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# Numerical investigation of laminar mixed convection flow between two parallel planes in the presence of rotating cylinders 

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#### Abstract

In this work, mixed convection and laminar flow between two parallel planes in the presence of two rotating circular cylinders is investigated numerically. A wall flux density is imposed on the top and bottom planes and the rotating cylinders' surfaces are assumed to be adiabatic. The study is performed for different values of Reynolds number $(50 \leq R e \leq 300)$, angular rotational velocity of the cylinders $(-300 \leq W(\mathrm{rad} / \mathrm{s}) \leq 300)$, angular velocity ratios of the cylinders $(0.25 \leq W 1 / W 2 \leq 4)$ and the distance between the cylinders. It is observed that the flow patterns and thermal field are affected considerably by the rotational motion of the cylinders and the Reynolds number variation of the longitudinal flow. In addition, Local Nusselt number computed shows how the existence of the two cylinders affects heat transfer enhancement with varying percentages from $20 \%$ to $52.94 \%$ compared to the case without cylinders. Augmentation of angular velocity ratio brings from 11.76\% to $31.76 \%$ of heat transfer improvement.


## 1 Introduction

Heat transfer by mixed convection (free and forced) with the presence of cylinders has many industrial applications such as: cylindrical cooling devices in the plastics industries, oil and gas pipelines. In addition, this phenomenon occurs in many engineering systems, for example; the cooling of electronic components, tubular and compact heat exchangers, nuclear reactors, airplanes, automobiles, buildings and so on. Many scientific works have been published to analyse the phenomenon of mixed convection in 2D geometries with the presence of vortex movements. In Khan et al 2004 [1] an integral approach on the equations of forced convection movement in the presence of cylinders between two parallel planes is performed in order to study the effects of the flow blocking by the cylinders on the fluid flow and heat transfer. Correlations are obtained for the coefficient of friction and the Nusselt number as a function of the Reynolds number, Prandtl and the blocking rate. Bharti et al 2006 [2] studied numerically using the finite volume method the heat transferin a forced convection flow between two parallel planes around a circular cylinder. Simulations are executed for a Reynolds number ranging between 10 and 45 and Prandtl number goes from 0.7 to 400 . Authors proposed simple correlations for the Nusselt number as a function of the dimensionless variables Re and Pr. Thus, the rate of heat transfer increases with an increase in the Reynolds and / or Prandtl numbers.In the work of G. Biswas and S. Sarkar 2009 [3] the mixed convection is

[^0]characterized in a longitudinal flow around a heated cylinder. They explained the vortex formation process behind a heated cylinder in a low Reynolds number flow under the influence of thermal buoyancy. The numerical study carried out by VAF. Costa and AM. Raimundo 2010 [4] is focused on the heat transfer by mixed convection in a square enclosure with a rotating cylinder centred inside. The overall thermal performances of the enclosure areanalysed via the global Nusselt number. Results show clearly how the rotating cylinder affects the thermal performances of the enclosure and in which way the thermo-physical properties of the cylinder are important to the overall process of heat transfer inside the enclosure.SH. Hussain and AK. Hussein 2011 [5] in their project managed numerical simulations using a finite volume scheme for laminarmixed convection flow in a two-dimensional square enclosure of width and height ( L ) and with the presence rotating circular cylinder of radius $\mathrm{R}=0.2 \mathrm{~L}$, enclosed inside the cavity. Findings of the study explain that the increase in Richardson and Reynolds numbers has an important role on the flow and temperature fields and that the locations of the rotating cylinders have an important effect on the improvement of the convection heat transfer in the square enclosure. F. Selimenfendigiel and HF. Oztop 2015 [6] presented a numerical study of mixed convection in a cavity driven by a cover filled with Ferro fluid in the presence of two rotating cylinders. The cavity is heated from below and cooled from the driven wall. Rotating cylindrical surfaces and the vertical side walls of the cavity are assumed to be adiabatic. A magnetic dipole source is placed under the bottom wall of the cavity. The results of this study revealed that the angular velocities of the cylinders in addition to the angular velocity ratio and the diameter ratios have a considerable effect on improving the heat transfer in the cavity. Improvements in heat transfer of around $181.5 \%$ are obtained for a clockwise rotation of the cylinder at $\mathrm{W}=-400 \mathrm{rad} / \mathrm{s}$ compared to the case of stationary cylinders. Moreover, increasing the angular velocity ratio from 0.25 to 4 brings about $91.7 \%$ enhancement in heat transfer. Kamel Yahiaoui et al 2017 [7] carried out a numerical study on twodimensional laminar mixed convection and heat transfer from a circular and rotating isothermal cylinder confined in a horizontal channel. The blocking rate and the Prandtl number are fixed at 0.05 and 0.7 respectively. The results are in very good agreement with those resulting from the previous studies at $\mathrm{Ri}=0$ and $\mathrm{a}=0$. The simulated mixed convection flux and heat transfer for Richardson number Ri and Reynolds number in the range from 1 to 40, demonstrating the influence of thermal buoyancy varies from 0 to 1 and for the rotation rate from $\alpha=0$ to $\alpha=4$. The importance of the circular cylinder rotation effect on the mixed convection and the measured local and average Nusselt numbers is illustrated. Besides, representative aerodynamic profiles and isothermal models are presented and discussed is this paper.

In the present study, a parametric analysis of the laminar flow of mixed convection in a two-dimensional geometry (flow between two parallel planes, in the presence of two movable cylinders) with an imposed wall flux density on the walls is studied numerically. More specifically, the effects of the parameters; Reynolds number, angular velocity of the rotating cylinders W , ratios of the angular velocities of the two cylinders as well as the distance between the two cylinders on the rate of heat transfer in terms of Nusselt number, were examined.

## 2 Problem description

The considered problem consists in the study of heat transfer for a fluid flow passed on two circular cylinders between parallel planes heated by a constant flow. A schematic representation of the considered geometry with the boundary conditions is presented in Figure 1. With $\mathrm{R}=0.2 \mathrm{~cm}$ is the cylinder radius, $\mathrm{A}=1 \mathrm{~cm}$ is the distance between the two cylinders and $\mathrm{H}=1 \mathrm{~cm}$ is the height between the two plates, the first cylinder is situated at $\mathrm{x}=6 \mathrm{~cm}$ from the inlet section. Moreover, q " is the heat flux density and $\mathrm{U}_{0}, \mathrm{~T}_{0}$ are the average velocity and the temperature at the inlet respectively. At the inlet section an inlet velocity profile is considered for all the computations.


Fig. 1 - Physical model

The system of interest is considered to be two-dimensional, laminar, incompressible and steady. The physical properties of the fluid are assumed to be constant, except for the density of the fluid in the buoyancy parameter, in which the Boussinesq approximation is adopted because the relative variations in density are very small. The dimensionless conservation equations are as follows:

Continuity equation:

$$
\begin{equation*}
\operatorname{div}(\vec{u})=\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}
\end{equation*}
$$

Momentum conservation equations:

$$
\begin{gather*}
U \frac{\partial U}{\partial X}+V \frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{R e}\left(\frac{\partial^{2} U}{\partial X^{2}}+\frac{\partial^{2} U}{\partial Y^{2}}\right)  \tag{2}\\
U \frac{\partial V}{\partial X}+V \frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{R e}\left(\frac{\partial^{2} V}{\partial X^{2}}+\frac{\partial^{2} V}{\partial Y^{2}}\right)+R i T \tag{3}
\end{gather*}
$$

Equation energy conservation:

$$
\begin{equation*}
U \frac{\partial T}{\partial X}+V \frac{\partial T}{\partial Y}=\frac{1}{\operatorname{RePr}}\left(\frac{\partial^{2} T}{\partial X^{2}}+\frac{\partial^{2} T}{\partial Y^{2}}\right) \tag{4}
\end{equation*}
$$

The dimensionless parameters that appear in the formulation are the Reynolds number ( $\operatorname{Re}=D_{h} . U_{0} / v$ ), the Richardson number $\left(\mathrm{Ri}=\mathrm{Gr} / \mathrm{Re}^{2}\right)$ and Prandtl number. The RePr product is the Peclet number, Pe. For height value of the Peclet number the heat molecular diffusion is negligible.

## 3 Grid study and validation

A mesh sensitivity study was carried out in order to choose the optimal number of nodes. For the computational domain discretization, triangular elements are utilized (figure 2.a). We tested six grids (3136), (13520), (19230), (45870), (74162), and (112530) for each we presented the average Nusselt number and the profile of the velocity according to the coordinate y.Figure 2 shows the variation of the local Nusselt number as a function of the number of elements for the 6 grid values. It is noted that the two values of the Nussetl numbers for (74162) and (112530) are almost identical. This result confirms that the 74162 number of elements is the favourable grid for carrying out the next project simulations in the present study.


Fig. 2 - Grid sensitivity; a) Computational domain discretization. b) Variation of the Nusselt number Nu with the number of elements for the 6 considered grid sizes.

Figure 3 shows the evolution of the axial velocity as a function of the transversal coordinate Y for the six selected meshes. We notice that the difference between the velocity values is almost negligible, therefore, the mesh network (74162) is adopted for the rest of the simulations.


Fig. 3 -Variation of the velocity profile as a function of Y coordinate for the 6 grids at ( $\mathrm{X}=13 \mathrm{~cm}$ ). a) complete profile b) the selected area.

For the validation of the used solver configuration, a comparison the local Nusselt number values obtained with the present solver in a longitudinal flow around a stationary heated cylinder (see Figure 4) with those obtained by (G.Biswas [3], Dennis et al [8 ], Jafroudi and Yang [9] and Apelt and ledwich [10]) in the same configuration.


Fig. 4 - The selected physical problem for validation with boundary conditions G.Biswas [3].
Table 1 compares the results of the present work showing the values of the mean Nusselt number calculated on the surface of the cylinder with those of G. Biswas [3], Dennis et al [8], Jafroudi and Yang [9] and Apelt and ledwich [10]. It is well remarked that the comparison is very satisfying and it revealed a very good agreement between the used solver configuration and the chosen literature work for validation.

Table 1 - Comparison of the mean Nusselt number computed at on the cylinder for different Reynolds numbers with previous studies.

| Re | Present solver G.Biswas[3] |  | Dennis et al [8] | Jafroudi et Yang [9] Apelt et Ledwich [10] |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 1,8698 | 1,8577 | 1,876 | 1,821 | 1,864 |
| 20 | 2,4359 | 2,4483 | 2,557 | 2,433 | -- |
| 30 | 2.9257 | 2,8877 | - | 2.850 | - |
| 40 | 3,4085 | 3,2531 | 3,480 | 3,200 | 3.255 |

## 4 Results and discussions

In the present project, the effect of various parameters on the thermal field and fluid dynamics have been studied numerically. The main parameters are: The Reynolds number which varied between 50 and 300 with a step of 50 , the
rotation angular velocity of the cylinders $\mathrm{W}(0$ to $300 \mathrm{rad} / \mathrm{s})$, the ratio of the angular speeds of the two cylinders $(0.25,0.5$, 1,2 and 4 ), in addition to the distance between the two cylinders.

### 4.1 Effect of the Reynolds number

In order to show the effect of the Reynolds number on the improvement of heat transfer (Nusselt number), various simulations were carried out with different values of Reynolds number $\operatorname{Re}(50,100,150,200,250$ and 300), the Richardson number and the angular velocity of the two cylinders are constant in this case ( $\mathrm{Ri}=1.25$ and $\mathrm{W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s}$ ). The local Nusselt number is calculated along the X direction (Figure 5) at different planes. Note that in this case the two cylinders rotate in the positive direction (clockwise).

Figure 5 shows the variation of the Nusselt number as a function of the longitudinal coordinate x for several values of the Reynolds number. Figure 5 b shows a zoom of the area near the cylinders.


Fig. 5 - Variation of the local Nusselt number for different Reynolds numbers $(\operatorname{Ri}=1.25, \mathrm{~W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s})$ a) complete profile b) the selected area.

The values of the Nusselt number are approximately identical far from the cylinders. Once you get closer to the cylinders, the values increase considerably upstream and downstream of the cylinders. The maximum values are located at the same x coordinate and they have an increasing trend as a function of the Reynolds number. The movement of the cylinders destroys the formation of the dynamic and thermal boundary layer. Which presents a barrier in front of momentum transfer and thermal transfer.

Figure 6 displays the variation of the local Nusselt number estimated at $x=6.5 \mathrm{~cm}$ located between the two cylinders as a function of the Reynolds number for a Richardson number Ri $=1.25$ and a constant rotation velocity of the two cylinders $\mathrm{W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s}$. The Nusselt number evolves exponentially as a function of the Reynolds number and the rate of improvement in heat transfer amounts to $40.32 \%$ at a Reynolds number equal to 300 compared to that calculated for a Reynolds number equal to 50 . The disturbance area known significant agitation due to the rotational movements of the cylinders which is combined with the main flow.


Fig. 6 - The evolution of the local Nusselt number as a function of Reynolds number at $\mathrm{X}=6.5$

$$
(\mathrm{Ri}=1.25, \mathrm{~W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s})
$$

Secondary flows help to affect the maximum velocity of the fluid in the channel. The maximum velocity provides a definite indication of the decrease or increase in the residence time. The increase in the previous favours the phenomena of transfers. Figure 7 presents the variation of the maximum velocity along the mean line between the two plates for several values of the Reynolds number. It seen that the maximum velocity decreases in the area near the cylinders due to the generation of secondary flows on the part of the cylinders in both the mobile and stationary study cases. This reduction is favourable for good agitation in the fluid of both two dynamic and thermal fields.


Fig. 7 - Distribution of the maximum velocity along the ox axis, for different Reynolds numbers.
About the structure of the flow, the streamlines are presented in figure 8 for several values of the Reynolds number and the Richardson number $\mathrm{Ri}=1.25$ with rotational velocity of the two cylinders $\mathrm{W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s}$. As the Reynolds number increases, the kinematics of the flow becomes complex and the number of vortices degenerates in the area close to the two cylinders, in the centre and on either side of the cylinders. At low Reynolds numbers, vortices form near the bottom wall and increasing Reynolds numbers gradually inhale them.


Fig. 8 - The streamlines for different Reynolds number $(\operatorname{Ri}=1.25, \mathrm{~W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s})$.

### 4.2 Effect of the cylinders' angular velocity

The values of the rotation velocity and the direction of rotation of the cylinders play a primordial role in the dynamic agitation of the flow and therefore the heat transfer increases or decreases according to the rotation protocol. In other words, the combination of movements on the part of the cylinders and the main flow can be a favourable or unfavourable factor in transfer phenomena.

We present in Figure 9 the evolutions of the local Nusselt number along the x coordinate for a Reynolds number equal to 100 and for a Richardson number equal to 1.25 and at different values of the rotation velocity of the cylinders in both, positive and negative ways. After a certain distance, the Nusselt number begins to be constant, then it becomes disturbed because of the existence of the cylinders and it begins to slow down again far from the zone of the disturbance. In the case of stationary cylinders, the Nusselt number is greater compared to the other cases because the fluid stays longer which allows it to exchange as much heat. The rotational movement in the flow releases the fluid outside the disturbed area. Therefore, when the rotation velocity high, the quantity of fluid expelled downstream of the channel is important whatever the rotation direction of the cylinders.



Fig. 9 - Variation of the Nusselt number for different values of $\mathrm{W}(\mathrm{Re}=100, \mathrm{Ri}=1.25, \mathrm{~W} 1 / \mathrm{W} 2=1) \mathrm{a})$ complete profile b) the selected area.

Figure 10 illustrates the evolution of the maximum velocity along the x direction for a Reynolds number equal to 100 and Richardson number equal to 1.25 at different values of the cylinders' rotation velocity in both directions, positive and negative. The flow slows down (maximum velocity is the smallest) in the case where the two cylinders are stationary. This gives more residence time to the fluid, and therefore to exchange with the hot walls enough amount of heat compared to the
other considered cases. The values of the maximum velocity increase in the disturbance zone with the increase of the angular velocity of the cylinders W. Moreover, the velocity length of establishment after the second cylinder also increases with W . And we also note that there is no effect of the cylinders 'direction of rotation on the maximum measured velocity.


Fig. 10 - Variation of the maximum velocity along x for different angular velocities of the cylinders a) complete profile b) the selected area.

$W_{1}=W_{2}=0$


Fig. 11 - The streamlines for different values of the cylinders' rotational velocity.
The structure changes considerably according to the rotation protocol and it changes according to the direction of rotation. Figure 11 shows the streamlines for a Reynolds number equal to 100 and a Richardson number equal to 1.25 for different values of the rotational velocities of the cylinders. In the case of stationary cylinders, vortices are formed after the
cylinders. These vortices are known as the Van Karman vortices and they depend essentially on the Reynolds number. The rotational movement of the cylinders destroys these vortices and they moved far from the cylinders' area. Thus, the cylinders decelerate the movement while its movement helps to release the fluid trapped between the two cylinders.

### 4.3 Effect of the angular velocities ratio

In the following, we are going to test the effect of the ratio between the two rotational velocities of the two cylinders on the evolution of the local Nusselt number along the longitudinal coordinate x. First, the angular velocity of the first cylinder is fixed at $\mathrm{W} 1=100 \mathrm{rad} / \mathrm{s}$ and the angular velocity of the second cylinder takes the following values: $(25,50,100,200$ and $400 \mathrm{rad} / \mathrm{s}$ ). The Reynolds number of the main flow is 100 and the Richardson number is 1.25 , see Figure 12.

The local Nusselt number decreases rapidly along the pipeline. Once the flow crosses the zone of disturbance, the Nusselt number is disturbed then it is restored far from the cylinders. It increases monotonically as a function of the first three ratios of the angular velocities $(0.25,0.5$ and 1$)$. This evolution knows a sudden increase then it decreases towards an asymptotic value. While for the other values, the local Nusselt number is presented by two peaks in the upstream and downstream areas of the cylinders. This is mainly due to the change in the structure of the flow. In addition, these peaks are inversely proportional to the ratio of the rotational velocities.


Fig. 12 - The variation of the local Nusselt number for different ratios of the angular velocities of the cylinders with $(\operatorname{Re}=100, \mathrm{Ri}=1.25, \mathrm{~W} 1=100 \mathrm{rad} / \mathrm{s}) \mathrm{a})$ complete profile b$)$ the selected area.
Figure 13 displays the evolution of the maximum velocity along the mean line of the pipe for several values of the two cylinders' angular velocities ratio and for a Reynolds number of the main flow equal to 100 , with $\mathrm{Ri}=1.25$. It is remarked that the maximum velocity increases with the decrease in this ratio so the secondary flow is attenuated.


Fig. 13 - Variation of the maximum velocity on the centre of the geometry for different ratios of the cylinders angular velocities with $(\operatorname{Re}=100, \operatorname{Ri}=1.25$ and $\mathrm{W} 1=100 \mathrm{rad} / \mathrm{s})$ a) complete profile b) the selected zone.

### 4.4 Effect of the distance between the two cylinders

Figures 14 and 15 present the evolution of the local Nusselt number and the maximum velocity, respectively, along the channel for two values of the distance between the cylinders, $\mathrm{A}=5 \mathrm{R}$ and $\mathrm{A}=2.5 \mathrm{R}$ with Reynolds number equal to 100 and Richardson number equal to 1.25 . The two cylinders are rotated in the same direction where $\mathrm{W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s}$. The Nusselt number increases with the decrease of the distance separating the two cylinders. Because in a more confined medium the secondary flow becomes more intense and consequently the heat transfer improves. For small distances between the two cylinders, the secondary flow is greater which explains the decrease in the maximum velocity.


Fig. 14 - Variation of the Nusselt number for different distances between the two cylinders $\mathrm{A},(\mathrm{Re}=100, \mathrm{Ri}=1.25$ and $\mathrm{W} 1=\mathrm{W} 2=100 \mathrm{rad} / \mathrm{s}) \mathrm{a})$ complete profile b) the selected area.


Fig. 15 - Variation of the maximum velocity at the centre of the geometry for different distances between the two cylinders $(\operatorname{Re}=100, \operatorname{Ri}=1.25)$, a) complete profile b) the selected area.

## Conclusion

In this present work, a numerical characterization of two-dimensional flow with the presence of two aligned cylinders considering for two scenarios, stationary and rotating are performed. The local Nusselt number and the maximum velocity are the two parameters that characterize the thermal and dynamic performances of the flow. These quantities are calculated as a function of the Reynolds number, the angular velocities of the cylinders and the Richardson number. It has been proven that for aligned cylinders, the heat transfer and the dynamics of the secondary flows are greater in the case where the cylinders are stationary. Also, the decrease in the distance between the two cylinders presents a factor favouring the intensification of transfer phenomena (dynamic and thermal). As a perspective, the location of the two cylinders and their number presents a promising solution improving heat transfer in this type of phenomenon.

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