Recueil De Mécanique
El Wancharissi University
of Tissemsilt

Research Paper DOI : 10.5281/zenodo.5917217 Open access

Dynamic analysis of a composite rotor

Sahli Souhila*, Bouzidane Ahmed and Benadda Mohamed

Research Laboratory of Industrial Technologies, Department of Mechanical Engineering, Univ. Tiaret, Algeria

ARTICLE INFO

Article history:

Received 08 June 21

Received in revised form 8 September 21

Accepted 17 October 21

Keywords:

Rotor dynamics ; Composite materials ; Finite element model ; Vibration analysis ; Energy expressions ; Natural frequencies.

ABSTRACT

Rotor dynamic is a branch of applied mechanics that plays a major role in keeping the vibrational energy as small as possible. It covers several topics, two of them are the base of this study: modelling and analysis. The use of composite materials in rotor dynamic analysis gives an important calculation to design efficient rotating composite shaft. The energy expressions of a composite rotor system are obtained by the use of rotating coordinate system. Lagrange's equation obtains the equation of motions, and the finite element method solves those equations to obtain the matrices of the rotor system with the use of the MATLAB program that can handle complex and large matrices. Those mathematical models are used in order to explain the dynamic behavior of composite rotor. This model is used to investigate the unbalance response of a rotating composite shaft supported by flexible journal bearings in transient regime. The numerical results show that composite shaft made of Boron/Epoxy have better dynamic characteristics than Graphite/Epoxy.

1 Introduction

One of the problems associated with design of composite drive shafts has been the accurate determination of the flexural critical speeds. As the drive shafts are quite long, their critical speeds are lower and may occur near the operating speed. In order to analyze the problems related to the lateral bending of composite shafts, equivalent modulus theory is commonly used.The theory is based on Kirchhoff's hypothesis for thin laminated beams. The equivalent moduli are found using classical laminate theory.

The knowledge of the dynamic behavior of composite rotors is of great importance in power production engineering and in adjacent fields. The subject of high-performance composite materials is involved in development modern design and manufacturing methods for industrial components.

Studies on composite shafts started in 1970's. The most important development of composite shafts has taken place in aerospace (helicopter) industry [1] [2] and automotive applications [3] [4]. Other applications include the use of composite shafts as quill shaft by Spencer [5], an aircraft power take off shaft by Garguilo [6], generator shaft by Raghava and Hammond [7], shaft for a cooling tower by Berg [8], a papermill by Cox [9] and naval propulsion systems by Wilhelmi et al [10]. The two U.S. patents by Worgan and Smith [11] and Yates and Rezin [12] indicate that the preliminary hurdles to a composite driveshaft design were overcome. Fromknecht [13] highlighted the possible benefits accruing from the use of composite shaft in mechanical power transmission.

Zinberg and Symonds [14] investigated the critical speeds of rotating anisotropic cylindrical shafts based on an equivalent modulus beam theory (EMBT), and dos Reis et al. [15] evaluated the shaft of Zinberg and Symonds by the finite element

* *Corresponding author. Tel.: +213790025916.*
E-mail address: sahli.sou97@gmail.com

method.Bauchau [16]: specialized in studies of high-speed graphite/epoxy shafts and the design of tapered composite shafts. The study of Bauchau deal with a numerical procedure using a generalized criterion of optimality. The shaft was modeled using a beam formulation including shear deformation and rotary inertia. The optimization was done under constant volume constraint. Different configurations are studied for various number of plies and orientation angle. In the optimal configuration the natural frequency increased by about 21 to 44%, and the bending stress decreased by about 48 to 59%.Spencer and Mcgee [17]: worked on the design of subcritical composite drive shafts in order to overcome its hurdles. Their report contains an analysis of the present steel shaft designand the two design approaches to a composite drive shaft. The first is a two-piece drive shaft; the second is a combined single drive shaft eliminating several other parts. The design section includes the design of a joint to interface between the steel couplings and the composite shaft.Zorzi and Gioradani [18]: by the experiments on an aluminum shaft and on a composite shaft, they made an achievement on matching the experimental and theoretical results.

• Lim and Darlow [19]: the reduction in weight of the rotor system led them to look forward if there is a possibility of supercritical operations. The optimal design of composite drive shafting is developed with the goal of minimizing, system weight. The study is illustrated with an application to a composite tail rotor drive shaft for advanced helicopters. It is also applicable to the design of composite synchronization drive shah for helicopters and other composite shah for aircraft, spacecraft and automobiles. The use and effectiveness of the optimal design procedure are demonstrated with an illustrative example.Reis and all [15]: used the finite element method to evaluate the critical speeds of composite shafts. A numerical procedure to evaluate the rotor dynamic performance of thin-walled filamentary wound laminated composite circular cylindrical shafts of any layup is presented. Numerical results, for the critical speeds and the unbalance response of a sample composite shaft, are obtained and compared with predictions based on classical methods of analysis and experimental results found in the literature.

Hoffman [20]: proved that carbon fiber is necessary to have a balance between length, diameter and natural frequency. Another problem of paramount importance in composite shaft design has been that of optimization of the material and geometric parameters. The optimization objectives are somewhat different in aerospace applications as compared to automotive driveshaft design. In automotive applications, cost is one of the major driving factors. Thus, detailed cost-sensitivity analyses are performed in order to get a cost-optimal design. The solution lies in using hybrid composite shafts, as shown by Hoffmann.

This provides the engineer with two important design variables to control, viz, the fiber winding angle and the mixing ratio of carbon and glass fibers. The additional variable of carbon-glass ratio greatly increases the range of designs alternatives.Hetherington and all [21]: also worked on the reduction in weight but of the tail drive rotor because they studied on composite helicopter. An experimental program is underway to investigate the dynamic behavior of supercritical composite drive shafts for helicopter applications. Design optimization results have shown that the system of least weight is achieved by the use of composite materials with a shaft that operates at a supercritical speed. Uncertainties in the ability to manufacture, balance, and safely operate supercritical composite shafts motivates the experimental program to examine their dynamic performance. Results are presented far experiments with both aluminum and optimized graphite/epoxy shafts.Singh and Gupta [22]: The shell modes involving cross sectional deformation of nonrotating shafts have been experimentally analyzed by modal testing by Singh and Gupta Tests showed the existence of coupling of higher flexural modes with shell modes. The mountingof a disc on the shaft resulted in suppression of some shell modes, reduction in flexural natural frequencies and increase in damping ratios of all modes.

Rotor systems play a vital role in processing machinery, aero engines, and wind turbines. Since moving parts can be worn during operation, the system performance is variable throughout the service period actually, especially for the rotor system with various defects. Unfortunately, the existing models about bearing defects are established neglecting the changes of dynamic behaviors, which may not reflect the practice well. Thus, a model that considers this variation is proposed by Yu et al. [23]. The effects of both distributed and localized defects are modeled under the excitation from the system itself.

Jadhav et al; [24] diagnose the damaged bearings for the detection of distributed defects based on Dimensional Analysis (DA) methodology validated by an experimental investigation.

In the field of rotor dynamic, there are several problems and obstacles can be found throughout the studies of vibrations which lead to the damage of the rotors. Our studies are specialized in the field of dynamic analysis of composite rotors, in order to solve those problems and to find solutions for the latter. Moreover, and as it's known that the composite materials are characterized by very high mechanical properties which make them able to with stand the applied solicitation more than a normal material.

2 Mathematical and Finite Element Models

The mathematical model of the rotor is derived from the Lagrange's equation which is obtained from the strain and kinetic energies expressions. The mathematical representation of the specific physical phenomena that involve rotating machines requires a reliable design tool.

The main phenomena that occur in rotor dynamics can be evaluated by the finite element method. This section focuses exclusively on the finite element model based upon the Timoshenko beam theory to obtain matrix equations. In this context, the present work is dedicated to a numerical investigation the unbalance response (dynamic behaviour) of a rotating composite shaft supported by flexible journal bearings.

2.1 Kinetic energy of the shaft, the disk and the unbalance

While in operation, rotors of machines have a great deal of rotational energy, and a small amount of vibrational energy [25], in order to study the dynamics of a system comprising one or more rotors, it is possible to write the equations of motion either in a fixed reference frame or in a reference frame rotating at the same speed as the rotor (figure 1) [26].
The “XYZ” triad is a fixed frame of reference and “xyz” triad is a rotating frame of reference with “X” and “x” being collinear and coincident with the undeformed rotor center line. Rotating frame is one which rotates about the longitudinal axes at angular Ω [27].

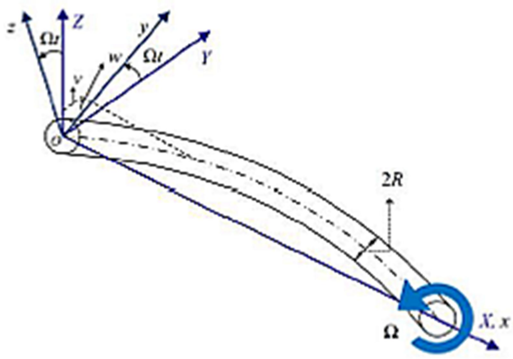

Fig. 1-A micro-rotating beam with circular cross section and the corresponding coordinate system [28].

A typical cross section of the rotor in a deformed state is defined relative to XYZ by the translations “v(x,t)” and “w(x,t)” in the Y and Z directions respectively to locatethe elastic centreline and small angle rotationsθy(s,t) and θz(s,t) about Y and Z respectively to orient the plane of the cross-section. The “xyz” triad is attached to the cross-section with the "x" axis normal to the cross-section[27].
The angular velocity vector of the rotational rigid-body motion of the element can be determined by considering three successive Euler angles :θx(x,t) , θy(x,t) and θz(x,t), whereθx(x,t)=Ωtbecause the torsional deformation of the element has been assumed to be negligible.
The expressions of kinetic energies are necessary to characterize the shaft, the disc and the unbalance [29], either a composite rotor consisting of “N” layer of orthotropic material, and according to the beam theory , for each ply of beam element of length “L” and constant cross-section [30] , the kinetic energy is given by the form :
For an element of the shaft [31]:

$$T_s = \frac{1}{2}\int_0^L \left[I_m\left(\dot{v}^2+\dot{w}^2\right) + I_d\left(\dot{\theta}_y^2+\dot{\theta}_z^2\right) - 2\Omega I_p \theta_y \dot{\theta}_z + I_p\Omega^2 \right] dx \quad (1)$$

With:

$$I_m = \pi\sum_{n=1}^{k}\rho_n\left(R_n^2 - R_{n-1}^2\right)$$

$$I_d = \frac{\pi}{4}\sum_{n=1}^{k}\rho_n\left(R_n^4 - R_{n-1}^4\right)$$

$$I_p = \frac{\pi}{2}\sum_{n=1}^{k}\rho_n\left(R_n^4 - R_{n-1}^4\right)$$

Where:

- $2\Omega I_p \theta_y \dot{\theta}_z$: this term represents the gyroscopic effect.
- $I_d\left(\dot{\theta}_y^2+\dot{\theta}_z^2\right)$: this one represents the effect of rotary inertia.
- “I_m”: the moment of mass inertia. “I_d”: the moment of diametrical inertia. “I_p”: the moment polar inertia.

For an element of the disk [31]: “The disk is supposed rigid, so, only its kinetic energy is considered, plus, both of the translational and the rotational energies are summed in the following form”:

$$T_d = \frac{1}{2} m_d \left(\dot{v}^2 + \dot{w}^2\right) + \frac{1}{2} I_{dx} \left(\dot{\theta}_z^2 + \dot{\theta}_y^2\right) + \frac{1}{2} I_{dy} \Omega^2 + I_{dy} \Omega \dot{\theta}_z \theta_y \quad (2)$$

With:

- $\frac{1}{2} m_d (\dot{v}^2 + \dot{w}^2)$: Kinetic energy of an element in translation in a plane.
- $\frac{1}{2} I_{dx} (\dot{\theta}_z^2 + \dot{\theta}_y^2)$: Kinetic energy of rotation of an element about the “y” and “z” axis.
- $\frac{1}{2} I_{dy} \Omega^2$: Constant term for disc rotation energy.
- $I_{dy} \Omega \dot{\theta}_z \theta_y$: Gyroscopiceffect (Coriolis).

For the unbalance: “The unbalance is defined by a mass “mu” situated at a distance “d” from the geometric center of the shaft. The unbalance mass is negligible in the relation of the rotor mass.”

$$T_u = m_u \Omega d \left(\dot{u} \sin \Omega t - \dot{w} \cos \Omega t\right) \quad (3)$$

2.2 The strain energy of the shaft

The deformation energy is not affected by the movement of the support because it only depends on the stresses and therefore on the deformation of the shaft in relation to the support (figure 2) [32].

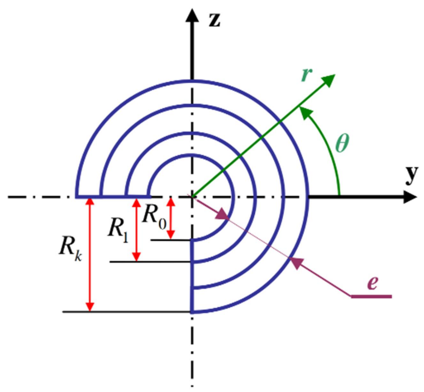

Fig. 2 - “k” shaft layers in composite materials [33].

With: “R0” is the inner radius of the shaft, “Rk” is the outer radius of the shaft and “e” is the thickness of the shaft.

The expression for the strain energy of the shaft is :

$$\begin{aligned} U &= \frac{1}{2} \int_V \{\sigma_{ij}\}^t \{\varepsilon_{ij}\} dV \\ &= \frac{1}{2} \int_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{rr}\varepsilon_{rr} + \sigma_{\phi\phi}\varepsilon_{\phi\phi} + \tau_{xr}\gamma_{xr} + \tau_{x\phi}\gamma_{x\phi} + \tau_{r\phi}\gamma_{r\phi})\, dV \end{aligned} \quad (4)$$

Where: “V” is the volume of the shaft, and, since: $\varepsilon_{\phi\phi} = \varepsilon_{rr} = \gamma_{r\phi} = 0$ the deformation expression becomes [34]:

$$U = \frac{1}{2} \int_V (\sigma_{xx}\varepsilon_{xx} + \tau_{xr}\gamma_{xr} + \tau_{x\phi}\gamma_{x\phi})\, dV \quad (5)$$

Using the equationin the cylindrical coordinate system (x; r; θ), the deformation energy expression becomes:

$$U = \frac{1}{2} \int_V (C'_{11}\varepsilon_{xx}^2 + 2kC'_{16}\gamma_{x\phi}\varepsilon_{xx} + kC'_{55}\gamma_{xr}^2 + kC'_{66}\gamma_{x\phi}^2)\, dV \quad (6)$$

Where: $2kC'_{16}\gamma_{x\phi}\varepsilon_{xx}$ accounts for the shear-normal coupling effect.

Replacing the relations for the cross section rotation where y = r cos φ and z = r sin φ, and integrating over the shaft cross sectional area by summing up the contribution of each orthotropic layer, the deformation energy in expanded form is given by [34]:

$$U=\frac{1}{2}A_{11}\int_0^L\left[\left(\frac{\partial\theta_z}{\partial x}\right)^2+\left(\frac{\partial\theta_y}{\partial x}\right)^2\right]dx+\frac{1}{2}k_sA_{16}\int_0^L\left[-\theta_y\frac{\partial\theta_z}{\partial x}-\frac{\partial\theta_z}{\partial x}\frac{\partial w}{\partial x}+\theta_z\frac{\partial\theta_y}{\partial x}-\frac{\partial\theta_y}{\partial x}\frac{\partial v}{\partial x}\right]dx+$$
$$\frac{1}{2}k_s\left(A_{55}+A_{66}\right)\int_0^L\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+\theta_z^2+\theta_y^2+2\theta_y\frac{\partial w}{\partial x}-2\theta_z\frac{\partial v}{\partial x}\right]dx \tag{7}$$

With:

$$A_{11}=\frac{\pi}{4}\sum_{n=1}^{k}C'_{11n}\left(R_n^4-R_{n-1}^4\right)$$
$$A_{16}=\frac{2\pi}{3}\sum_{n=1}^{k}C'_{16n}\left(R_n^3-R_{n-1}^3\right)$$
$$A_{55}=\frac{\pi}{2}\sum_{n=1}^{k}C'_{55n}\left(R_n^2-R_{n-1}^2\right)$$
$$A_{66}=\frac{\pi}{2}\sum_{n=1}^{k}C'_{66n}\left(R_n^2-R_{n-1}^2\right)$$

Where: “k” is the number of layers and “n” is the index of layers.

2.3 Finite Element Model

There are two methods which are often used for the dynamic analysis of rotors: “The transfer matrix method (TMM)” and “ The finite element method (FEM)”, both of these methods have been used extensively for modeling and analyses of rotor systems, but, the finite element method is known among the most important and efficient methods for modelling and solving complex problems in engineering sciences, especially in rotor dynamics [32, 35].
A finiteelement model of the rotor was created using twonoded Timoshenko beam elements with gyroscopic effects included. It has been seen that the finite element method consists mainly in setting out the matrices and solving the equations of [36], it is applied to the solution of the governing partial differential equation of the Timoshenko beam with rotary and gyroscopic effects.Elemental matrices are presented for the mass, stiffness, and gyroscopic effect[35].

2.3.1 Equation of motion

The general equation of motion is obtained by applying Lagrange’s equations on the energy expressions of the rotor element by outputting the different characteristic matrices of
the system (mass matrix, stiffness matrix and damping matrix), the Lagrange’s equation is given by [32]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}=F_{q_i} \tag{8}$$

Where: “ q_i ”: Independent generalized coordinates. With: “i” is the number of degrees of freedom. And “ F_{q_i} ”: The generalized forces vector.

The anisotropic properties of composite materials and their lightness can be used to optimize composite shafts in order to improve their dynamic behavior, the differential equation that represents the dynamic behavior of a composite flexible rotor system is as follows [37]:

$$[M]\{\ddot{q}\}+[D+\Omega G]\{\dot{q}\}+[K]\{q\}=\{W+F_u\} \tag{9}$$

Where:

- [M]: Mass matrix. [D]: Damping matrix. [K]: Stiffness matrix. [G]: Gyroscopic effect.

- Ω : The shaft rotation speed. $\{W\}$: The weight of the rotating parts. $\{F_u\}$: The unbalance forces, and, the vector $\{q\}$ contains the generalized displacements.

2.4 Numerical models

The finite element model of a composite shaft is calculated by using a numerical example contains two composite materials “boron/epoxy” and “graphite/epoxy”, which are considered for the vibration analysis [38]. The results of those two composite materials can be compared to each other, their calculations is obtained from MATLAB program developed to perform the vibration analysis of composite shaft. In this study, each element will be modelled and represented by eight degrees of freedom with taking the gyroscopic effect into account.
The table below shows some mechanical properties of the two composite materials:

Table 1 - Properties of composite materials.

Properties		Materials	
		Boron/epoxy	Graphite/epoxy
Young modulus	E11	211 GPa	139 GPa
	E22	24 GPa	11 GPa
Shear modulus	G12=G13	6.9 GPa	6.05 GPa
	G23	6.9 GPa	3.78 GPa
Poisson coefficient (ν)		0.36	0.313
The density (ρ)		1967 Kg/m³	1578 Kg/m³

2.5 Composite rotor simulation model

The calculation program developed in this work, is a model written in MATLAB/ SIMULINK, this program consists in solving the dynamic equations of a composite rotor system, and, the SIMULINK model is developed in order to solve the equations of motion [39, 40].

The simulation model of composite rotor has made to determine the natural frequencies of a rotational composite rotor with different boundary conditions and different physical and geometric parameters.

Resolution system

The different programming steps are as follows:

- Reading the data of all the necessary elementary physical and geometric parameter of the system (shaft, disk and bearings).
- Reading all the values of the integrals.
- Formation of elementary matrices .
- Formations of global matrices [M], [K], [G] and [C] with taking the boundary conditions into account.
- Formations of reduced matrices [Mr], [Kr], [Gr].
- The programme gives the results of the eigenvalues (ω).

3. Results and discussions

The MATLAB/SIMULINK program is written to perform the vibration analysis of the uniform composite shaft in order to validate the finite element model.

3.1 Comparison between two composite materials

3.1.1 Campbell Diagram

The Campbell diagram is one of the most important tools for understanding the dynamic of the rotating machines, it is used to evaluate the critical speed at different operating [41]represents the critical speeds as function of rotating speed for both of composite materials.

The critical speeds obtained from the present example are shown in the table 2:

Table 2 - Critical speeds of the two different composite materials

Materials	Graphite/Epoxy	Boron/Epoxy
Critical speeds (Krpm)	14.5	16.5

In this example, Campbell diagram maps two critical speeds which are caused by the influence of the shear deformation effect, the composite shaft made of “graphite/epoxy” has the first critical speed “N_{CG}”, and the composite shaft made of “boron/epoxy” has the second critical speed “N_{CB}”.

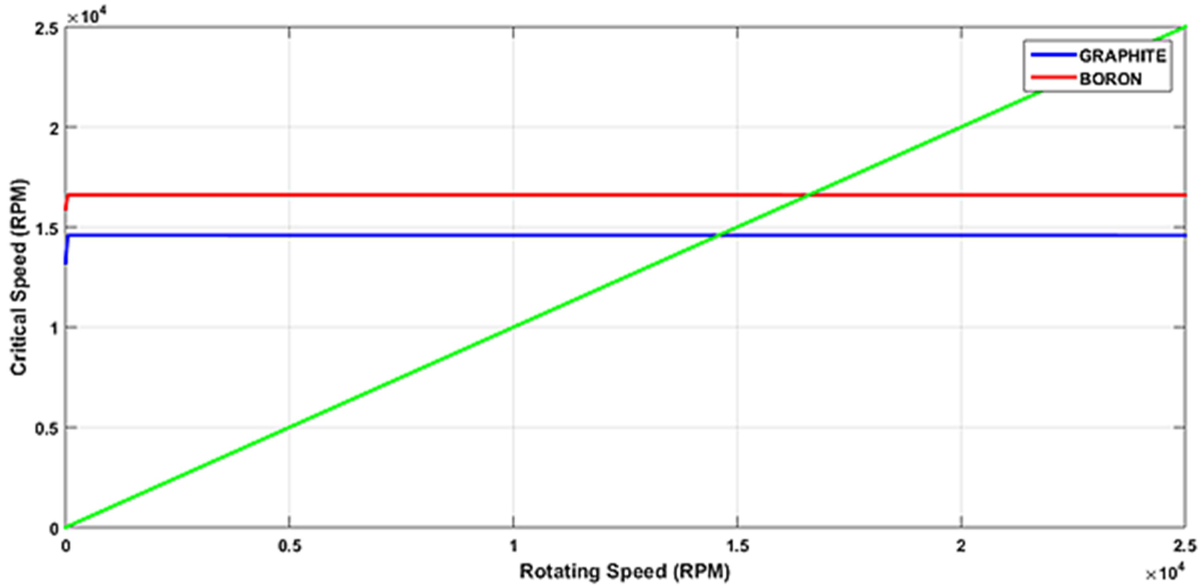

Fig.3 - Campbell diagram of critical speeds as function of rotating speeds.

3.1.2 Mode shape

As it is known that rotating machinery has to rotate to do useful work, the plot below shows what happens to the first mode of this rotor system once it is spinning [42].
Figure 4illustrates the vibration mode shape on different parts of the structure.

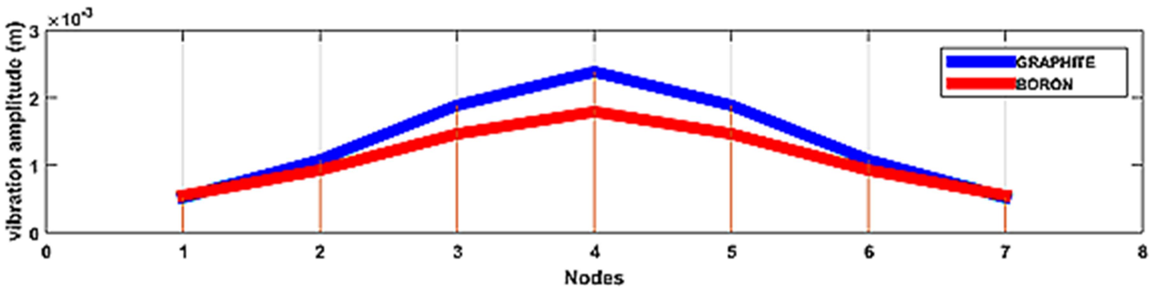

Fig. 4 - The first mode shape.

Note that the vibration amplitudes of both composite materials are obtained through only the first mode, and it is noticed that those amplitudes are higher at the level of the disk. The second and the third mode are not considered on this rotor system, which means that the using of composite materials on the rotor dynamics is important.

3.1.3 Vibration amplitudes of disk

The vibration amplitudes for both of composite materials as function of rotational speed are illustrated in the figure 5.

First of all, this plot has two zones, the rigid zone on the operating range, and it is before the peaks, and the flexible zone which is after the peaks. Note that the "graphite/epoxy" material has the first vibration amplitude peak "A_{CG}" which is bigger than the second peak "A_{CB}" of the "boron/epoxy".

$$A_{CG} = 2.45\times10^{-3}m\,,\quad A_{CB} = 1.75\times10^{-3}m$$

$$A_{CG} > A_{CB}$$

The difference between these amplitude values is obtained with a reducing ratio which is determined as follows:

$$\delta_B = \frac{x}{A_{CG}}\times100 \quad ,\quad x = A_{CG} - A_{CB} = \left(2.45-1.75\right)\times10^{-3} = 0.7\times10^{-3}m$$

$$\Rightarrow \delta_B = \frac{0.7\times10^{-3}}{2.45\times10^{-3}}\times100 = 28.57\,\%$$

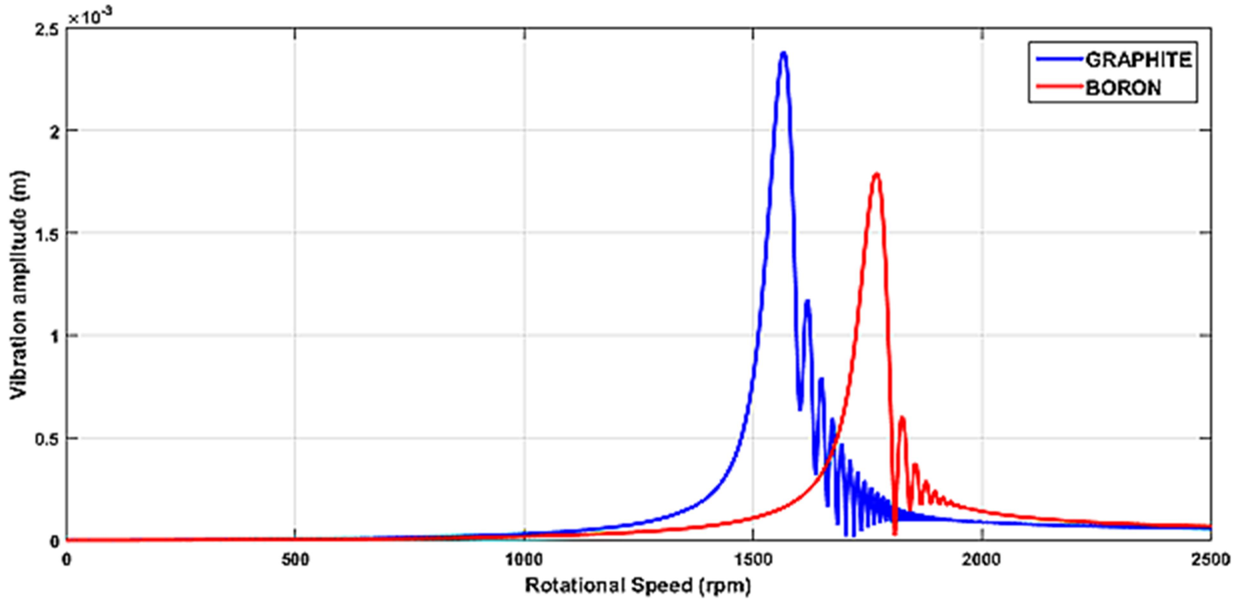

Figure 5 - Vibration amplitudes of a rigid disk for the two composite materials.

3.1.4 Vibration amplitude of the bearings

For the bearings, the evolution of the vibration amplitudes as function of the rotational speed is presented in the figure 6.

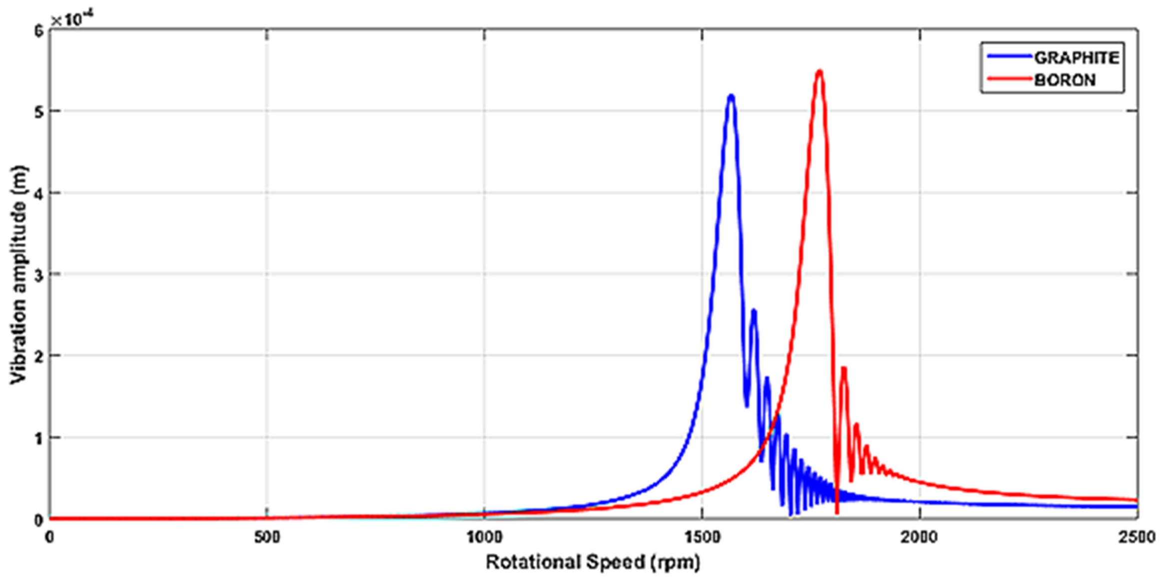

Fig. 6 - Vibration amplitudes of the bearings for the two composite materials.

It is noticed that “graphite/epoxy” material has the first vibration amplitude peak of **5.2 10^{-4} m** which is lower than the “boron/epoxy” one that has a peak of **5.5 10^{-4} m** this difference is around to **5.45%** on the other hand, it is noticed that the damping ratio of the “graphite/epoxy” is much lower than the “boron/epoxy” one, so that can be explained the stiffness of Boron is very higher than Graphit stiffness

$$"\zeta_B > \zeta_G" \quad \Leftrightarrow \frac{D_B}{2\sqrt{K_B m_B}} > \frac{D_G}{2\sqrt{K_G m_G}}$$

And since: $K_B >> K_G$, so: “$D_B > D_G$”
Where: “D” is the damping.

3.1.5 Transmitted force

The figure 7 represents the variation of the amplitude of the transmitted force as function of the rotational speed.

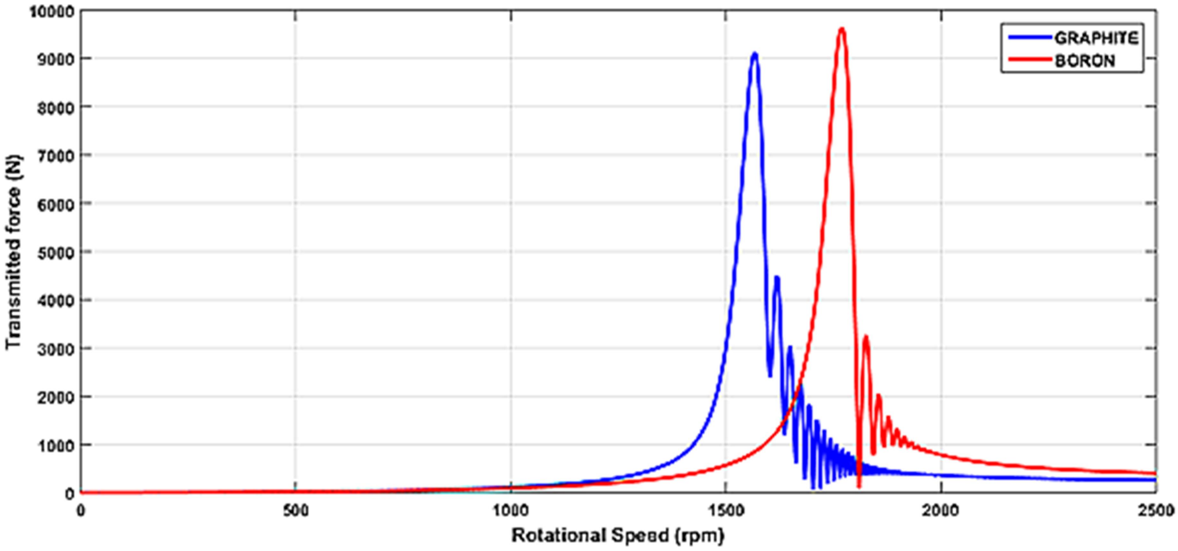

Fig. 7 - Transmitted force due to rotation unbalance.

This plot shows that there are two amplitude’s peaks of the transmitted force at speeds close to a critical speed. The first peak is for the “graphite/epoxy” and the second one is for the “boron/epoxy”. Note that those peaks are close to each other so the reduction ratio between them is literally negligible.
Those transmitted force amplitudes are obtained by the hydrostatic forces which are given by the following equations:

$$\left.\begin{aligned} F_{Tx} &= K_x\ x + D_x\ \dot{x} \\ F_{Ty} &= K_y\ y + D_y\ \dot{y} \end{aligned}\right\} \Rightarrow \left\{ F_T = \sqrt{F_{Tx}{}^2 + F_{Ty}{}^2} \right\}$$

In order to have a significant reduction in the transmitted force, it has to be a proper design of a bearing support system [43].

After the interpretation of the previous plots, the results illustrate that the shafts made of “boron/epoxy” have higher mechanical properties (stiffness, damping and damping ration) than the shafts made of “graphite/epoxy”, which can explain that the “boron/epoxy” gives better results.
So, in the next interpretations and analysis, only “boron/epoxy” material is considered.

3.1.6 The unbalance effect on dynamic behaviour

The figures 8 and 9 below represent orbits of vibration amplitudes as function of different eccentricity values (e=30µm, e=50µm, e=70µm), in the middle shaft and in the bearing journal respectively at the speed of shaft made by “boron/epoxy”.

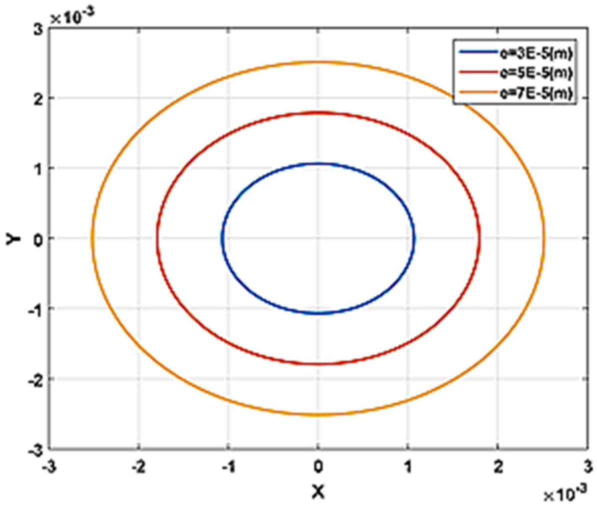

Fig. 8 - The effect of the unbalance eccentricity on vibration amplitude orbits in middle shaft at "N_{CB}".

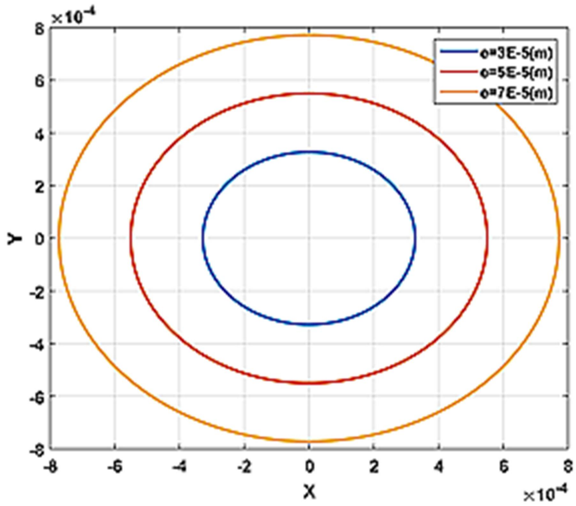

Fig. 9 - The effect of the unbalance eccentricity on vibration amplitude orbits in bearing journal at "N_{CB}".

The figure 10 represent orbits of bearing transmitted forces as function of different eccentricity values (e=30µm, e=50µm, e=70µm), at the speed of shaft made by "boron/epoxy".

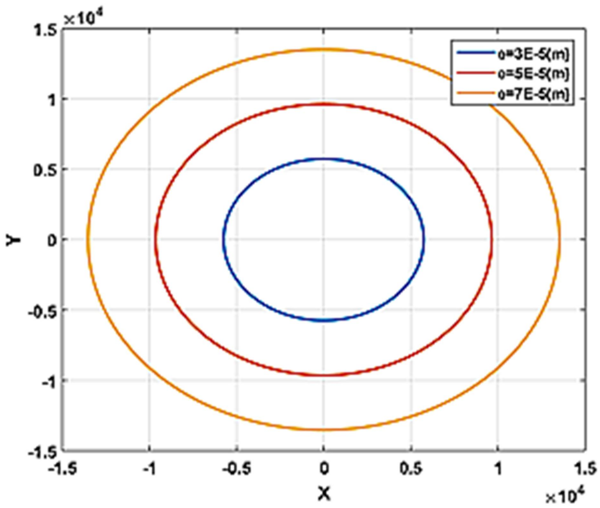

Fig. 10 - The effect of the unbalance eccentricity on bearing transmitted force orbits at "NCB".

The results show that the vibration amplitudes in middle shaft and in bearing journal, increase with unbalance eccentricity, as well as the bearing transmitted forces, and this is due to the increase of unbalance forces.

Rotor dynamics analysis including determination of critical speeds, vibration amplitudes and steady-state response, is conducted through numerical simulation works.

4. CONCLUSION

Vibration analysis of a rotating composite shaft using the finite element model that is based on the Timoshenko beam theory, is presented in this work with the help of the “LAGRANGE” formulation to solve the equation of motions.

The utilization of the finite element model in the area of rotor dynamics has yielded highly great results, and it has been successful in solving problems with complicated geometry and without the need to accept many simplifying assumptions, and the MATLAB programming helps to have fast calculations with high efficiency. There are several materials that shafts can be made of, among them, we chose in this study the graphite/epoxy and the boron/epoxy, on the purpose of making a comparison between them, so, the results of the numerical model obtained show that:

- ✓ Rotors made of composite material don’t need to reach the second and the third mode shape to obtain the vibration amplitudes, the first mode shape is enough.
- ✓ Concerning about “boron/epoxy” material, it has higher mechanical properties (stiffness, damping and damping ratio) than the “graphite/epoxy”.
- ✓ Transmitted forces to bearings due to rotation unbalance, have been influenced by the unbalance eccentricity, and they have a direct correlation between them, as the transmitted forces increases with the increase of the eccentricity.

REFERENCES

[1] R. Kraus, and M. Darlow, Design and experimental studies of composite power transmission shafting. In : *AHS, Annual Forum, 43 rd, Saint Louis, MO*. (1987) pp. 733-738.

[2] M. D. Bielefield, «Fabrication of braided RTM driveshaft tubes for the RAH-66 Comanche,» Annual forum proceedings-American helicopter society, vol. 50 (1994) pp.1001-1001.

[3] S. Kliger, Howard andd H. S. Kliger. Design and Material Implications of Composite Driveshafts. 1980.

[4] W. Hoffmann, T. Schafferus, and M. ROUILLOT. The second generation of composite propshafts; Die zweite Generation von Faserverbund-Kardanwellen. *AutomobiltechnischeZeitschrift*, 1994, vol. 96.

[5] B. Spencer, «Advances in Power transmission using filment wound composites,» Tomorrow's Material : Today, vol. 34 (1989) pp.1108-1119.

[6] D. Gargiulo, «Design and development of a power takeoff shaft,» Journal of Aircraft, vol. 23, n°112 (1986) pp. 876-880.

[7] R. S. Raghava, and E. F. Hammond, Development of a filament wound composite shaft for an aircraft generator. In : *Composites go to market. SPI Annual conference preprint. 39*. 1984. p. 11. F. 1-11. F. 5.

[8] K. Berg, «Composite driveshafts in cooling towers,» J. Cool. Tower Inst, vol. 10, n° %11, pp. 36-37, 1989.

[9] J. Cox, «Carbon fiber shafts,» Papermaker, vol. 57, 1994, n°110, p. 25,.

[10] G. F. Wilhelmi, W. M. Appleman, and T. C. Loo Francis, Composite shafting for naval propulsion systems. *Naval Engineers Journal*, vol. 98 n°4 (1986) pp. 129-136.

[11] G. P. Worgan, and D. R. Smith, *Carbon fiber drive shaft*. U.S. Patent No 4,089,190, 16 mai 1978.

[12] D. N. Yates, and D. B. Rezin, *Carbon fiber reinforced composite drive shaft.* U.S. Patent No 4,171,626, 23 oct. 1979.

[13] T. Fromknecht, «Composite technology in plings and shaftings for power transmission,» ASME International power transmission and gearing conference, vol. 43 (1992) pp. 575-581.

[14] H. Zinberg, and M. F. Symonds, The development of an advanced composite tail rotor driveshaft. In : *Proceedings of the 26th Annual Forum of the American helicopter Society,* 1970, *Washington, United States*.

[15] H. L. M. Dos Reis, R. B. Goldman and P. H. Verstrate,Thin-walled laminated composite cylindrical tubes: part III—bending analysis. *Journal of Composites, Technology and Research*, , vol.9 n°2 (1987) pp.58-62.

[16] O. A. Bauchau, Optimal design of high speed rotating graphite/epoxy shafts. *Journal of Composite Materials*, vol. 17 n°2 (1983) pp.170-181.

[17] J. McGee, B. Spencer, and D. Shy, *Feasibility Study on the Design of Reinforced Plastic Components for the LVTP (7) Vehicle Shafts*. BRUNSWICK CORP LINCOLN NE DEFENSE DIV, 1984.
[18] E. S. Zorzi, and J. G. Giordano, Composite shaft rotordynamic evaluation. In : *ASME, Design Engineering Technical Conference, Cincinnati, OH*. 1985. p. 1985.
[19] J. W. Lim, and M. S. Darlow, Optimal sizing of composite power transmission shafting. *Journal of the American Helicopter Society*, , vol.31 n°1(1986) pp.75-83.
[20] W. Hoffman, «Fibre composite in the driveline,» *Plastics and rubber international,* vol.14 n° 15 (1989) pp.46- 49.
[21] P. L. Hetherington, R. F. Kraus, et DARLOW, Mark S. Demonstration of a supercritical composite helicopter power transmission shaft. *Journal of the American Helicopter Society*, vol.35 n°1 (1990) pp.23-28.
[22] S. P. Singh, H. B. H. Gubran, and K. Gupta, Developments in dynamics of composite material shafts. *International Journal of Rotating Machinery*, vol.3 n°3(1997) pp.189-198.
[23] H. Yu, Y. Ran, G. Zhang, *et al.* A time-varying comprehensive dynamic model for the rotor system with multiple bearing faults. *Journal of Sound and Vibration*, vol.488 (2020) p.115650.
[24] P. M. Jadhav, S. G. Kumbhar, R. G. Desavale, *et al.* Distributed fault diagnosis of rotor-bearing system using dimensional analysis and experimental methods. *Measurement*, vol.166 (2020) p.108239.
[25] E. Kramer, Dynamics of rotors and foundations, Springer science, 2013.
[26] E. Chatelet, D. Lornage andG. Jaquet-Richardet, Georges. A three dimensional modeling of the dynamic behavior of composite rotors. *International Journal of Rotating Machinery*, vol.8 n°3 (2002) pp.185-192.
[27] G. Maurya, *Analysis of gyroscopic effects in rotor disc systems,* Doctoral dissertation, 2013.
[28] M. &. A. M. Hashemi, «Analytical study of three-dimensional flexural vibration of micro-rotating shafts with eccentricity utilizing the strain gradient theory,»*Meccanica,* vol. 51 n°16 (2016) pp.1435-1444.
[29] M. Hachemi, *Application de l'ODS à l'analyse des problemes de vibration des machines tournantes,* Doctoral dissertation, 2012.
[30] W. J. Chen, J. E. Littleton, Introductionto dynamics of rotor-bearing systems, Trafford Victoria, 2007.
[31] S. B. Arab et al. "A finite element based on Equivalent Single Layer Theory for rotating composite shafts dynamic analysis." *Composite Structures* 178 (2017) pp.135-144.
[32] R. Subbiah, J. E.Rotor and structural dynamics of turbomachinery, Switzerland: Springer, 2018.
[33] A. Boukhalfa, *Comportement vibratoire des arbres tournants en matériaux composites,* Docttoral dissertation, 2013.
[34] R Sino, TN Baranger, E Chatelet, G Jacquet«Dynamic analysis of a rotating composite shaft,» *Composite science and technology,* vol.68 n°12 (2008) pp.337--345.
[35] N. Rieger, Rotordynamics 2: Problems in turbomachinery, Springer, 2014.
[36] M Hashemi, M Asghari , «Analytical study of three-dimensional flexural vibration of micro-rotating shafts with eccentricity utilizing the strain gradient theory,» *Meccanica,* vol.51 n°16 (2016) pp.1435-1444.
[37] A. A. Cavalini, T. A. M. Guimaraes, A. M. Thiago, D. A. Silva, R. M. G. Bruno. Analysis of the dynamic behavior of a rotating composite hollow shaft. *Latin American Journal of Solids and Structures*, vol.14 (2017) pp. 1-16.
[38] M. Al Muslmani, *Rotordynamic analysis of tapered comosite driveshaft using conventional and hierchical finite element formulations,* Doctoral Dissertation, 2013.
[39] S. Merdaci, Analytical solution for free vibration analysis of plates functionally graded FGP with porosities. Recueil de mécanique, Vol.4(2) (2020) pp.397-408.
[40] B. Mohamed, *Conception d'un paier intelligent à patins hydrostatiques contrôlés par des valves électorhéologique pour contrôler les vibration de rotor.,* Doctora dissertation, 2020.
[41] N.Dumitru, E.Secară, M.Mihălcică, «Study of rotor-bearings systems using Campbell diagram,» *In preccedings of the 1st international conference on Manufacturing Engineering, Quality and Production Systems.,* vol.2
[42] E Swanson, CD Powell, S Weissman, «A Practical review of rotating machinery critical speeds and modes,» vol.39 n°15 (2009) pp.16-17.
[43] R. Tiwari, Rotor systems : analysis and identification, CRC Press, 2017.