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Entropy Generation in electrically conducting elasto-viscous fluid in a porous medium over a stretching sheet subject to a transverse magnetic field

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Abstract — An analysis is performed for entropy generation in a steady laminar boundary layer flow of an electrically conducting elasto-viscous fluid in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet, The effects of viscous dissipation, internal heat generation of absorption and work done due to deformation are considered in the energy equation. The variations in the velocity, temperature field and entropy generation with change in various dimensionless parameters, such as viscoelastic parameter (K), combined parameter (C), Prandtl number (*Pr*) are presented. It was found that the temperature varies significantly with the Prandtl number and the magnetic parameter and the entropy generation decreases with the increase of the Prandtl number, so the optimum design and efficient performance of the flow system can be enhanced by the ability to clearly identify the source and location of entropy generation.

Keywords: *Entropy generation, Magnetic field, Porous medium, Second grade fluid, Stretching sheet.*

I. Introduction

The design of thermal systems can be achieved by optimization of entropy generation in the systems. This issue has been the topic of great importance in many engineering field such as heat exchangers, cooling of nuclear reactors, MHD power generators, geophysical fluid dynamics, energy storage systems and cooling of electronic devices, etc. Entropy generation is associated with thermodynamics irreversibility, which is common in all types of heat transfer processes. Different sources of irreversibility are responsible for entropy generation such as heat transfer across finite temperature gradient, characteristics of convective heat transfer, magnetic field effect, viscous dissipation effect etc [1].

An intensive effort, both theoretical and experimental, has been devoted to problems of non-Newtonian fluids. The study of MHD flow of viscoelastic fluids over a continuously moving surface has wide range of applications in technological and manufacturing processes in industries.

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This concerns the production of synthetic sheets, aerodynamic extrusion of plastic sheets, cooling of metallic plates, etc [2].

The objective of this paper is to study the entropy generation in a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet.

II. Formulation of the problem

In two-dimensional Cartesian coordinate system (x, y) we consider an incompressible, electrically conducting fluid of second grade, through the porous medium subject to a transverse magnetic field over a stretching sheet with the plane $y = 0$, then the fluid is occupied above the sheet $y > 0$.

The x -axis is taken in the direction of the main flow along the plate and the y -axis is normal to the plate with velocity components u, v in these directions. Under the usual boundary layer approximations, the flow is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\lambda B_0^2}{\rho}u-\frac{\nu}{k_1}u$$

$$+\frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right] \qquad (2)$$

λ: electric conductivity, $\Omega^{-1}.m^{-1}$
B_0: Uniform magnetic field strength, $Wb.m^{-2}$
ρ: Density of the fluid, $kg.m^{-3}$
α_1: Positive constant
k_1: Permeability of porous medium
ν: Kinematic viscosity of the fluid, $m^2.s^{-1}$
The boundary conditions are given by:

$$y=0,\quad B>0,\quad u=Bx,\quad v=0 \qquad (3.a)$$

$$y\to\infty,\quad u\to u_\infty,\quad \frac{\partial u}{\partial y}\to 0 \qquad (3.b)$$

The energy equation is:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2$$

$$+\alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+Q(T-T_\infty) \qquad (4)$$

C_P: Specific heat of the fluid, $J.kg^{-1}.K^{-1}$
T: Temperature, K
k: Thermal conductivity of the fluid, $W.m^{-1}.K^{-1}$
μ: Dynamic viscosity of the fluid, $kg.m^{-1}.s^{-1}$
Q: Rate of internal heat generation or absorption $W.m^{-3}.K^{-1}$
The relevant boundary conditions are:

$$y=0;\quad T=T_S=T_\infty+A\left(\frac{x}{l}\right)^2 \qquad (5a)$$

$$y=\infty;\quad T\to T_\infty \qquad (5b)$$

III. Analytical solution

A similarity solution exists in this situation if we introduce a transformation [3]:

$$u=Bx\,f'(\eta)$$
$$v=-(B\nu)^{1/2}f(\eta)$$
$$\eta=(B/\nu)^{1/2}\,y \qquad (6)$$

B: Extensibility factor, s^{-1}
f: Dimensionless function
η : Dimensionless variable, $\eta=(B/v)^{1/2}$
From (2) and (6), we have:

$$f'^2-ff''=f'''+K(2f'f'''-f''^2-ff^{IV})-Cf' \qquad (7)$$

K: Viscoelastic parameter; $K=\alpha_1 B/\mu$
C: Combined parameter, $C=\lambda B_0^2/\rho B+\nu/k_1B$
Now let us seek a solution of Eq. (7) in the form [3], [4]:

$$f(\eta)=\frac{1}{m}(1-e^{-m\eta}) \qquad (8)$$
$$m=\sqrt{(1+C)/(1+K)}$$

The boundary conditions are

$$\eta=0\ ,f=0\ \text{et}\ f'=1 \qquad (9a)$$
$$\eta\to\infty\quad f'\to 0, f''\to 0 \qquad (9b)$$

On substituting (8) into (6) and using boundary conditions (9a) and (9b) the velocity components take the form:

$$u=Bxe^{-m\eta} \qquad (10a)$$
$$v=-(Bv)^{1/2}(1-e^{-m\eta})/m \qquad (10b)$$

Defining the dimensionless temperature

$$\theta(\eta)=\frac{T-T_\infty}{T_S-T_\infty} \qquad (11)$$

So the energy equation can be written as:

$$\theta''+\Pr f\,\theta'-\Pr(2f'-\beta)\,\theta=-\Pr E[(f'')^2+Kf''(f'f''-ff''')] \qquad (12)$$

$$\theta(0)=1 \qquad (13a)$$
$$\theta(\infty)=0 \qquad (13b)$$

Pr: Prandlt number, $\Pr=\mu Cp/K$
E: Eckert number, $E=B^2l^2/ACp$
β: Heat source/sink parameter, $\beta=Q/B\rho Cp$
Introducing the dimensionless variable [5]

$$\xi=-re^{-m\eta}\quad ;r=\Pr/m^2 \qquad (14)$$

Substituting (14) into (12) we find

$$\xi\frac{d^2\theta}{d\xi^2}+(1-r-\xi)\frac{d\theta}{d\xi}+\left(2+\frac{\beta r}{\xi}\right)\theta=-\frac{\Pr E(1+K)\xi}{r^2} \qquad (15)$$

$$\theta(-r)=1 \qquad (16a)$$
$$\theta(0^-)=0 \qquad (16b)$$

Eq. (15) can be transformed into the standard confluent hypergeometric equation or the Kummer's equation [3], [6], the solution is given by:

$$\theta(\xi)==(1+H)\left(\frac{\xi}{-r}\right)^{\frac{r+s}{2}}\frac{M\left(\frac{r+s-4}{2},s+1,\xi\right)}{M\left(\frac{r+s-4}{2},s+1,-r\right)}-H\left(\frac{\xi}{-r}\right)^2 \qquad (17)$$

Where:

$$s=r\sqrt{1-\frac{4\beta}{r}};\quad H=\frac{E\Pr(1+K)}{4-2r+\beta r}$$

$$M(a,b,\xi) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n}{(b)_n n!} \xi^n$$

M is the Kummer's function,

$(a)_n = a(a+1)(a+2) \ldots .(a+n-1)$

$(b)_n = b(b+1)(b+2) \ldots (b+n-1)$

The solution (17) can be rewritten, in terms of η as:

$$\theta(\eta)=(1+H)e^{-\frac{(r+s)m\eta}{2}} \frac{M\left(\frac{r+s-4}{2},s+1,-re^{-m\eta}\right)}{M\left(\frac{r+s-4}{2},s+1,-r\right)} - He^{-2m\eta} \quad (18)$$

IV. Second law analysis

The local volumetric rate of entropy generation in the presence of a magnetic field and porous medium is given by [7]:

$$S_G = \frac{k}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\lambda B_0^2}{T_\infty} u^2 + \frac{\mu}{k_1 T_\infty}\left(\vec{V}\right)^2 \quad (19)$$

For prescribed boundary condition, the characteristic entropy generation rate is:

$$S_{G_0} = \frac{k(\Delta T)^2}{l^2 T_\infty^2} \quad (20)$$

So the entropy generation number is given by:

$$N_S = \frac{S_G}{S_{G_0}} = \frac{4}{X^2}\theta^2 + Re_l \theta'^2 + \frac{BrRe_l}{\Omega} f''^2 + \frac{Br}{\Omega}(Ha^2 + Da^2) f'^2 - \frac{BrDa^2}{\Omega X^2 Re_l} f^2 \quad (21)$$

X: Dimensionless axial distance, X =□x/l

Re_l :Reynolds number based on the characteristic length, $Re_l = u_l l/\nu$

Br: Brinkman number $Br = \mu u_x^2/k\Delta T$

Ω : Dimensionless temperature difference, $\Omega = \Delta T/T_\infty$

Ha: Hartman number, $Ha = B_0 l\sqrt{\lambda/\mu}$

Da: Darcy number, $Da = l/\sqrt{k}$

V. Results and discussion

The effects of the viscoelastic parameter K on the longitudinal velocity $f'(\eta)$ and the transverse velocity $f(\eta)$ are illustrated on Fig.1 and Fig.2. As it can be seen, for a fixed value of η, both $f'(\eta)$ and $f(\eta)$ increase as viscoelastic parameter increase since as fluid viscosity decreases, the momentum boundary layer becomes thinner, leading to an increase in the fluid velocity gradient. And also this can be explained by the fact that, as the viscoelastic parameter decreases hydrodynamic boundary layer adheres strongly to the surface, which in turn retards the flow in the longitudinal and the transverse directions.

Fig.3 depicts the temperature profiles $\theta(\eta)$ as function of η for different values of the Prandtl number. As it can be noticed, $\theta(\eta)$ decreases with η whatever is the value of the Prandtl number, for a fixed value of η, the temperature $\theta(\eta)$ decreases with an increase in Prandtl number which means that the thermal boundary layer is thinner for large Prandtl number and the thermal conductivity is decrease when Prandtl increases which can reduce strongly the temperature profiles.

Fig.4.illustrates the effect of the Prandtl number on the entropy generation number N_S. The entropy generation number decrease when Prandtl increases, this is due to the fact that according to fig. 3 the temperature which is one of the sources of entropy generation decrease sharply with the increase of the Prandlt number.

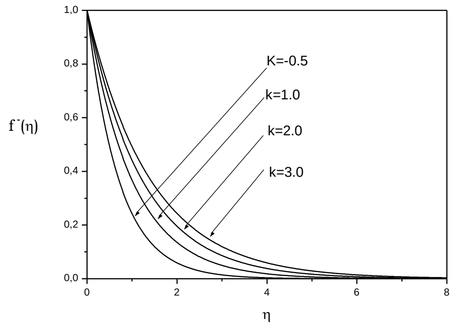

Fig.1. Effect of the viscoelastic parameter on the longitudinal velocity(C=1).

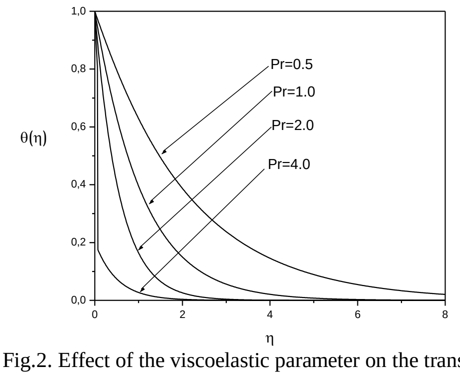

Fig.2. Effect of the viscoelastic parameter on the transverse velocity (C=4.0).

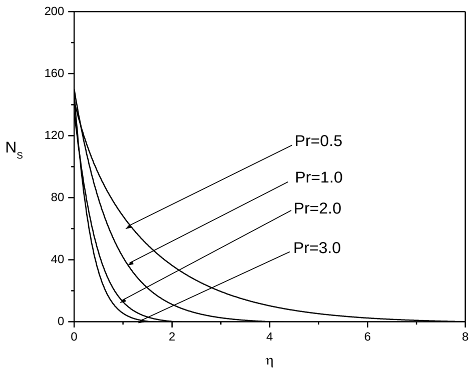

Fig.3. Effect of the Prandtl number on the emperature. *(C=1.0, K=1.0, E=0.1, β=0.01).*

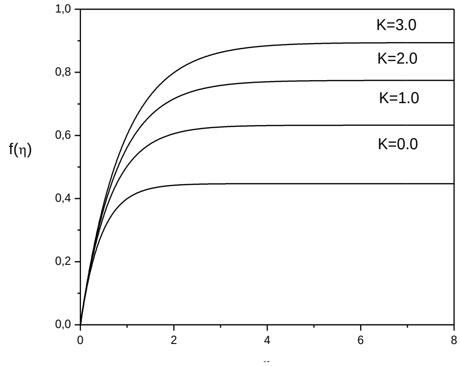

Fig.4. Effect of the Prandtl number on the entropy generation number *(C=4.0, K=1.0, β=0.01, Re_l=10.0, Ha=1.0, X=0.2, E=0.1, Da=1.0, $Br\Omega^{-1}$=1*

IV. Conclusions

From the results the following conclusions could be drawn:

a. The velocities depend strongly on the combined and the viscoelastic parameters.
b. The temperature varies significantly with the Prandlt number.
c. The entropy generation decreases with the increase of the Prandlt number
d. The surface act as a strong source of entropy generation and heat transfer irreversibility
e. The optimum design and efficient performance of the flow system can be enhanced by the ability to clearly identify the source and location of entropy generation.

The present results show that optimization of entropy generation in the flow system can be achieved with appropriate choice and combination of the various thermo physical parameters.

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