par la comparaison avec des résultats disponibles dans la littérature scientifique.

Mots clés **:** Coudes, Guides d'ondes rectangulaires, Méthode des éléments finis, Discontinuités, Matrices S.

INTRODUCTION

Bends in rectangular waveguide technologies play an essential role in satellite and communication systems, channelling of signals between different components in system.
H and E plane bends have significant impact on the performance of sophisticated network such as those necessary for antenna beam forming.

In the past years, the numerical solutions of waveguide discontinuity problems have been studied using different methods. In many works, the usual approach is based on finite element method (FEM) in combination with integral equations [1,2,3], others with modal expansions [4,5,6] and still others use the time domain finite difference approach (FDTD) [7]. Weisshaar [8] presented an accurate method based on method of moments; the modes of rectangular waveguide are used as basis functions in the solution of Helmholtz equation.
In this paper, we present a simple and accurate procedure to determine the modal scattering of rectangular waveguide bends at E and H plane belongs to the class that use a modal expansion to represent the electromagnetic field into the guiding structures connected to the junctions.

We are interested in this work to analyse various waveguide bends in E and H plane using the standard WR 75 waveguides; the bend angle is kept fixed at 90° and 180°.

The details of formulations are given in this paper. Our results are compared with available published data.

FORMULATION

We consider an H plane waveguide multiport junction, as shown in figure1, constitutes of the discontinuity region (Ω)connected to rectangular waveguides. The junction is excited by TE_{10} mode. Since this junction is symmetric along the y direction, only the TE_{m0} modes can be excited into the waveguide and the electromagnetic problem is formulated with the electric field component Ey.

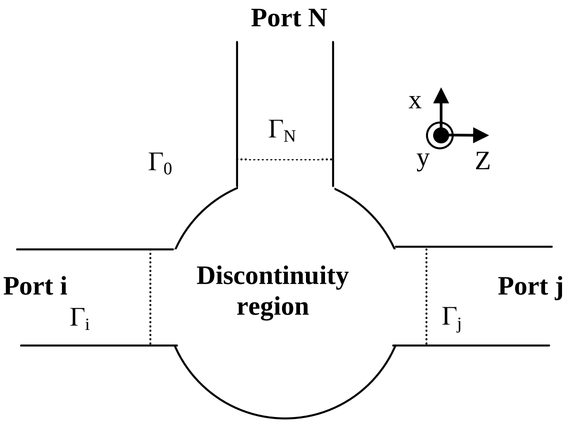

Figure 1: Multiport H plane junction of rectangular waveguide

The discontinuity region (Ω) is delimited by perfectly conducting wall Γ_0 and by reference planes Γ_k (k= 1, …,N) located far away from the junction, where it is assumed that all the higher modes have been strongly attenuated and only the dominant mode is present.
The components of the electric and magnetic fields in the waveguide tangential to the port (k), when the port (j) is fed with the TE_{10} mode, can expressed as :

$$E^{(k)}_{y\_go}(x^{(k)}) = \delta_{kj} e_1^{(j)}(x^{(j)}) e^{j\beta_1^{(j)} z^{(j)}} + \sum_{m=1}^{\infty} B_m^{(k)} e_m^{(k)}(x^{(k)}) e^{-j\beta_m^{(k)} z^{(k)}} \quad (1)$$

$$j\omega\mu_0 H^{(k)}_{x_{go}}(x^{(k)}) = \frac{\partial E^{(k)}_{y_{go}}}{\partial z^{(k)}} \quad (2)$$

$e_m^{(k)}$ (m = 1, 2, …) are the basic functions of the TE_{m0} modes into the waveguide k.

$$e_m^{(k)}(x^{(k)}) = \frac{2}{\sqrt{a^{(k)}}\,b}\sqrt{\frac{k_0 Z_0}{\beta_m^{(k)}}}\sin\left(\frac{m\pi}{a^{(k)}}x^{(k)}\right) \quad (3)$$

k_0 and z_0 are respectively the free space propagation constant and the characteristic impedance.
$\beta_m^{(k)}$ is the propagation constant of TE_{m0} modes into the waveguide (k) with dimensions ($a^{(k)} \times$ b).

Inside the region (Ω), the electric field E_y of these modes is a solution of the Helmholtz equation.

$$\nabla_t.\left(\frac{1}{\mu_r}\nabla_t E_y\right) + k_0^2\varepsilon_r E_y = 0 \quad (6)$$

The associated boundary conditions are :

$$E_y = 0 \text{ at } \Gamma_0 \quad (7)$$

$$\frac{\partial E_y}{\partial n} = 0 \quad (8)$$

at magnetic wall.
The continuity conditions of fields at each port give:

$$E_y\big|_{\Gamma_k} = E^{(k)}_{y_{go}} \quad k = 1,..., N \quad (9)$$

$$H_x\big|_{\Gamma_k} = H^{(k)}_{x_{go}} \quad k = 1,..., N \quad (10)$$

The boundary value problem described by the scalar Helmholtz equation (6) with boundary conditions can be written in a weak form by using the weighted residual procedure:

$$\iint_\Omega \nabla_t W.\frac{1}{\mu_r}\nabla_t E_y d\Omega - k_0^2\iint_\Omega \varepsilon_r W E_y d\Omega - \sum_{k=1}^{N}\int_{\Gamma_k} W\frac{\partial E^{(k)}_{y_{go}}}{\partial z^{(k)}} d\Gamma_k = 0 \quad (11)$$

Where W are weighting functions.
These equations constitute the system of equations to solve in H plane by using the finite element method (FEM).
The solution on each element (e) is sought among the approximating functions $\overline{E}_y^{(e)}$ of the form:

$$\overline{E}_y^{(e)}(x,y) = \sum_{j=1}^{N^{(e)}} \overline{E}^{(e)}_{y_j}\alpha_j^{(e)}(x,y) \quad (12)$$

Where
$\overline{E}^{(e)}_{y_j}$ *and* $\alpha_j^{(e)}(x,y)$ $(j = 1,..., N^{(e)})$ are respectively the coefficients and the interpolation functions.
$W_i^{(e)} = \alpha_i^{(e)}$ $(i = 1,..., N^e)$, and the residue $R_i^{(e)}$ on element (e) is expressed as:

$$R_i^{(e)} = \frac{1}{\mu_r}\iint_{\Delta^{(e)}} \nabla_t\alpha_i^{(e)}.\nabla_t\overline{E}_y^{(e)} d\Omega - k_0^2\varepsilon_r\iint_{\Delta^{(e)}}\alpha_i^{(e)}\overline{E}_y^{(e)} d\Omega - \sum_{k=1}^{N}\int_{\Gamma_k^{(e)}}\alpha_i^{(e)}\frac{\partial E^{(k)}_{y_{go}}}{\partial z^{(k)}} d\Gamma_k \quad i = 1,..., N^{(e)} \quad (13)$$

or in matrix form:

$$\frac{1}{\mu_r}\left[S^{(e)}\right].\left[\overline{E}_y^{(e)}\right] - k_0^2\varepsilon_r\left[T^{(e)}\right].\left[\overline{E}_y^{(e)}\right] + \sum_{k=1}^{N}\left\{\left[C_k^{(e)}\right].\left[B_k\right] - \left[H_k^{(e)}\right]\right\} = \left[R^{(e)}\right] \quad (14)$$

$\lfloor S^{(e)} \rfloor$ and $\lfloor T^{(e)} \rfloor$ are the local matrices of scalar nodal elements, and $\lfloor E_y^{(e)} \rfloor$ is the vector of nodal unknown coefficients of elements (e). $[B_k]$ is a column vector whose jth entry is the amplitude of the transmitted mode j at port (k), and $\lfloor C_k^{(e)} \rfloor$ and $\lfloor H_k^{(e)} \rfloor$ come from the contour integrals at the ports. $\lfloor C_k^{(e)} \rfloor$ is a (3*M) matrix :

$$C_{km}^{(e)} = j\beta_m^{(k)} \int_{\Gamma_k^{(e)}} \alpha_i^{(e)} e_m^{(k)} d\Gamma_k^{(e)} = \qquad (15)$$

$$j\beta_m^{(k)} \frac{2}{\sqrt{a^{(k)}b}} \sqrt{\frac{k_0 z_0}{\beta_m^{(k)}}} \int_{\Gamma_k^{(e)}} \alpha_i^{(e)} \sin\left(\frac{m\pi}{a^{(k)}} x^{(k)}\right) d\Gamma_k^{(e)}$$

and $H_k^{(e)}$ is a (3×1) column vector:

$$H_{k_i}^{(e)} = j\beta_1^{(\iota)} \delta_{kl} \int_{\Gamma_k^{(e)}} \alpha_i^{(e)} e_1^{(\iota)} d\Gamma_k^{(e)} \qquad (16)$$

$$= \delta_{k\iota} j\beta_1^{(\iota)} \frac{2}{\sqrt{a^{(\iota)}b}} \sqrt{\frac{k_0 z_0}{\beta_1^{(\iota)}}} \int_{\Gamma_k^{(e)}} \alpha_i^{(e)} \sin\left(\frac{\pi}{a^{(\iota)}} x^{(\iota)}\right) d\Gamma_k^{(e)}$$

where (ι) is the port fed and $\delta_{k\iota}$ is Kronecker's delta.

The continuity boundary conditions at each port can be assembled into a global system and the residue annihilated yields the total system of equations to be solved.

Folowing the discussion in [9,10], we get the matrix equation:

$$\begin{bmatrix} [A] & [D] \\ [C] & [F] \end{bmatrix} \begin{bmatrix} [B] \\ [E] \end{bmatrix} = \begin{bmatrix} [E^{inc}] \\ [H^{inc}] \end{bmatrix} \qquad (17)$$

where $[A]$ is a diagonal matrix, $[D]$ and $[C]$ are sparse rectangular matrices, and $[F]$ is the sparse and symmetric finite element matrix.

The formulation of the electromagnetic problem in the case of homogenous E plane junctions follows a path similar to that outlined in the H plane case.

RESULTS

The bend is a simple structure, used to change the direction of a waveguide.

The various waveguide 90 degree bends (figure 2) are:

a) Square bends;

b) Fully mitered bends;

c) Partially mitered bends;

d) Circular bends.

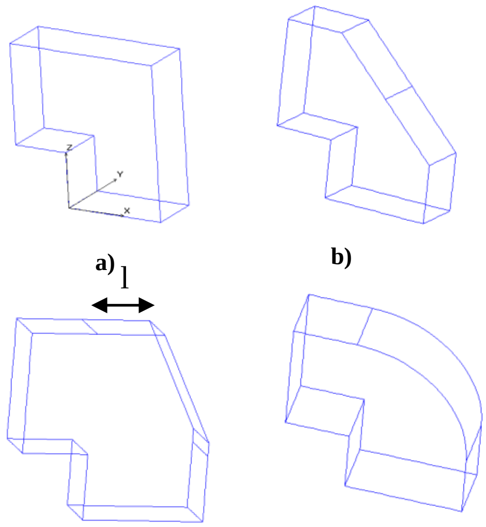

Figure 2: Different configurations of waveguide bends

The analysis with the standard WR 75 (a=18.35mm, b=9.175mm) in both H and E plane is shown in figure 3, figure 4, figure 5 and figure 6. The references planes have been placed at a distance equal to 2 mm from the interior edge of the bend and l =1.5 mm is the position the miter defined in figure 2.C.

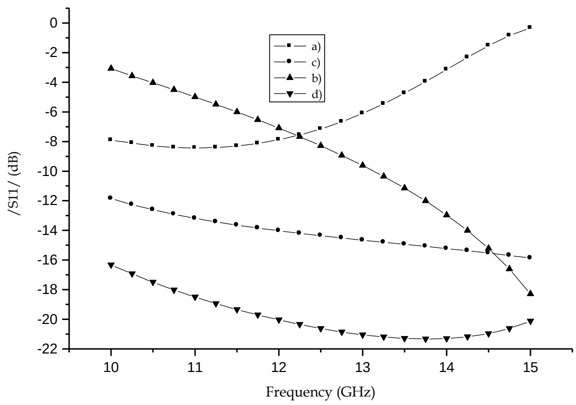

Figure 3: Return loss of H plane bends a) square bend. b) Fully mitered bend. c) Partially mitered bend. d) circular bend.

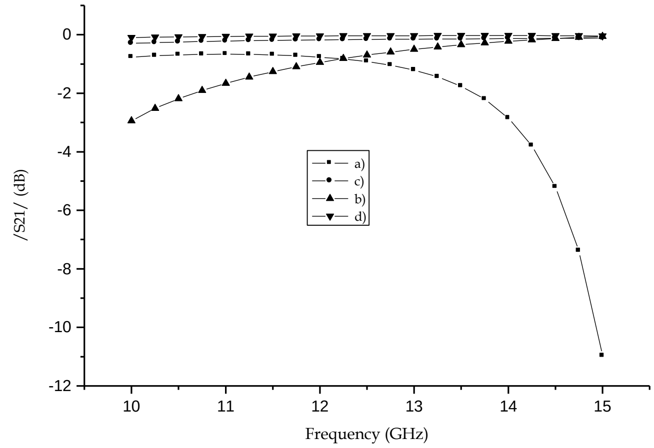

Figure 4: Transmission coefficient of H Plane bends

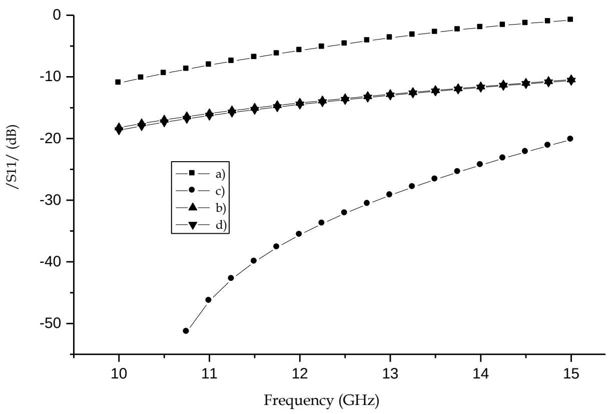

Figure 5: Return loss of E plane bends

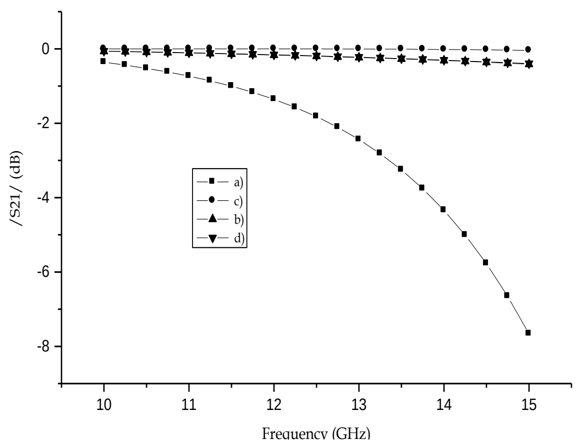

Figure 6 : Transmission coefficient of E plane bends

- Figure 3 shows the reflection coefficient S_{11} for different H plane bends, The square bend ''a'' has the worst behaviour, and the circular bend has the best but it is difficult to manufacture.

- The transmission coefficient S_{21} (figure 4) show that the square bend is well at low frequencies, fully mitered bend displays an opposite behaviour and the circular bend present the best response of all.

- The return loss and transmission coefficients of E plane (fiure5 and figure 6) show that the circular bend is the best and the square bend present the worst response.

Good agreement with the results of [9, 11] is obtained for 10 TE_{m0} modes in each ports of structure.

Now, we take the standard WR 75 but with dimensions (a=19.05mm, b=9.525mm) in order to compare our results with those of reference [12].

Figure 7 and figure 8 show results of analysis of partially mitered bend in E plane (figure 2.c) for l equals 3.81 mm, 6.35 mm and 8.89 mm. These diagrams show that low return loss S11 is over a wide frequency band for l= 8.89 mm which is the best response.

This analysis is in good agreement with the data provided in [12].

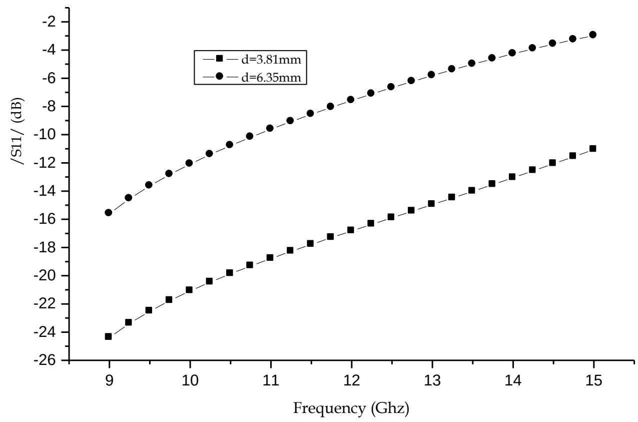

Figure 7: Return loss of E plane mitered bends f or l =3.81 mm and l =6.35 mm

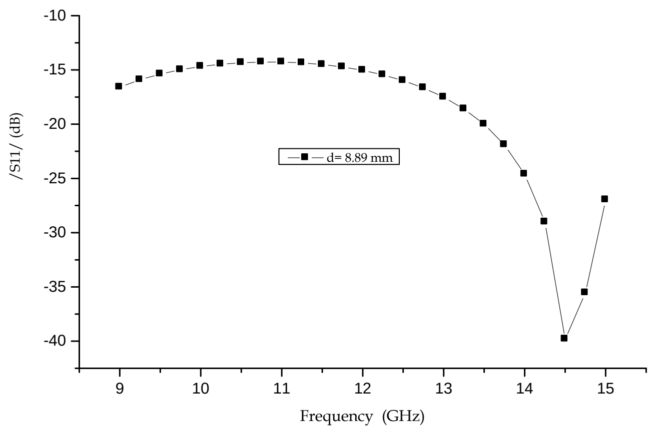

Figure 8: Return loss of E plane mitered bends for l = 8.89 mm

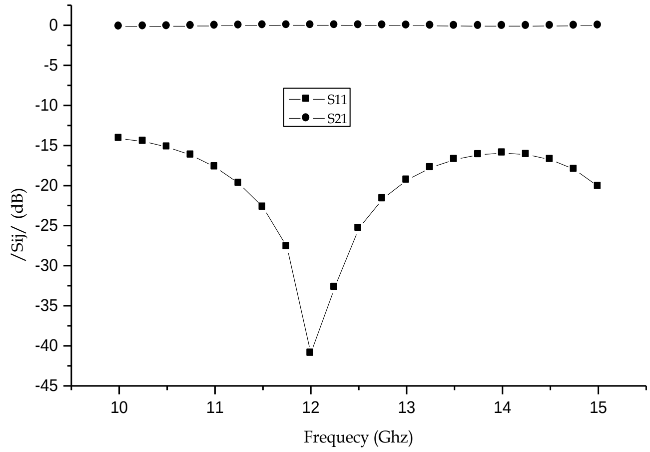

Figure 10: Parameters Sij of 180° E plane mitered bends for l = 3.81 mm

Many applications require cascaded structures as:

Cascaded of two mitered waveguide bends (figure 2.c) constitute waveguide 180 degree bends in E plane (figure 9).

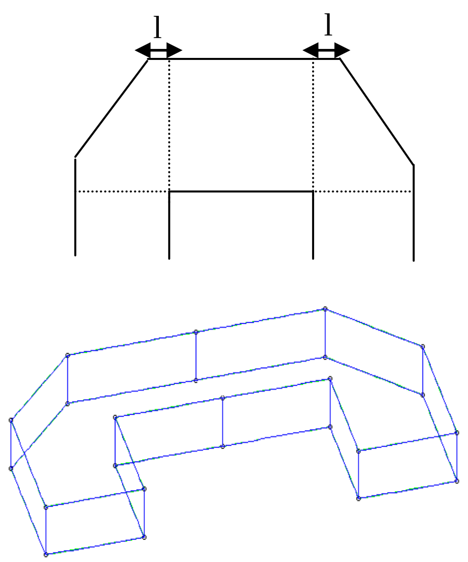

Figure 9 : 180 degree E plane Partially mitered bends with l =3.81 mm

Figure 10 shows the scattering characteristics of 180 degree E plane mitered bend with WR 75 (a = 19.05 mm, b = 9.525 mm) and l =3.81 mm.
Good agreement with reference [13] is obtained with 20 TE_{m0} modes.

CONCLUSION

An analysis, based on Finite Element Method (FEM), has been developed in this article which is capable of treating a wide variety multiport junctions homogenous or inhomogeneous in E and H plane.
The efficiently and versatility of this method is verified by different configurations of waveguide bends in both E and H plane.
Numerical results obtained from the present work are in excellent agreement with those published in the following references.

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