Complementarity and Substitution between the Neoclassical and Endogenous Growth Theories

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Résumé :

Cet article tente de définir la relation de complémentarité et de substitution entre la Théorie Néoclassique et celles de la Croissance Endogène. Certes ces dernières se sont basée sur les résultats de la précédente et ont essayé de la compléter, mais elles ont, tout de même, apporté plusieurs changements radicaux, surtout en ce qui concerne l'origine du progrès technique et l'imperfection de la concurrence.

Mots-clés: Théorie Néoclassique, Croissance Endogène, Point de stabilité.

Abstract:

This article attempts to define the process of complementarity and substitution between the Neoclassical and Endogenous Growth Theories. Certainly the latter based on the results of the former and tried to complete it, but they brought many radical changes, especially in regard of the origin of technical progress and releasing the assumption of perfect competition.

Keywords: Neoclassical Theory, Endogenous Growth, Steady-state

Introduction:

The field of economic growth has enormously evolved since its foundation by political economists of the Eighteenth Century; the pace of evolution itself increased over time. The Classical Theory reigned over a century and a half; the Neoclassical Model was the prevalent for a few decades; while Modern Theories are updated almost every decade. In most cases, each new theory is based on the old one; some assumptions and techniques are maintained, others are renewed. The same process occurred between Neoclassical and Endogenous Growth Theories, the latter were built on the previous but included many changes.

Through this paper, we'll try to answer a simple question: The Endogenous Growth Theories have only completed the Neoclassical Theory or they have completely replaced it? To investigate the relation, first, the Neoclassical Model is developed through its different versions, then, an Endogenous Growth Model is introduced, and at each step, similarities and differences are highlighted. Results and comments are presented at the conclusion.

1. The Neoclassical Growth Theory [1]

The Neoclassical Growth Theory was first developed by Frank Ramsey in the 1920s, but it was Robert Solow who, in 1956, suggested its most popular version in a seminal paper on economic growth and development entitled "*A Contribution to the Theory of Economic Growth*". The key aspect of the Solow model is the Neoclassical form of the production function, a specification that assumes constant returns to scale, diminishing returns to each input, and positive and smooth elasticity of substitution between the inputs. This production function is combined with a constant-saving-rate rule to generate a simple general equilibrium model of the economy.[2]

1.1. Presentation of the Neoclassical Model

In his model, Solow lifted the hypothesis of production techniques' rigidity maintained by Harrod. Moreover, he assumes that at every moment, *ex ante* decisions of saving and investing coincide; coordination problem is, then, solved.[3]

More simplifying assumptions are, then, formulated: [4]

- The world consists of countries that produce and consume a single, homogeneous good (output);
- There is no international trade in the model (since there is only one good);

- Technology available to firms in this simple world is unaffected by the actions of the firms, it is given exogenously.

1.1.1. Characteristics of the Neoclassical Model

We say that a production function is Neoclassical if it satisfies the following properties: [5]

* Constant returns to scale: $F(\lambda K, \lambda L) = \lambda . F(K, L)$ for all $\lambda > 0$, the definition of scale includes only the two rival inputs, K and L;

* Positive and diminishing returns to private inputs: for all $K > 0$ and $L > 0$, $F(.)$ exhibits positive and diminishing marginal products with respect to each input:

$$\frac{\partial F}{\partial K} > 0 \,, \quad \frac{\partial^2 F}{\partial^2 K} < 0$$
$$\frac{\partial F}{\partial L} > 0 \,, \quad \frac{\partial^2 F}{\partial^2 L} < 0$$

The Neoclassical Theory assumes that, *ceteris paribus*, each additional unit of capital delivers positive additions to output, but these additions decrease as the number of machines rises. The same property is assumed for labor;

* Inada conditions: The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0, and approaches 0 as capital (or labor) goes to infinity:

$$\lim_{K \to 0} \left(\frac{\partial F}{\partial K}\right) = \lim_{L \to 0} \left(\frac{\partial F}{\partial L}\right) = \infty$$
$$\lim_{K \to \infty} \left(\frac{\partial F}{\partial K}\right) = \lim_{L \to \infty} \left(\frac{\partial F}{\partial L}\right) = 0$$

* Essentiality: An input is essential if a strictly positive amount is needed to produce a positive amount of output: $F(0, L) = F(K, 0) = 0$. This property implies also that output goes to infinity as either input goes to infinity: $F(\infty, L) = F(K, \infty) = \infty$.

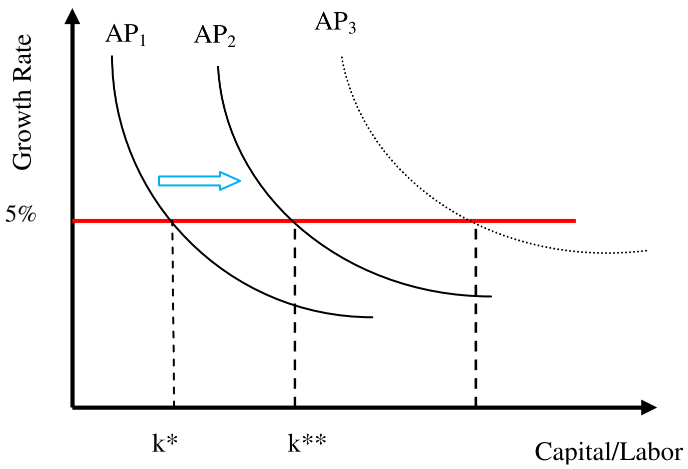
1.1.2. Sustainability of Economic Growth

During the late 1950s, just after Solow has explained his theory, the United Kingdom was experiencing a slow population growth (1%), a respectable per capita income (£6000), an acceptable saving rate (20 % of income), and most of all big advances in technological research. [6] As a whole, the economy enjoyed new levels of prosperity and growth. But, will the growth last?

The Neoclassical Growth Theory says that the prosperity will last (population growth rate is not enough high to lower wages, as predicted by the Classical Growth Theory), but growth will stop unless technology keeps advancing (because of diminishing returns to capital stock).

According to the Neoclassical Growth Theory, all economies have access to the same technologies, and capital is free to roam the globe. So, growth rate and income levels per person around the world will converge. In reality, some signs of convergence can be seen among rich countries, but not in the world as a whole.[7] The theory emphasizes the free market as the key allocation mechanism. The decision whether to consume or invest resources is then dependant on market price signals, not on orders from governments. It assumes perfect competition, mobile resources, fixed technology and prices determined in free markets. An important constraint to economic growth in this model is that investment in capital is subject to the law of diminishing returns. So, the level of investment is determined by the rate of return anticipated on a new investment project compared to the going interest rate in the financial markets.[8]

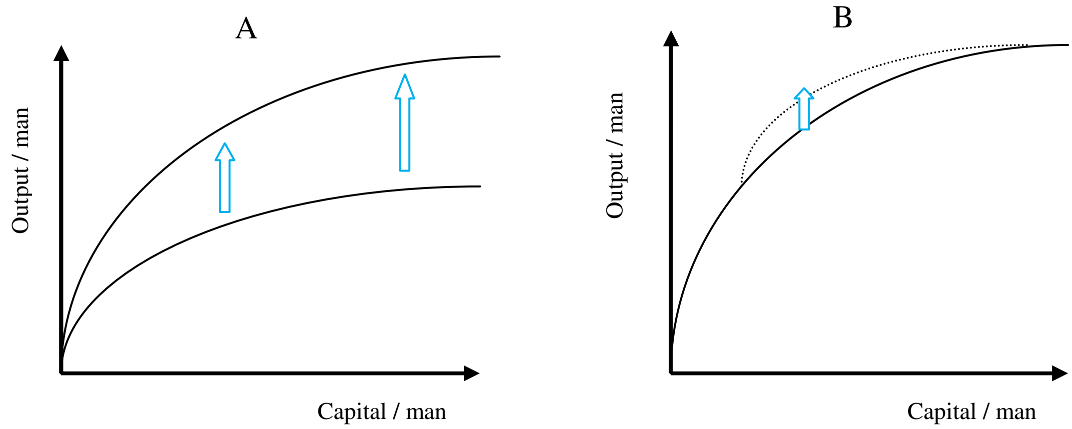
Figure 1: Exogenous Technical Change



Source: Cleaver T "Economics. The Basics" Routledge, 2004, p 180.

Given an unchanging population growth rate, if capital stock grows faster than the labor force does, then the rate of return on capital will fall as a result of diminishing returns. Capital stock must grow just fast enough to equal that needed to equip the labor force. In the meanwhile, living standards can improve in the short-run only until per capita capital reaches its steady-value; therefore living standards are condemned to stay the same. No further growth is possible unless exogenous technological growth occurs. In that case, capital productivity may increase. This effect is represented in **Figure 1** by the shift that occurs on curve AP. The downward slope still remains, but the steady-state capital per person will be reached at k^{**}.

Figure 2: The effects of technological progress



Source: Atkinson A.B. & Stiglitz J.E "A New View of Technological Change" The Economic Journal, vol. 79, n° 315, 09.1969, p 573.

The recent literature on technical progress has almost entirely been based on the assumption that its effect can be represented as shifting the production function outwards as illustrated in **Figure 2 (A)**. Technical advance is assumed to raise output per head for all possible techniques. The theory seems to have missed one of the most important points of the activity analysis (Mrs. Robinson's blue-print approach): that if one brings about a technological improvement in one of the blue-prints this may have little or no effect on the other blue-prints. If the effect of technological advance is to improve one technique of production but not other techniques of producing the same product, then the resulting change in the production function is represented by an outward movement at one point and not a general shift, as we may see in **Figure 2 (B)**.

Most of the growth models discussed in this article has the same basic general-equilibrium structure.[9] First, households own the inputs and assets of the economy, including ownership rights in firms, and choose the fractions of their income to consume and save. Each household determines how many children to have, whether to join the labor force, and how much to work. Second, firms hire inputs, such as capital and labor, and use them to produce goods that they sell to households or other firms. Firms have access to a technology that allows them to transform inputs into output. Third, markets exist on which firms sell goods to households or other firms and on which households sell the inputs to firms. The quantities demanded and supplied determine the relative prices of the inputs and the produced goods.

1.2. The Solow Basic Model

This first version of the Solow Model is a very simple one, it emphasizes the Classical Growth Theory since it involves no technological progress. However, understanding the extended models requires beginning with the simplest one.

1.2.1. The Model

The Solow Model is built around a production function and a capital accumulation equation. The production function describes how inputs such as buildings, cars, farmers, and seeds are used to produce output. An important remark is that these inputs are rivalries: they can't be used by multiple producers simultaneously. Inputs are grouped into two categories, capital, K, and labor, L. Y denotes output. We assume that production function takes a Cobb-Douglas form and is given by:

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha} \quad (1)$$

Where α is a constant between 0 and 1.

Note that: $f'(k) = \alpha k^{\alpha-1} > 0$, $f''(k) = -\alpha - (1-\alpha)k^{\alpha-2} < 0$, $\lim_{k\to\infty} f'(k) = 0$, $\lim_{k\to 0} f'(k) = \infty$. Thus, the Cobb-Douglas form satisfies the properties of a Neoclassical production function.

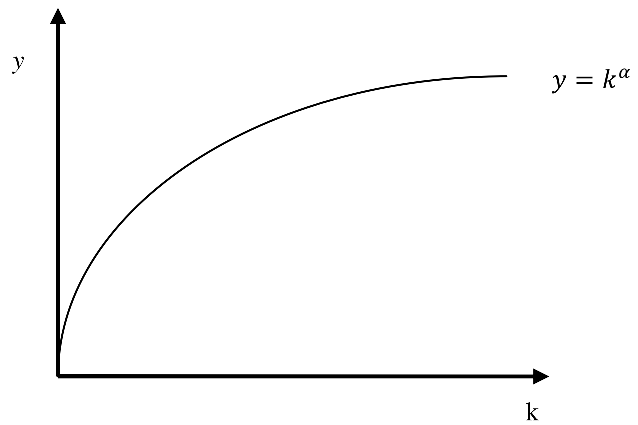
Per capita variables are more representative for cross-countries comparisons (according to the World Bank database, in 2013, GDP of Russia was equal to 35 times that of Luxembourg, but since the numbers of inhabitants are largely different, GDP per person in Luxembourg was more than 11 times the per capita GDP of Russian citizens), and since the definition of constant returns to scale applies for all values of λ, it also applies to $\lambda = 1/L$.

The production function from (1) can be written in terms of output per worker $y \equiv Y/L$, and capital per worker $k \equiv K/L$: [10]

$$y = f(k) = k^{\alpha} \quad (2)$$

Figure 3 shows that the production function is exposed to the diminishing returns rule; each additional unit of capital we give to single worker increases its output by less and less.

Figure 3: A Cobb-Douglas production function



Source: Jones C.I "*Introduction to Economic Growth*" W.W. Norton & Company, USA, 1998, p 21.

The second key equation of the Solow Model is the capital accumulation equation: [11]

$$\dot{K} = sY - dK \tag{3}$$

$\dot{K}$ is the continuous time version of $K_t - K_{t-1}$, which is the change in the capital stock per period. The over-dot denotes a derivative with respect to time: $\dot{K} = \partial K/\partial t$.

sY is the gross investment. Solow assumes that workers/consumers save a constants fraction s of their combined incomes. The economy is closed, so that saving equals to investment:

$$I(t) = S(t) \equiv Y(t) - C(t).$$

dK is the depreciation that occurs during the production process. The standard functional form that we use implies that a constant fraction, d, of the capital stock depreciates every period (regardless of how much output is produced). Before wearing out completely, all units of capital are assumed to be equally productive.

Labor input varies over time because of population growth, changes in participation rates, shifts in the amount of time worked by the typical worker, and improvements in the skills and quality of workers. For simplification, Solow assumes that everybody works the same amount of time with the same constant skill. Population growth reflects the behavior of fertility, mortality, and migration.[12] He assumes also that the labor force growth rate $\dot{L}/L$ is constant and equals to the population growth rate, given by the parameter n. The exponential growth is seen in the following relation: [13] $L(t) = L_0 e^{nt}$

Using some mathematical transformation (taking logs and derivation) on equation (3), we get:

$$\frac{\dot{k}}{k} = \frac{sY}{K} - n - d$$

$$\frac{\dot{k}}{k} = \frac{sy}{k} - n - d$$

The capital accumulation equation in per worker terms is, then, written:

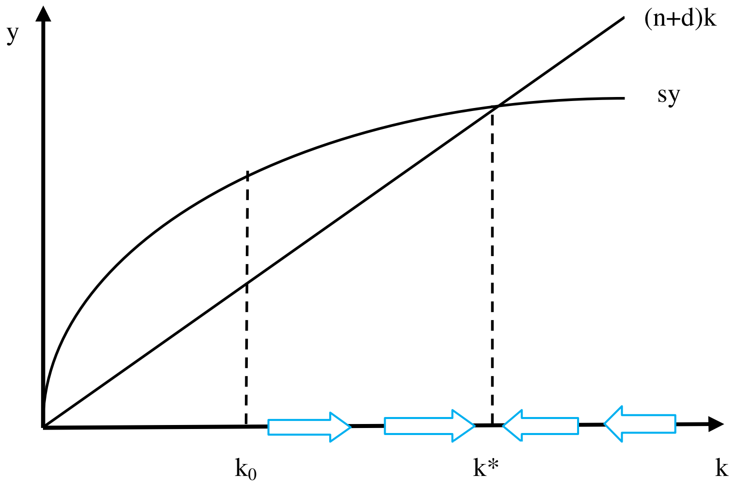
$$\dot{k} = sy - (n + d)k$$

The term $(n + d)$ is the effective depreciation rate for the capital-labor ratio k.

Investment per worker sy increases k, depreciation per worker dk reduces it. So does the population growth, in term nk, because there are more workers for the same amount of capital.

Let's suppose an economy that starts with a given stock of capital per worker k_0, a given population growth rate n and investment rate s. The fundamental question is "How economy grows?"

Figure 4: The Basic Solow Diagram



Source : Jones C.I, op.cit, p 25.

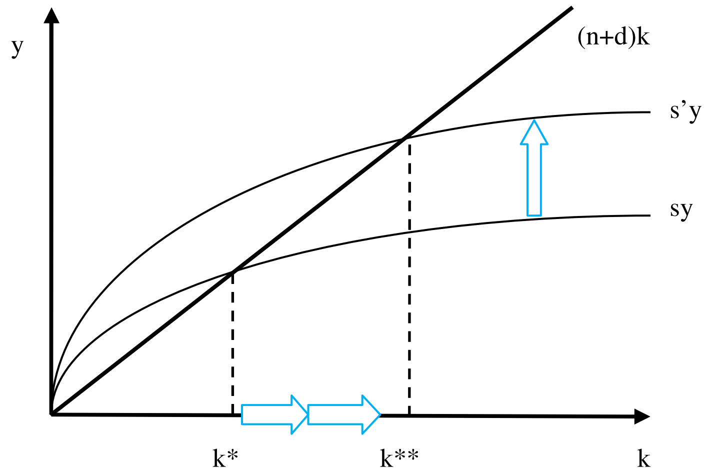
Figure 4 represents the Solow Diagram in which two curves can be distinguished. The first curve is the amount of investment per person, sy, assumed to have the same shape as the production function. The second curve is the amount of new investment per person required to keep the amount of capital per person constant, represented in the line $(n+d)k$. The difference between these two curves represents the change in the amount of capital per worker. When this difference is positive and the economy increases its capital per person, starting from k_0, we say that *capital deepening* occurs. When the difference is zero, at point k^*, but capital stock K keeps growing, due to population growth, we say that *capital widening* occurs. k^* is called the *Steady State* Capital-Labor ratio. If capital stock per worker is larger than k^*, the amount of investment per worker provided by the economy is not enough to keep the capital-labor ratio constant, $\dot{k}$ is then negative which means that capital per worker declines until it reaches k^*.

1.2.2. Comparative Statics

Comparative statics are used to examine the response of the model to changes in the values of various parameters,[14] such as investment rate and population growth rate.

***An increase in the investment rate*:** Consider an economy in its steady-state value of capital stock per worker. If the consumers decide to increase permanently their investment rate from s to s', the investment per worker curve shifts upward from sy to $s'y$ as shown in **Figure 5**. The positive difference between the sy and $(n+d)k$ curves causes a new capital deepening situation until capital stock per worker reaches its new steady state k^{**}.

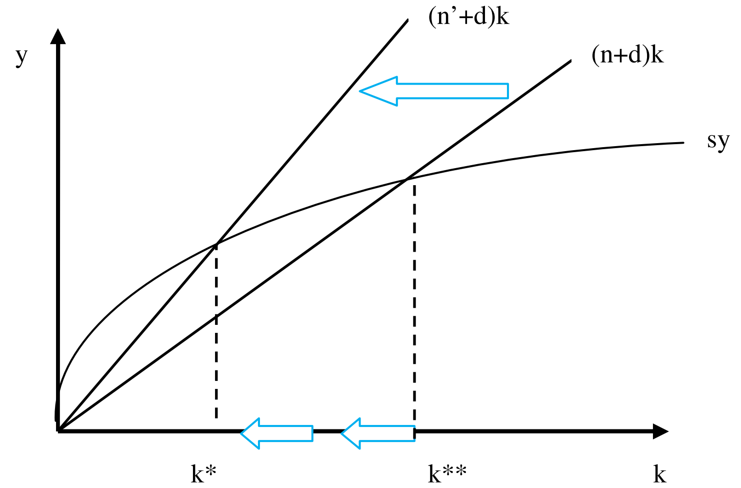
Figure 5: An increase in the investment rate



Source : Jones C.I, op.cit, p 27.

***An increase in the population growth rate*:** Consider an economy that has already reached its steady state. For some reason (migration or better health care), the population growth rate increases from n to n'. The slope of the $(n+d)k$ curve increases, the curve itself shifts upward to $(n'+d)k$. In result, the curves difference is negative which means a decline in the amount of capital stock per worker. The new steady-state point will be at k^{**} in the **Figure 6**.

Figure 6: An increase in population growth rate



Source : Jones C.I, op.cit, p 28.

1.2.3. Solving for Steady-state Magnitudes

By definition, the steady-state quantity of capital per worker is defined by the condition that $\dot{k} = 0$. By substituting (2) in (3): [15] $\dot{k} = sk^{\alpha} - (n + d)k$

Setting the right-hand side of the equation equal to zero yields:

$$k^* = \left(\frac{s}{n + d}\right)^{1/(1-\alpha)}$$

Substituting this in the production function reveals the steady-state quantity of output per worker: $y^* = \left(\frac{s}{n+d}\right)^{\alpha/(1-\alpha)}$

This equation reveals the Solow Model's answer to the question "Why are we so rich and they so poor?". Countries with high savings rates accumulate more capital per worker, and as a result, more output per worker. Countries with high population growth rates tend to be poorer; savings in these countries go simply to keep the capital-labor ratio constant in the face of a growing population.

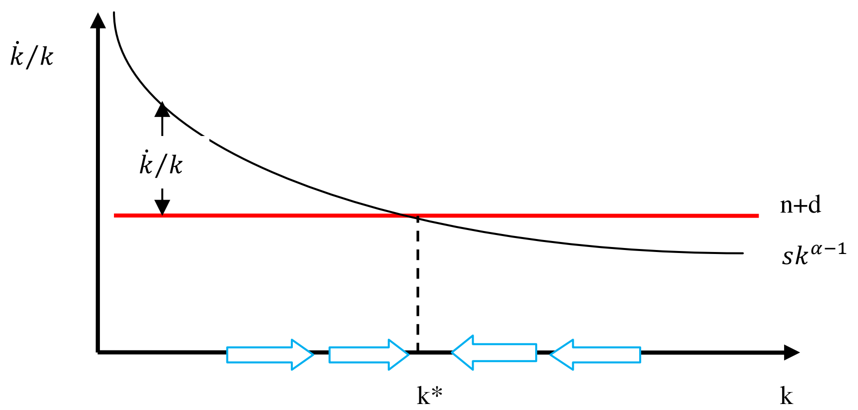
In the Solow Basic Model, there is no per capita growth since output per worker is constant in the steady-state. Output itself is growing, but only at the rate of population growth.

As a result, the basic model fails to predict the fact that economies exhibit sustained per capita income growth. The reason comes from the capital accumulation equation: [16]

$$\dot{k}/k = sk^{\alpha-1} - (n + d) \qquad (4)$$

The transition dynamics implied by equation (4) are plotted in **Figure 7**.

Figure 7: Dynamics of Transition



Source : Jones C.I, op.cit, p 32.

The $sk^{\alpha-1}$ curve has a downward slope since the average product of capital y/k decreases by the increase in capital per worker k. $(n+d)$ is given exogenously, then it is plotted as a horizontal line. The difference between the two lines represents the growth rate of the capital stock per worker, $\dot{k}/k$. The further an economy is below its steady-state value of k, the faster the economy grows.

1.3. Technology and the Solow Model

To generate sustained growth in per capita income, Solow introduces technological progress, A, to his model. Technological progress occurs when A increases over time.

The first issue is how to introduce exogenous technological progress into the model. Three popular propositions are due to Hicks (1932), Harrod (1942), and Solow (1969). [17]

1.3.1. Introducing technological progress

Hicks says that a technological innovation is neutral (Hicks-neutral) if the ratio of marginal products remains unchanged for a given capital-labor ratio. It can be written: $Y = A.F(K, L)$.

Harrod defines an innovation as neutral (Harrod-neutral) if the relative input shares, $(K.F_K)/(L.F_L)$, remain unchanged for a given capital-output ratio. Mathematically: $Y = F[K, L.A]$. This form is called labor-augmenting technological progress because it raises output in the same way as an increase in the stock of labor.

Finally, Solow defines an innovation as neutral (Solow-neutral) if the relative input shares, $(L.F_L)/(K.F_K)$, remain unchanged for a given labor/output ratio. This definition can be shown to imply a production function of the form: $Y = F[K.A, L]$.

However, Acemoglu (2002) refutes the neutrality of technical progress, he insists on the fact that, in most situations, it benefits some factors of production more than other. [18]

Barro and Sala-i-Martin (2004) prove that the only labor-augmenting (Harrod-neutral) technological change turns out to be consistent with the existence of a steady state, that is, with constant growth rates of the various quantities in the long run. [19] Thus, for the rest of this article, technological progress will be introduced as labor-augmenting.

The production function becomes, then: [20]

$$Y = F(K, AL) = K^{\alpha}AL^{1-\alpha}$$

A is supposed to be exogenous, so it is growing in a constant rate:

$$\frac{\dot{A}}{A} = g \Leftrightarrow A = A_0 e^{gt}$$

g is a parameter representing the growth rate of technology.

The capital accumulation equation is still the same as in the basic model:

$$\dot{K}/K = s\frac{Y}{K} - d \tag{5}$$

The production function is rewritten in terms of output per worker:

$$y = k^{\alpha}A^{1-\alpha}$$

After taking logs and differentiating:

$$\dot{y}/y = \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A} \tag{6}$$

From equation (5), we notice that the growth rate of K will be constant if Y/K is constant (Y and K growing at the same rate). This situation is called a Balanced Growth Path.

Using the notation g_x (growth rate of a variable X), along a balanced growth path: $g_y = g_k$. By substituting in (6): $g_y = g_k = g$

The model with technology reveals that technological progress is the source of sustained per capita growth. The variable k is no longer constant in the long-run, so we use a new variable $\tilde{k} \equiv K/AL \equiv k/A$ called the ratio of capital per worker to technology (or capital per effective unit of labor) which is constant along the balanced growth path.

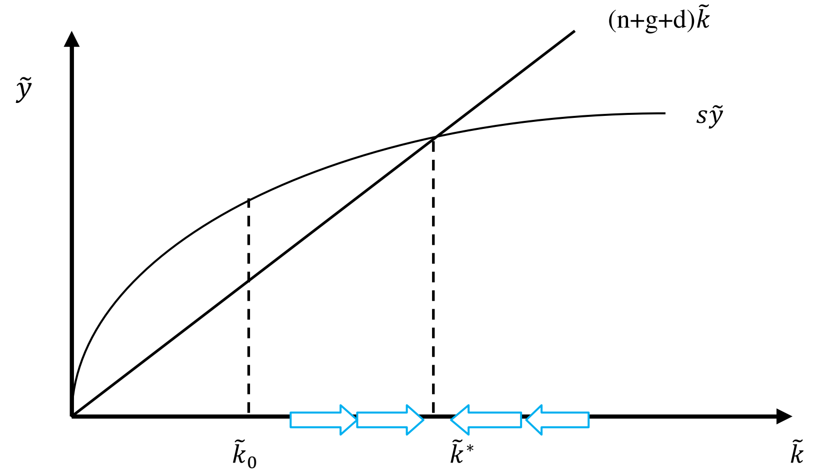
Rewriting the production function in terms of $\tilde{k}$: $\tilde{y} = \tilde{k}^{\alpha}$ (7)

Where $\tilde{y} \equiv Y/AL \equiv y/A$ is called "output-technology" ratio (or output per effective unit of labor). Using the same methodology as in the basic model, we get the capital accumulation equation: [21]

$$\dot{\tilde{k}} = s\tilde{y} - (n + g + d) \quad (8)$$

The Solow diagram with technological progress represented in **Figure 8** is very similar to the Basic Solow diagram vis-à-vis the analysis. However, the economic interpretation may be slightly different since the variables are expressed in terms of per effective unit of labor (no longer in terms of per worker units).

Figure 8: The Solow diagram with technological progress



Source: Jones C.I, op.cit, p 35.

Steady-state output-technology ratio is determined by the production function and the condition that $\dot{\tilde{k}} = 0$:

$$\tilde{k}^* = \left(\frac{s}{n + g + d}\right)^{1/(1-\alpha)}$$

Substituting this in the production function:

$$\tilde{y}^* = \left(\frac{s}{n + g + d}\right)^{\alpha/(1-\alpha)}$$

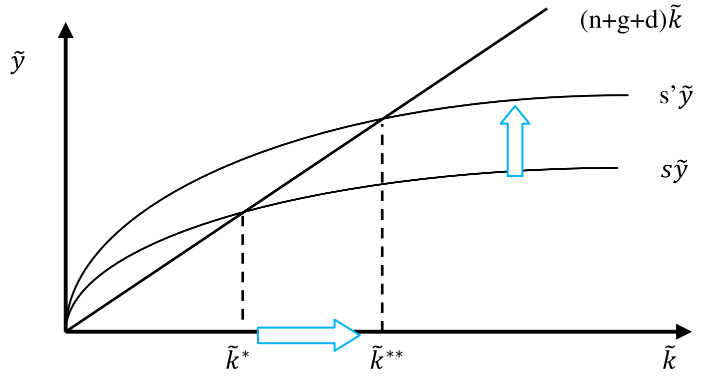
Rewriting this in terms of output per worker:

$$y^*(t) = A(t)\left(\frac{s}{n + g + d}\right)^{\alpha/(1-\alpha)}$$

1.3.2. Comparative Statics

Output per worker along the balanced growth path is determined by technology, the investment rate and the population growth. However, these two last parameters affect the long-run output per worker level but do not affect its growth rate. To explain the idea, let's consider an economy in its steady-state, and then the investment rate s permanently increases to s'. **Figure 9** shows the economy's reaction, unsurprisingly almost the same as in the basic model.

Figure 9: An increase in the investment rate



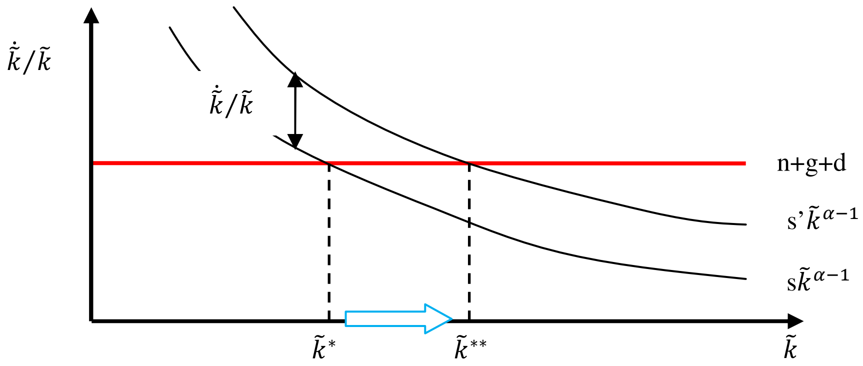
Source : Jones C.I, op.cit p 37.

To consider the impact on growth, we rewrite the equation (8) such as: [22]

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s\frac{\tilde{y}}{\tilde{k}} - (n+g+d)$$

Figure 10 illustrates the transition dynamics implied by this equation.

Figure 10: An increase in the investment rate: Transition dynamics

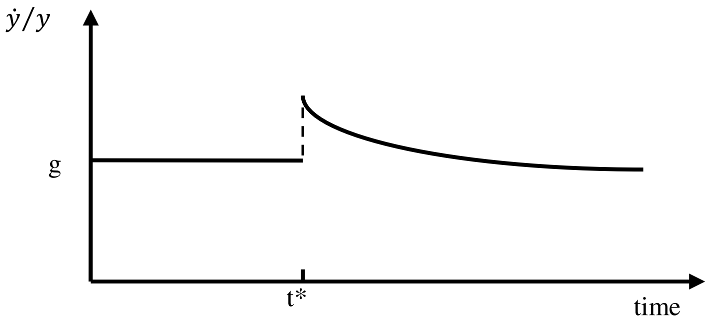


Source : Jones C.I, op.cit, p 38.

The investment rate change from s to s' raises the growth rate temporarily until the economy reaches its new steady-state $\tilde{k}^{**}$; Since g is constant, faster growth of $\tilde{k}$ implies that output per worker grows more rapidly than technology: $\dot{y}/y > g$ as we can see in **Figure 11**.

At the time of the policy change, t^*, output per worker starts growing faster until the output-technology ratio reaches its new steady-state. This shock test reveals the fact that a permanent policy change, in the Solow model, may have a permanent effect on the per capita output level, but only temporary effect on its growth rate.

Figure 11: The effect of an increase in investment rate on growth



Source : Jones C.I, op.cit, p 38.

In his 1957 article, Solow breaks down output growth into capital growth, labor growth and technological growth. He begins by postulating a production function such as: [23]

$$Y = BK^{\alpha}L^{1-\alpha}$$

B is a Hicks-neutral productivity term, commonly referred to as Total Factor Productivity. After taking logs and differentiating the equation above, we get the key-formula of growth accounting: $\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} + \frac{\dot{B}}{B}$

This means that output growth is a weighted average of capital and labor growth, plus the growth rate of B.

1.4. The Solow Extended Model

In their 1992 article, George Mankiw, Paul Romer and David Weil, using empirical evidence, conclude that the Solow Model performs very well. Although, the model can be improved if extended to include human capital since the latter emphasizes the impact of saving and population growth rates on income growth rate. [24] For the rest of this section, modifications applied by Neoclassical economists at that time are discussed.

Consider an economy where output Y is produced by combining physical capital, K, with skilled labor, H, according to a constant returns to scale Cobb-Douglas production function: [25]

$$Y = K^{\alpha}(AH)^{1-\alpha} \tag{9}$$

A is exogenous labor-augmenting technology.

1.4.1. Human Capital Accumulation

Let u denotes the fraction of an individual's time spent learning skills instead of working. Then, L becomes the total amount of labor used in production (total population – population learning skills). H is generated according to the following formula:

$$H = e^{\psi u}L \tag{10}$$

Where ψ is a positive constant. If $u = 0$, then, $H = L$ that is all labor is unskilled.

By increasing u, a unit of unskilled labor increases the effective units of skilled labor H. The amount of the increase may be calculated by taking logs and derivatives on equation (10):

$$\frac{d\log H}{du} = \psi$$

This last equation states that a small increase in u increases H by the percentage ψ.

Physical capital is accumulated by investing some output instead of consuming it: [26]

$$\dot{K} = s_K Y - dK$$

s_K is the investment rate of physical capital.

The production function from equation (9) is rewritten in terms of output per worker:

$$y = k^{\alpha}(Ah)^{1-\alpha} \tag{11}$$

By the same way, equation (10) becomes: $h = e^{\psi u}$. Where u is exogenously assumed constant. The model is solved by considering "state variables" since h, g and y/k are constant along the balanced growth path. Dividing equation (11) by Ah, the result would be:

$$\tilde{y} = \tilde{k}^{\alpha}$$

This is almost the same as in equation (7), same form but different terms.

Following the same previous reasoning, the capital accumulation equation in terms of state variables would be: [27] $\dot{\tilde{k}} = s_K\tilde{y} - (n+g+d)\tilde{k}$

We notice that adding human capital to the model does not change the basic flavor of the model.

1.4.2. Solving for Steady-state Magnitudes

The model is solved by setting $\dot{\tilde{k}} = 0$, that yields: [28]

$$\frac{\tilde{k}}{\tilde{y}} = \frac{s_K}{n+g+d}$$

Substituting this in the production function:

$$\tilde{y}^* = \left(\frac{s_K}{n+g+d}\right)^{\alpha/(1-\alpha)}$$

Rewriting this in terms of output per worker:

$$y^*(t) = \left(\frac{s_K}{n+g+d}\right)^{\alpha/(1-\alpha)} hA(t) \qquad (12)$$

In the steady-state, per capita output grows at the rate of technological progress g, as in the Solow Basic Model.

The Neoclassical Growth Model came to break almost two centuries of Classics reign in the field. Although its basic model was very similar to its predecessors', it opened the door to more sophisticated extensions that ameliorated its fit to the reality. However, one point that Neoclassics failed to highlight is the growth rate of technology. Assuming it the same across countries or different but growing at the same pace takes the model away of reality.

Technological progress is not the only factor that grows exogenously and affects growth. Population growth and investment rates too are defined outside the model, which drives us to say that the whole model depends on exogenous factors.

Modern growth theories aim to resolve this problem by introducing technology, investment rate, population growth, and many other factors (each at a time) to be determined inside the model.

2. Endogenous Growth Theories

As we saw in the previous section, the Neoclassical Growth Model depends on many exogenous factors. As their name predicts, exogenous factors are determined outside the model which means that the latter cannot make predictions about growth since it cannot make predictions about those factors either. Assuming the factors fixed or growing at a fixed rate simplifies the model but reduces its reliability.

The origin of the Endogenous Growth Models goes back to the beginning of the Twentieth Century with Schumpeter who stated their main ideas, and then the Neumann's "AK" Model on which modern models are built. Modern Endogenous Growth Models (as their name predicts too) investigate growth using factors determined inside the model. Including all the factors that may affect growth in the same model would be impossible to solve (at least with current techniques), so every model includes a limited but diversified set of factors, such as technological progress (Romer 1986; Grossman & Helpman 1991)[29], human capital (Lucas 1988), products diversification (Romer 1990), creative destruction (Aghion & Howitt 1992) and many others. The Romer Model is the one to be developed in this section.

The initial wave of the new research (Romer 1986, Lucas 1988, Rebelo 1991) built on the works of Arrow (1962), Sheshinski (1967), and Uzawa (1965) and did not really introduce a theory of technological change. In these models, growth may go on indefinitely because the returns to investment in a broad class of capital goods do not necessarily diminish as economies develop. Spillovers of knowledge across producers and external benefits from human capital are parts of this process, but only because they help to avoid the tendency for diminishing returns to the accumulation of capital.

The incorporation of R&D theories and imperfect competition into the growth framework began with Romer (1987, 1990) and included significant contributions by Aghion and Howitt (1992) and

Grossman and Helpman (1991). In these models, technological advance results from purposive R&D activity, and this activity is rewarded by some form of monopoly power. The rate of growth and the underlying amount of inventive activity may be, however, affected by some distortions related to the creation of the new goods and methods of production. In these frameworks, the long-term growth rate depends on governmental actions, such as taxation, maintenance of law and order, provision of infrastructure services, protection of intellectual property rights, and regulations of international trade, financial markets, and other aspects of the economy. [30]

2.1. The Technology-based Growth Model (Romer 1986) [31]

In 1986, Paul Romer proposed a model that offers an alternative view of long-run prospects for growth. In a fully specified competitive equilibrium, per capita output can grow without bound, possibly at a rate that is monotonically increasing over time. [32]

The Romer Model endogenizes technological progress by introducing the search for new ideas; firms that invest in R&D are motivated by the monopoly benefits they obtain, mainly through patents (perfect competition assumed by Neoclassics does not hold any more). It is also based on externalities between firms: each firm's investment increases its productivity, other firms' productivity too. [33]

The Romer's Growth Theory assumes that average productivity does not fall as capital per person increases. And so long as projects returns are higher than the rate of time preference, then businesses will continue to invest, and growth will continue to raise incomes, engender more ideas, more technology, more growth, and so on… [34] This means that, due to technological spillovers, production inputs are no longer subjects to diminishing returns, which gets us out from the Neoclassical Growth Theory framework.

The aggregate production function in the Romer Model describes how the capital stock, K, and productive labor, L_Y, combine to produce output, Y, using the stock of ideas, A: [35]

$$Y = K^{\alpha}(AL_Y)^{1-\alpha} \tag{13}$$

Equation (13) exhibits constant returns to scale in K and L_Y. With A as an input into production, then there are increasing returns.

Capital accumulates as people in the economy forego consumption at a given rate, s_K, and depreciates at the exogenous rate d, exactly as in Solow model:

$$\dot{K} = s_K Y - dK$$

Labor (the whole population) grows exponentially: $\frac{\dot{L}}{L} = n$

2.2. Technological Progress

Technology is no more exogenous as in the Neoclassical Growth Theory; it is the stock of knowledge (or number of ideas) that has been so far invented. Then, $\dot{A}$ would be the number of ideas produced at any given period: $\dot{A} = \bar{\delta} L_A$ (14)

Where L_A is the number of people attempting to discover new ideas (Labor is producing either new ideas or output: $L_A + L_Y = L$). Following Schumpeter, the allocation of labor between production and R&D remains constant over time. [36] These discoveries are made at the rate $\bar{\delta}$.

The rate at which new ideas are produced may be modeled in the sort: [37]

$$\bar{\delta} = \delta A^{\phi} \tag{15}$$

Where δ and ϕ are constants ($\phi > 0$: research productivity increases with the stock of ideas already discovered, $\phi < 0$: ideas are increasingly difficult to discover over time, $\phi = 0$: research productivity is independent of the stock of knowledge).

The average productivity of research depends on the number of people searching for new ideas at any point of time. Having more persons involved in researches may cause effort duplication; this may be modeled by transforming the term of labor making ideas: L_A^{λ} ($\lambda = 1$ means that there is no idea produced twice). Hereafter, we will assume $\phi < 1$.

Combining these assumptions with equations (14) and (15) yields the Romer general production function for ideas: [38] $\dot{A} = \delta L_A^{\lambda} A^{\phi}$ (16)

Assuming a constant fraction of the population is permanently producing ideas, the Romer Model follows the Neoclassical Model in predicting that all per capita growth is due to technological progress: $g_y = g_k = g_A$

Dividing both sides of equation (16) by A yields: [39]

$$\frac{\dot{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}} \quad (17)$$

Along a balanced growth path, the growth rate of technology is constant g_A if and only if both numerator and denominator of right-hand side of equation (17) grow at the same pace.

Taking logs and derivatives of this equation yields:

$$0 = \lambda \frac{\dot{L}_A}{L_A} - (1 - \phi)\frac{\dot{A}}{A} \quad (18)$$

Along a balanced growth path, the growth rate of researchers must be equal to the growth rate of the whole population: $\dot{L}_A / L_A = n$. Substituting this in equation (18) yields:

$$g_A = \frac{\lambda n}{1 - \phi}$$

In the Neoclassical Growth Model, a higher population growth rate reduces the level of income along a balanced growth path; more people means that more capital is needed just to keep K/L constant. The additional effect of endogenous technology progress theory is that people are the key input to the creative process; a larger population generates more ideas, then higher productivity levels.

The proved relationship between the growth rates of output and population emphasizes a new question: if the population stops growing, would long-run growth cease?

The answer is provided in the production function for ideas brought by Romer in his 1990 paper, he assumes $\lambda = 1$ and $\phi = 1$. That is: $\dot{A} = \delta L_A A$

Extracting technology growth rate: $\frac{\dot{A}}{A} = \delta L_A$

According to Romer, even a constant research effort would generate a sustained growth of the level of ideas.

In the United States, as in most industrialized countries, research efforts has increased enormously during the last half century, however the annual average growth rate of the U.S. economy has been around 1.8 %. The prediction of the original Romer formulation is then rejected.

A similarity between the Technology Model and the Neoclassical one is that government policy and change in investment rate have no long-run effect on economic growth.

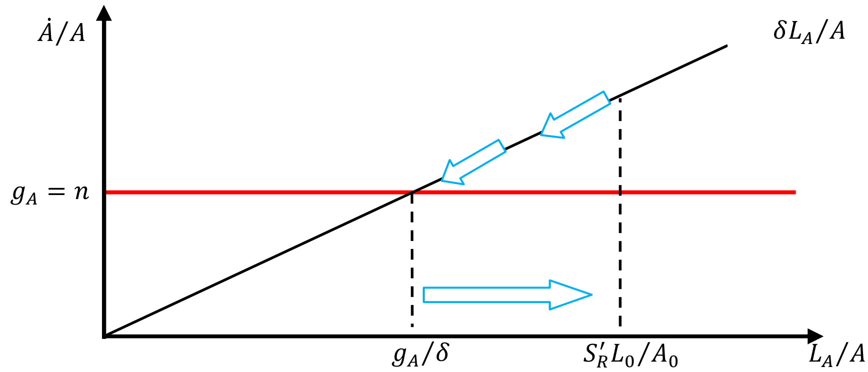
2.3. Comparative Statics

Comparative statics for the ideas-based models can be analyzed by the same techniques as for the Neoclassical Model. The change in investment rate has no impact on technology level, then its analysis will be skipped (exactly the same results as in Solow's Model). We are more interested in the impact caused by change in the fraction of population searching for new ideas (commonly called R&D intensity). For the study, let's assume $\lambda = 1$ and $\phi = 0$. Equation (17) is rewritten as: [40] $\dot{A}/A = \delta \frac{s_R L}{A}$ (19)

Where s_R represents the R&D intensity ($L_A = s_R L$)

In **Figure 12**, the economy begins in a steady-state where it grows at the rate of technological progress $g_A = n$ (as discussed earlier). L_A/A is then equal to g_A/δ. If R&D intensity permanently increases from s_R to s'_R at $t = 0$, the ratio L_A/A raises to a higher level. Researchers produce an increased amount of ideas, then the growth rate of technology increases. At the new point, $g_A > n$ so the ratio L_A/A declines over time, bringing down the technology growth rate until the economy returns to its initial steady-state where $g_A = n$.

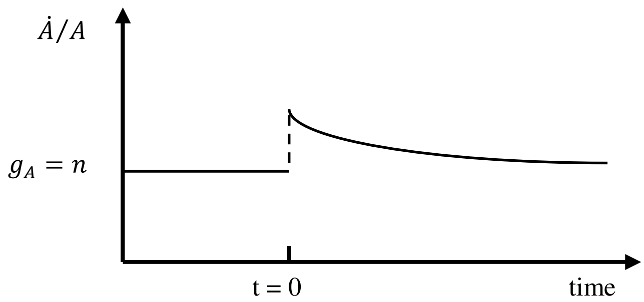
Figure 12: Technical progress: An increase in the R&D share



Source : Jones C.I, op.cit, p 99.

Therefore, a permanent increase in the fraction of population researching for new ideas temporarily raises the growth rate of technology, but not on the long-run, as shown in **Figure 13**. This result was contested by Guellec and Ralle (1993) through their empirical study, [41] where they estimated an increase of 0.18 % in technology growth rate if the number of research doubles, still the very low effect may be not too contradictory to the Romer theory.

Figure 13: $\dot{A}/A$ over time



Source : Jones C.I, op.cit, p 100.

A permanent increase in s_R in the Romer model generates transition dynamics that are qualitatively similar to the dynamics generated by an increase in the investment rate in the Solow Model.

2.4. Solving for Steady-state Magnitudes

The rest of the model may be solved in a Solow framework[42], the long-run growth rate of the model is constant, and so is the ratio y/A which is given by the following equation:

$$\left(\frac{y}{A}\right)^* = \left(\frac{s_K}{n+g_A+d}\right)^{\alpha/(1-\alpha)} (1 - s_R) \tag{20}$$

The term $(1 - s_R)$ adjusts for the difference between output per worker and output per capita.

Equation (19) can be solved for the level of A along a balanced growth path:

$$A = \frac{\delta s_R L}{g_A}$$

Combining this equation with (20) yields:

$$y^*(t) = \left(\frac{s_K}{n + g_A + d}\right)^{\alpha/(1-\alpha)} (1 - s_R)\frac{\delta s_R}{g_A} L(t)$$

Unsurprisingly, Endogenous Growth Models try to reduce their reliability on exogenously determined parameters, which represents the main fragility of the Neoclassical Growth Model. In the above discussed model, Romer mostly succeeded in determining the source of technological change inside the model, he considers that technology grows with regard to the growth rate of population, to the fraction of population dedicated to making research, and to the efficiency of researchers (by avoiding repeated ideas).

3. Conclusion

Each time that an existing Growth Theory comes to a dead end by failing to explain sustain-ability, a new theory surges and try to resolve that problem. Classics failed to explain how an economy may keep growing on the long term just by coupling Capital and Labor, so Neoclassics presented a growth theory where many factors are introduced, such as saving, population growth, and capital stock depreciation. After that, technology progress was considered into that model aiming to raise its predictability.

The failure of Neoclassic Growth Models was that almost all those factors newly introduced were determined exogenously outside the model, were considered the same across all economies, or estimated to be growing at a same rate in spite of external conditions. Furthermore, production inputs were introduced under a diminishing returns rule which permits a long term growth but at a slowing pace. These two principal points were the reason to which Endogenous Growth Theory appeared.

Endogenous Growth Theory emphasizes the fact that all production factors must be deter-mined inside the production function. Actually, including all factors in the same function would create a mathematical dilemma with too many variables in one function. For that reason, we may find that every theory focuses on a well defined set of factors (the remaining factors kept exogenous) : technology, human capital, diversification…

The Romer Growth Theory, although based on the results brought by Neoclassics, it provided a new and better explanation of growth. The theory starts from the principle that inputs are not subjects to diminishing returns because of technological externalities. Technological progress itself is no longer exogenous; indeed, it depends on efforts made by firms and rewarded by monopolistic benefits. In last, a higher population growth rate augments rather than slows down per capita growth rate. These findings are mostly substituting those made by Neoclassics without denying their importance.

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[1] In spite of the rich literature existing on the subject, and in order to unify terms and signs used, and the methodology of developing models, it was decided to use the methodology of Jones (1998) for its simplicity. Mathematical developments of models were deliberately neglected since we are mostly concerned with their economic interpretation.

[2] Barro R.J. & Sala-i-Martin X "*Economic Growth*" 2nd ed., Massachusetts Institute of Technology, 2004, p 17.

[3] Guellec D. & Ralle P "*Les Nouvelles Théories de la Croissance*" 5th ed., La Découverte, Paris, 2003, p 31.

[4] Jones C.I "*Introduction to Economic Growth*" W.W. Norton & Company, USA, 1998, pp 18-19.

[5] Barro R.J. & Sala-i-Martin X, op.cit, pp 27-28.

[6] Mattheus K., Parkin M. & Powell M "*Economics*" 6th ed., Addison-Wesley, 2005, pp 691-692.

[7] Ibid., p 693.

[8] Cleaver T "*Economics. The Basic*" Routledge, 2004, pp 177-178.

[9] Barro R.J. & Sala-i-Martin X, op.cit, p 23.

[10] Ibid., p 28.

[11] Jones C.I, op.cit, p 22.

[12] Barro R.J. & Sala-i-Martin X, op.cit, p 23.

[13] Solow R.M "A Contribution to the Theory of Economic Growth" *The Quarterly Journal of Economics*, vol. 70, n° 1, 02.1956, p 67.

[14] Jones C.I, op.cit, pp 26-28.
[15] Ibid., pp 28-29.
[16] Ibid., p 31.
[17] Barro R.J. & Sala-i-Martin X, op.cit, p 52.
[18] Acemoglu D "Directed Technical Change" *The Review of Economic Studies*, vol. 69, n° 4, 10.2002, pp 781-809.
[19] Barro R.J. & Sala-i-Martin X, op.cit, p 53.
[20] Jones C.I, op.cit, pp 32-34.
[21] Ibid., pp 35-36.
[22] Ibid., p 37.
[23] Ibid., p 41.
[24] Mankiw N.G., Romer P. & Weil D.N "A Contribution to the Empirics of Economic Growth" *The Quarterly Journal of Economics*, vol. 107, n° 2, 05.1992, pp 407-408.
[25] Jones C.I, op.cit, p 48.
[26] Ibid., p 49.
[27] Mankiw N.G., Romer P. & Weil D.N, op.cit, p 416.
[28] Jones C.I, op.cit, p 50.
[29] Even models considering endogenous technological progress are divided into R&D-based and Learning by Doing-based models.
[30] Barro R.J. & Sala-i-Martin X, op.cit, pp 19-20.
[31] Indeed, the basic idea of the Romer model was presented in his 1986 paper, but many sophistications were then introduced (Romer 1987; 1990).
[32] Romer P "Increasing Returns and Long-Run Growth" *Journal of Political Economy*, vol. 94, n° 5, 10.1986, pp 1002-1037.
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[34] Cleaver T, op.cit, p 183.
[35] Jones C.I, op.cit, pp 90-91.
[36] Aghion P., Akcigit U. & Howitt P "What Do We Learn from Schumpeterian Growth Theory?" Chap. 1 in "*Handbook of Economic Growth*" by Aghion P. & Durlauf S., vol. 2, part B, Elsevier, 2014, p 518.
[37] Jones C.I, op.cit, p 92.
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[39] Jones C.I, op.cit, p 94.
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