EMPIRICAL ESTIMATION OF THE CONSUMPTION FUNCTION IN U.K. USING TIME-SERIES DATA

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Introduction

The main aim of the present part is to present and discuss actual time-series estimates of the various forms of the consumption function in U.K. We will test some concepts relevant to the econometric modelling of economic time-series Data by re-estimating some of the Models yet designed for an extended period of time (1950 - 1986) especially : Hendry (1974), Ball (1975), and Davidson (1978) in their investigation denoted by (DHSY). Also, we shall develop an econometric relationship and see whether this fulfils the theory criteria and we shall apply a variety of tests on the various estimated models.

The data used is described as follows : Y_t, C_{dt}, C_N, represent respectively : personal disposable income, Consumer's expenditure on durable goods, consumers' expendure on all other goods and services. The main series used are taken from Economic trends (1987 Annual supplement) and are quarterly seasonally unadjusted in £ million at 1980 prices. The quarterly value and log value are denoted by the letters (Q) and (ln) before variables respectively. But we start first by presenting and commenting on some problems which researchers could face in measuring MPC.

Problems in measurement

As we will see later, a remarkable difference in estimating the parameters of consumption function especially the MPC, has been noticed in analysing the results which have emerged from the past works on consumption function. Yet, (DHSY) have mentioned this difference in their famous paper [13]. "The studies listed as H,B,W. satisfy the requirement that appoximately the same data set (C, Y) is involved in all three cases. Nevertheless, the results differ in many respects and are conditioned by very different auxiliary hypothesis". Many of the mentioned differences can be reconciled by adjusting for various biases in the parameter estimates.

The estimator Biasness problem

One of the most commonly estimated by biased methods is the function of the form :

$$C_t = a + bY_t + dC_{t-1} + \mu_t$$

Where μ_t represents a random error term. This term occurs in all stochastic equations because it is not possible to explain all the variance of the dependent variable, given by :

$$\sigma^2_x = \frac{\sum_{t=1}^{N}(x_t - \bar{x})^2}{N-1}$$

where : x is the random variable, $\bar{x}$ is the mean value, N is the number of observations.

The size of $1/(N-1) \sum_{t=1}^{N} \mu^2_T$ is generally less than half percent of variance of consumption so this is not a major problem but the assumption that μ_t is a random and that it is uncorrelated with any of the independent variables in the equation is the most likely to cause biased parameters estimates. The best known method of estimation used in empirical work is the ordinary least squares (OLS)[14]. The OLS method minimizes the sum of the squares of the error terms over the sample period.

$$\text{Min. } \sum_{t=1}^{N} \mu_t^2$$

One of the fundamental assumption of OLS is that μ_t is not correlated with any of the independent variables in the equation, otherwise the estimates will be biased. Empirically this condition is fulfilled by stating that the independent variables are predetermined at time t.Exogenous variables (variables which are used to determine the value of other variables in the system but are not themselves determined within the system) such as time trends, dummy variables, foreign trade prices meet these conditions. Assume that exogenous and lagged variables are known at time t and not correlated with the random error term μ_t.

Given the consumption function of the form:

$$C_t = a + bY_t + dC_{t-1} + \mu_t$$

and

$$C_t + I_t + G_t = Y_t$$

Therefore:

$$Y_t = a + bY_t + dC_{t-1} + \mu_t + I_t + G_t$$

Clearly, we can notice the dependence between Y_t and μ_t which makes the necessary assumption of unbiasness unfulfilled and gives us biased estimators.

To clarify this point we suggest an example based on the case of the Korean war. When consumption increases because of a change in the factors affecting the random error term. Here, the consumption would rise even if income is unchanged. However income would be almost certain to increase in the absence of unusual offsetting factors, because consumption is one of the determinants of income. The direction of casuality in this case is that income rose because consumption rose. However, the opposite would happen if we considered the statistical analyses.

In the form in which the consumption function is being estimated, the rise in consumption would be due to the corresponding increase in income. This will bias the parameter estimate of the income term upward. Such a problem would not appear if consumption and income were independent.

To solve the problem of the simultaneity bias appearing in OLS we have to find independent variables which are not correlated with (μ) such as using the identity : $Y = C + S$ where : S denotes saving.

Substituting $\dot{Y}$ into the consumption function the result then would be:

$$C_t = a + b(C_t + S_t) + dC_{t-1} + \mu_t$$

$$C_t = \frac{1}{1-b}(a + bs_t + dC_t + \mu_t)$$

Since personal savings are not a component of aggregate demand or income, the OLS condition is fulfilled, but another problem emerges in another direction. The problem is due to errors of measurement. Saving in the national income accounts are calculated as a residual between income and consumption. So, any errors of measurement that tend to increase the reported value of consumption relative to its true value will understate the amount of savings. This introduce a negative relation between consumption and savings.

In their article[18], A. Ando and F. Modigliani have explained how this bias may be severe enough to explain otherwise nonsensical results in which the estimate of MPC is less than zero. The case in which we find this method of solving the problem of simultaneity bias really is unrecommended.

Another method known as two stage least squares (TLS) is used in this context. We first calculate a regression of income on all the exogenous and lagged variables in the system, for our simple case, this would be :

$$Y_t = \alpha C_{t-1} + \beta(I_t + G_t) + \epsilon_t$$

$\hat{Y}_t$ is Y_t calculated from this regression and the values of $\hat{Y}_t$ would be asymptomatically independent of μ_t because they depend only on known numbers at time t, and thus, they satisfy the OLS assumption. Next the regression equation for consumption is calculated substituting the $\hat{Y}$ parameter estimates obtained from the equation :

$$C_t = a + b\hat{Y}_t + dC_{t-1} + \mu_t$$

will be unbiased at least in large samples.

The multicollinearity problem

The problem occurs generally when several variables in a given function have very similar trends through the sample period[14]. In such cases we can not determine which of the independent variables cause the change of the dependent variable. For the consumption function both income and lagged consumption follow smooth upward patterns. Either variable could be used to explain a large part of the movements in consumption and we can not sort out the independent contribution of each variable. If this is the case the sum of the coefficients is approximately the same when different estimates are calculated but the magnitudes of the individual coefficients for income and lagged consumption are likely to vary appreciably. Then it will be impossible to find the short run MPC and the time pattern of adjustment to a change in income. The estimate of the income coefficient may be low one time and high the next. In the book "Model Building in the human sciences" by H. Wold, L.R. Klein presents in his paper "problems in the estimation of interdependent systems".

An example of these where he obtained for the same consumption function estimated by different methods for the period 1929 – 1962 (OLS, TLS, limited information, full information) different pictures and extremely different estimates as we can see from the following results:

$$C_t = -4.93 + \underset{(0.065)}{0.559}Y_t + \underset{(0.070)}{0.445}C_{t-1}$$

$$C_t = -4.94 + \underset{(0.066)}{0.560}Y_t + \underset{(0.071)}{0.444}C_{t-1}$$

$$C_t = -5.33 + \underset{(0.069)}{0.634}Y_t + \underset{(0.074)}{0.364}C_{t-1}$$

$$C_t = -3.41 + \underset{(0.074)}{0.352}Y_t + \underset{(0.088)}{0.663}C_{t-1}$$

Where : C_t is the total purchase of consumer goods and services in billions of constant dollars.

Y_t is the disposable personal income in billions dollars

The serial correlation problem

We know that consumption describes a smooth upward trend with relativity minor fluctuations and consumption of this period and the previous period will be highly correlated. If this is the case where C_t and μ_t are correlated, it is highly probable that C_{t-1} and μ_t will also be correlated[17], this can be shown as following.

Assume that the consumption function is transformated from equation in which consumption depends on infinite distributed lag of income.

$$C_t = a \sum_{i=0}^{\alpha} \lambda^i Y_{t-i} + e_t$$

Applying Koyck transformation so the error term will be neglected.

$$\lambda C_{t-1} = a(1-\lambda)(Y_t+\lambda Y_{t-1}+\lambda^2 Y_{t-2} +.....+ \lambda^{\infty} Y_{t-\alpha})+\lambda e_{t-1}$$

$$C_t = aY_t + \lambda C_{t-1} + (e_t - \lambda e_{t-1})$$

The error term designated as μ_t is a composite error term of the form $(e_t - e_{t-1})$.

Since C_t and e_t are correlated, C_{t-1} and e_{t-1} and thus C_{t-1} and μ_t are also correlated. This again invalidates the fundamental assumption of least-squares and the resulting parameter estimates will again be biased. This problem can be solved either by the use of an instrumental variable estimator or by iterative generalised least squares. Hence, with the benefit of hindsight it appears that OLS may not have been the best method of estimation to use in our empirical work.

Another method used in our empirical part is to use ratios. Since there is virtually no trend in the consumption/income ratio, often statistical problems are also reduced.

The use of ratio will reduce the magnitude of all problems discussed above. First, most of simultaneity bias is eliminated by making the c/y ratio primarily a function of lagged variables.

A simultaneous upward or downward movement in both consumption and income will tend to keep the C/Y ratio constant rather than move both consumption and income in the same direction, which was the main cause of this bias. It is also clear that ratios remove the common trends of all the variables. The movement of each independent variable are now dissimilar enough to estimate the relative importance of each term much more clearly. The distributed lag bias is also much smallèr in ratio form because the C/Y ratio does not follow as smooth a pattern as does the level of consumption. Although present and lagged C/Y terms may still be correlated, more of this will probably be due, to economic reasons and less to statistical correlation.

Some pitfalls in measuring MPC

There have been hundreds of consumption functions empirically estimated, below we shall set some rules for all consumption functions and examine the theoretical results that have been advanced within the frame work of them.

(a) Every consumption function should include present personal disposable income and past patterns of consumer behaviour as independent variables. The latter are usually represented by lagged consumption.

(b) All variables should be measured in constant currencies. There are both economic and statistical reasons for this. Consumers make decisions in terms of their real consumption relative to their real income.

(c) The correct should be used for personal disposable income. The use of the consumption deflator as the correct deflator can be defended on two grounds. One argument is the purchasing power, personal disposable income thought to be deflated by a price index measuring the amount of goods that can be bought in physical terms. Unless consumer savings go directly into the purchase of capital goods, an unlikely occurence given existing financial institutions, then the consumer price index is to be preferred over these indices. Including elements of capital good formation, such as the overall price deflator. The second argument stems from the new classical determination of consumer behaviour in which individual consumption functions are homogeneous of degree zero in consumer prices and income. The same basic relationship is thought to hold at a macroeconomic level.

(d) Functions that claim to explain long-run consumer behaviour should show that the long-run MPC is equal to the APC. This relationship is stressed by the relative income hypothesis, the permanent income hypothesis and the life cycle hypothesis, the permanent income hypothesis and the life cycle hypothesis. It is hard to justify the claim that once an individual receives extra income he will always continue to spend a smaller percentage of this income than of all his other income. Enventually this extra income must be combined with other sources of income and treated in the same way.

The question of when "eventually" arrives is an empirical question to be determined by the lag structure of the consumption function being estimated. Failure to obtain this result may indicate either that the independent variables do not explain consumption adequately, or that the parameters are biased due to the statistical methods used and do not measure what they are supposed to measure.

Recent work on consumption function

In recent years much attention has been paid to the possible influences of the price level and rate of inflation on consumer expenditure. In the models of the previous part it was assumed that any changes in the price level could be ignored. For example suppose we added a price variable to a simple life cycle hypothesis estimating equation and obtained :

$$C_t = \alpha + \beta Y_t + \gamma W_t + \delta P_t + \epsilon_t$$

where C_t, Y_t and W_t are real per capita consumption, income and wealth respectively and P_t is an index of consumer prices which has also been used to deflate the other variables. Economic theory suggests that the price coefficient, δ, should be zero. A rise in the price level with real income and real wealth remaining constant must imply an equiproportionate rise in money income and in money wealth and hence should lead to no change in consumption expenditure. If δ was positive then this would imply that consumers were showing the phenomenon known as "Money illusion". A positive δ means that a rise in P_t, with Y_t and W_t, constant, result in a rise in consumption. Consumers must be treating the equiproportionate rise in money income and money wealth as if it were a rise in real income and real wealth and not noticing the rise in prices.

Branson and Klevorick (1969) estimated a consumption function basically simmilar to the above equation although it contains fairly complex lags on the income and price variables. Using quarterly U.S. Data for the periode 1955-1965, they found their equivalent of δ to be significantly greater than zero and concluded that a significant degree of money illusion existed in the U.S. consumption function.

Juster and Watchel (1972) have considered the effects of high reats of price change on the consumption function. They found that high inflation rates are historically associated with variable inflation rates. Hence, if consumers do not expect a similar variability in money, future real income will be subject to greater uncertainty during times of high inflation and this will lead to greater precautionary saving. Deaton (1978) also considers inflation rates but argues that it is accelerating inflation that reduces consumer expenditure. For example, suppose that past inflation has been of 5% per year, that consumers expect this inflation rate to continue but that inflation rate has in fact accelerated to 10%. A consumer purchasing specific good, will find its price higher than expected but, because his expectations are still based on the past inflation rate he will not realize that the prices of all goods have risen to the same extent. The consumption function in Deaton's model is of the Friedman type and the model is estimated using quarterly U.S.

Data for the period 1954–1974 and quarterly U.K. Data for 1955–1974. The expected inflation rate is assumed to be unchanged so that changes in the APS are made to depend on the actual rate of inflation. Deaton does find that for both countries changes in the APS are positively related to the inflation rate.

Finally Davidson, Hendry, Sabra and Yeo (1978) studied postwar U.K. quarterly data concentrating mainly on the dynamic properties and lag structure of the relationship between disposable income and non-durable consumption rather than the economic behaviour underlying it. Their final preferred model is:

$$
\begin{aligned}
C_t - C_{t-4} = {} & \underset{(0.044)}{0.47}(Y_t - Y_{t-4}) - \underset{(0.05)}{0.21}\,\Delta_1 (Y_t - Y_{t-4}) \\
& - \underset{(0.02)}{0.10}(C_{t-4} - Y_{t-4}) + \underset{(0.003)}{0.01}\, D_t \\
& - \underset{(0.07)}{0.13}(p_t - p_{t-4}) - \underset{(0.15)}{0.28}\,\Delta_1 (p_t - p_{t-4})
\end{aligned}
$$

$$R^2 = 0.77 \qquad S = 0.0061 \qquad d = 1.8$$

Where P_t is the log of the implied consumption deflator (an index of consumer prices) and all variables are in log terms. Also Δ denotes period change in variables. D_t is a dummy variable. Which is projected as +1, −1 in 1973.1 and 1973.2 respectively and zero in the other quarters.

<u>The Data</u>

Let QYR denote personal real disposable income, QCDN consumers' expenditure on non-durable goods, and QCD consumers'expenditure on all other goods and services, all variables being in constant prices. The main series used in this report are taken from Economic trends (1986 Annual supplement) and are quarterly, seasonnally unadjusted in £ million at 1980 prices. Fig. 1 shows the time series of QYR and QCDN for the period 1950.1 – 1985.4.

The salient features of the data are the strong trends in both QCDN, QYR, the magnitude and stability of the seasonal pattern in QCDN compared to the input series QYR, the regularity of the outpout series QCDN, compared to the input series QYR, and the marked change in the behaviour of the QYR series after 1972.

Detailed scrunity reveals the presence of "Business cycles" which are more clearly seen in the transformed series QCDN4 = $QCDN_t - QCDN_{t-4}$ (Fig. 2). Fig. 3 shows that, the average propensity to consume (V denotes APC) has fallen steadily over the sample period from around 0.93 to 0.8. Although this evidence is still consonant with a long-run income elasticity of expenditure close to unity.

The estimation process

Method of estimation

In the following analysis one could easely notice that ordinary least squares method is mainly being used as the chosen instrument in the estimation process. This in spite of the well-known disadvantages of such method presented in the previous sections. OLS method is biased when the error is correlated with the regressor, which is the case in simultaneous equations, then some of the effect of the error term gets attributed to the regressor. Another problem emerges from applying OLS method. That is the inconsistancy of the estimators (in the case of consumption function and as we will see later, using OLS leads to an overestimation of the marginal propensity to consume. Such oversestimation can not be corrected by using a larger sample). In fact, taking in consideration the computing facilities which are available to us (all our empirical work has been developed using micro TSP programme.) We had a choice between several estimation methods (OLS, TLS, IV) and our choice of OLS method is based on the following justifications :

While OLS yields estimators that are biased and inconsistent, it should not be totally rejected as an estimation instrument for simultaneous equations systems. In many cases OLS estimators tend to exhibit efficiency and insensitivity to specification error and in many empirical works OLS has been used in estimating several specific simultaneous equations systems.

In his book "Monte Carlo Methods", Smith (1973) states that OLS method has the advantage of simplicity and low variance and might still be utilized in preliminary work, it is also appropriate if the model is recursive or approximately recensive. More generally, OLS is appropriate if the matrix of endogenous variables contains many zeros. Smith also found that OLS tends to improve relative to the limited-information estimators as the size of the model increases.

As stated by Ray C. Fair in "spefication, estimation and analysis of macroeconometric models (Havard University press, 1984, P.243)" and according to his results presented in p. 241, all the estimates using several methods (OLS, 2SLS, FIML, 3SLS) are fairly close to each other except for FIML estimates.

Although a consistent estimate can be obtained using an instrumental vriable (IV) that is highly correlated with the regressor and uncorrelated with the error term. This in fact is not easy to apply because of the problems associated to it, especially the problem of obtaining a suitable set of instrumental variables that are uncorrelated with the error term and correlated with the explanatory variables.

As stated by M.D. Intriligator in "Econometric models, techniques and applications (North Holland Publishing company, Amsterdam – Oxford)", when there is choice of an instrumental variable, the estimates are usually very sensitive to the particular instrumental variables chosen leading to a genuine problem of choice of such variables.

Many writers agree with the fact that while many estimation methods can be applied to simultaneous equations models with lagged endogenous variables, they generally fail to satisfy all assumptions of unbiasness. If the stochastic disturbance terms exhibit serial correlation, the case when past disturbance which influence the lagged endogenous variables influence current disturbances, so the lagged endogenous variables will be correlated with current endogenous variables causing biased and inconsistent estimators.

The legitimacy of treating current dated variables as "given" has generated substantial debate in econometrics. Statistically the precise condition for treating Z_t, as weakly exergenous in the conditional model for Y_t in the form:

$$F\ (x_1\ x_2 \ldots x_n/\ ) = \prod_{t=1}^{n} F(\ Y_t/Z_t,\ x_{t-1} \ldots x_1\ \alpha_1)$$

$$\prod_{t=1}^{n} F(Z_t/x_{t-1} \ldots x_1, \alpha_2)^{**}$$

are that the parameters of interest can be obtained from α_1 alone and that no information about α_1 is lost by ignoring α_2. If so, then the model for Z_t does not require estimation and a fully efficient analysis can be conducted just from the conditional model, so the modelling exercice is both cheaper and correct. Thus, the partition of α into α_1 and α_2 requires that the two subsets of parameters are uncorrelated, it is not sufficient simply to choose not to model Z_t. (For more details see: David F. Hendry (1983), Econometric modelling : The "Consumption function" in Retrospect).

Serial correlation and the appropriate test.

Through most of the following empirical work, the test for serial correlation will be mainly based on Durbin-Watson statistic not as most appropriate test but as approximate guide. In the later stage the more appropriate lagrange multiplier test is used.

To test for first-serial correlation consider the most important type of the linear relation between successive stochastic disturbance terms:

$\mu_i = \alpha\mu_{i-1} + v_i \quad ; |\alpha| < 1$

Now, consider the null hypothesis:

$H_0: \alpha=0$

The Durbin-Watson statistic is given by the formula:

$$\hat{d} = \frac{\sum_{i=m}^{n} (\hat{\mu}_i - \hat{\mu}_{i-1})^2}{\sum_{i=1}^{n} \hat{\mu}_i^2}$$

where $\hat{\mu}_i$ is the least squares residual.

$\hat{d}$ can be approximated as :

$$\hat{d}\ 2(1- \frac{\sum_{i=m}^{n} (\hat{\mu}_i - \hat{\mu}_{i-1})^2}{\sum_{i=1}^{n} \hat{\mu}_i^2})$$

As stated by many researchers and writers Durbin-Watson can be a good approximate test for firt-serial correlation (see for more details: Thomas H. Wannacott "Introductory statistic for Business and Economics, " third edition, p.694 and M.D. Intriligator "Econometric models, techniques and applications ", p.159).

But in case of more general forms of autocorrelation :

$$U_t = \delta_1 U_{t-1} + \ldots \delta_m U_{t-m} + V_t$$

The use of d - statistic is very limited in term of well testing presence or absènce of autocorrelation since δ_1 shows just a part of the problem, and testing the significance of δ_2, $\delta_3 \ldots \delta_m$ needs a more appropriate method such as Breusch-Godfrey test known as LM test which is used through our investigation when an acceptable model is found.

We started our analysis toward finding out the Best econometric relationship between consumption expenditure and disposable income in U.K. by using annual Data series for the period 1958-1985. The simple equation implied by the absolute income hypothesis is of the form:

$$C_t = \beta_0 + \beta_1 Y_t$$

Using OLS to regress C_t on Y_t yields :

(1) $\hat{C}_t = 964.252 + 0.8871\ Y_t$

$R^2 = 0.999 \qquad d = 0.681$

where C_t is the consumer's expenditure in 1986 prices and £ millions, Y_t is the total personal disposable income in 1986 prices and £ millions. The above equation is typically the Keynesian consumption function. Where we can notice the high value of R^2,this is totally expected since both C_t, Y_t show a strong upward trend through the sample period. The low value of Durbin-Watson statisfic (d=0.681) may be considered as evidence of mis-specification in our equation.