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Sensitivity study of load-dependent Ritz vectors on modal and seismic responses of cable stayed bridges

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ABSTRACT

In the present article, 3D Finite Element Model (FEM) of a bridge structure under load dynamics is performed in order to assess the sensitivity study of Load-Dependant Ritz vectors (LDR) on modal and seismic responses of cable stayed bridges. In this context, two techniques are examined in the present study for solving structural dynamics problems; the Traditional Modal Superposition (TMS) technique and that of Load-Dependent Ritz orthogonal vectors (LDR). The latter is based on a very efficient algorithm allowing the systematic generation of Load-Dependent Ritz orthogonal vectors (LDR), the accuracy of this method is significantly influenced by the selection of LDR vectors used for the modeling of the structural behavior. The cable-stayed bridge connecting two districts in eastern Algeria, characterized by an expected Peak Ground Acceleration (PGA) equal to 0.275g in accordance with Algerian seismic design code is selected in order to perform critical modal properties such as, frequencies, shapes of the required vibration modes and effective mass participation as well as the dynamic response of the cable stayed bridge under earthquake loadings in three orthogonal directions (longitudinal, transversal and vertical). The results of this study reveal that the LDR vectors method which has the important advantages of short Central Processing Unit (CPU) time as compared to traditional modal method is very efficient for modal and seismic analyses of cable stayed bridges.

1 Introduction

The seismic analysis of cable stayed bridge structures under seismic loadings represents an important step for practical considerations in the design procedures of this type of structures. The behaviour and the performance of these structures during a major earthquake are key elements in the safety evaluation of structures situated in seismically active regions. In general, earthquake response analysis of these structure type is very time consuming. Previous studies of Traditional Modal Superposition (TMS) technique and that Load Dependant Ritz Vectors (LDRV) with advantages, e.g. [1, 2] on dynamic

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behaviour of configuration various of extended bridge structures such as, ordinary highway bridges, Multi-span and simply supported box girder bridges, railway bridges and cable bridges, have shown that in general the computational quantity did not only affected by (N × N) system of structural stiffness and mass matrices, but also by the seismic excitation duration, the nonlinearity sources and material properties of the systems in the case of nonlinear time history analysis.

The determination of the free vibration properties of extended engineering structures such as bridges, dams and pipelines represents a very important step in the understanding of the dynamic behaviour and the performance of the structures, there are a number of numerical techniques that can determine the modal properties and the corresponding vibration mode shapes of linear elastic structural systems [3]. Among these numerical techniques, the Load Dependant Ritz Vectors (LDRV) method is generally considered a very reliable procedure which has been implemented in many structural analysis computer programs and represent an efficient approach to the reduction of large three dimensional structural systems such as Soil Structure Interaction, Fluid Structure Interaction systems and offshore platforms in which Traditional Modal Superposition (TMS) methods are found to be costly due to the large numerical effort required to solve the eigenvalue problem. Another important advantage of the LDRV is the possibility to carry out dynamic analyses of medium size structures on relatively inexpensive micro-computers e.g. [1, 2].

Note that using the LDRV method only vectors which are excited by the spatial load pattern, whereas some of the exact eigenvectors may be nearly orthogonal to the spatial load pattern and therefore do not significantly participate in the response. It has been shown that the Load-Dependent Ritz orthogonal vectors (LDRV) have many potential advantages in structural dynamics over modal parameters. For linear and nonlinear dynamic analyses, the response quantities of interest can be approximated more effectively by a smaller number of Ritz vectors than the eigenvectors [1, 2, 4, 5]. The technique of LDR vectors is based on a very efficient algorithm allowing the systematic generation of load-dependent Ritz orthogonal vectors for efficient dynamic analysis of large complex bridge systems, the accuracy of this method is significantly influenced by the selection of LDRV used for the modelling of the structural behaviour.

The reference [6] used the load-dependent Ritz vectors in structural damage detection. Ziqi and Der Kiureghian [7] used in conjunction with the multiple-support response spectrum (MSRS) combination rule for analysis of two real bridges to investigate and compare the accuracy and efficiencies of the original and extended MSRS method versus the new MSRS method using the two strategies for generating the LDR vectors. The LDR vector method has been applied to time-history structural response analysis of configuration various of box girder bridge structures [2], response spectrum analysis with multi-component seismic excitation [8], wave propagation problem [9], dynamic analysis of complex soil structure systems [10], it has been implemented in widely used commercial finite element software, e.g., [11, 12]

Motivated by the above perspectives, in the present research work, 3D Finite Element Model (FEM) of an extended structure of Mila cable stayed bridge under load dynamics is assessed in order to perform the sensitivity study of Load-Dependant Ritz vectors (LDR) on modal and seismic responses of cable stayed bridges. The analytical procedure of generating Ritz vectors is then presented. In this context, two techniques are examined in the present study for solving structural dynamics problems; the Traditional Modal Superposition technique (TMS) or eigenvectors and that of Load-Dependent Ritz orthogonal vectors LDR vectors. The cable-stayed bridge connecting two districts in eastern Algeria is selected in order to perform critical modal properties such as, frequencies, shapes of the required vibration modes and effective mass participation as well as the dynamic response of the cable stayed bridge under earthquake loading in three orthogonal directions (longitudinal, transversal and vertical). The results of this study reveal that the LDR vectors method which has the important advantages of short Central Processing Unit (CPU) time as compared to traditional modal method is very efficient for modal and seismic analyses of cable stayed bridges.

1.1 Description of the bridge structure

The highway bridge structure shown in Fig. 1(a) is selected as the reference case of the sensitivity analysis of LDRV method on modal and seismic responses of cable stayed bridges. It considered of one of the first cable-stayed bridges built in Algeria, and it is of strategic importance and built in order to connect Jijel to Constantine districts in Eastern Algeria, characterized by an expected Peak Ground Acceleration (PGA) equal to 0.275g in accordance with Algerian seismic design code [13]. This bridge located in a zone of moderate seismicity, is selected as an example of cable stayed bridges in order to perform the modal and seismic analyses of cable stayed bridges using the traditional modal superposition technique and that of load dependant Ritz vectors.

The bridge has an overall length of 502m and consists of three continuous spans in Prestressed Concrete with a mid-span length of 280m and two end spans of 111m length each as indicated in Fig. 1 (b). The superstructure consists of a longitudinally Prestressed Concrete (P.C.) deck, 13.30m wide and 2.04m height, (see Fig.1 (c)).

The bridge is supported by two H shaped concrete towers (see Fig. 1(d)), double plane semi-harp-type cables and two abutments. The two intermediate towers of heights equal to 111.85m and 142.85m have identical hollow circular cross sections(see Fig. 1(e)), resting on rigid spread footings and are fixed at their upper ends. High Damping Rubber Bearings (HDRB) are only located at right abutment.

There are 88 cable members, 44 supporting the main span and 22 supporting each side span. The stays consist of parallel wires in steel of 7mm diameter and the number of wire cable ranges from 59-145.

The overall description of the Mila cable stayed bridge is shown in Figure 1, bellow.

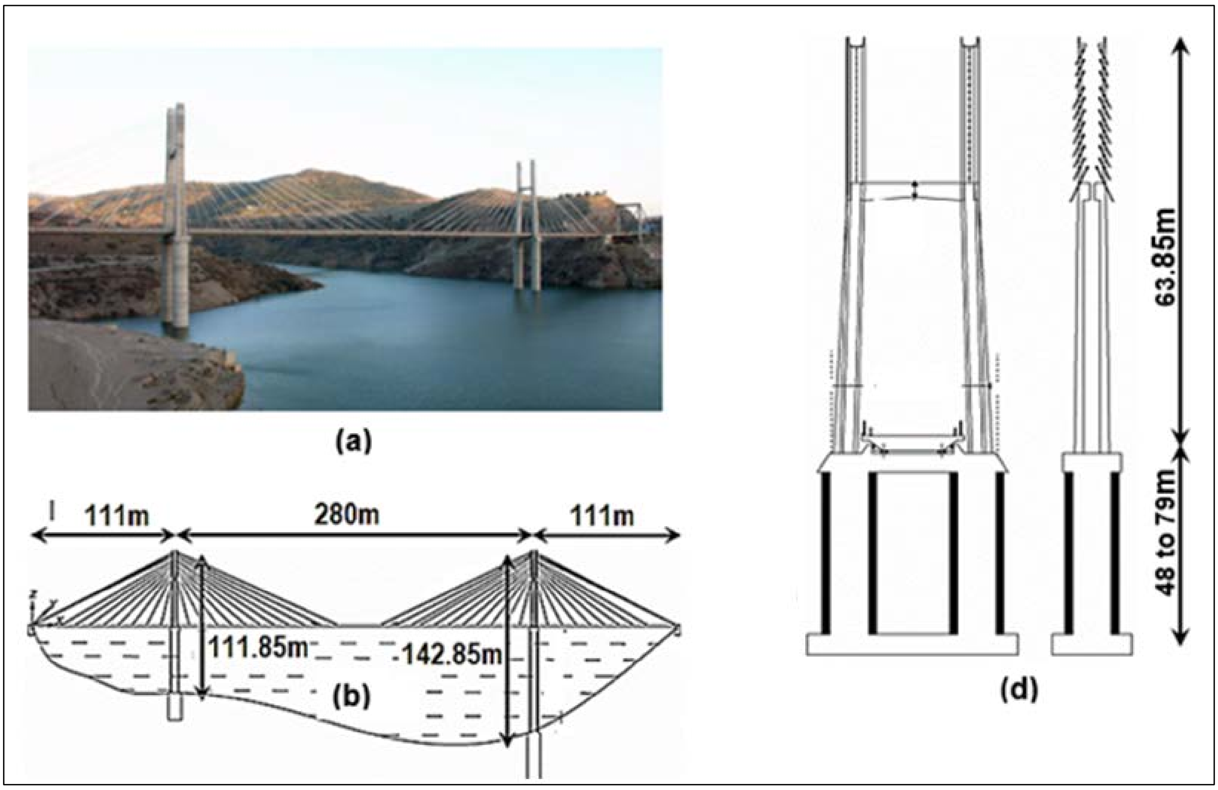

Fig. 1 –The Mila cable stayed bridge overview

1.2 Modeling of the study bridge structure

The bridge structure considered in this study is analytically modelled as a lumped mass system composed of 778 degrees of freedom and it represented by 3D-FE model as illustrated in the Figure 2. The 3-D FEM of the bridge is represented by 134 frame elements and 88 cable elements modelling stays. The non-linear behaviour of these stays due to their sags is taken into account by using an equivalent modulus of elasticity, e.g., ([1, 14]) and can be written as.

$$E_{eq} = \frac{E}{1+\frac{E\gamma^2 d^2}{12\sigma^3}} \tag{1}$$

Where E, $\boldsymbol{\gamma}$ and $\boldsymbol{\sigma}$ represent the modulus of elasticity weight density and tensile stress of stays respectively. The parameter $\boldsymbol{d}$ depicts the horizontal projection of stays.

The physical and mechanical properties of the components of the study bridge are listed in Table 1.

The entire cable stayed bridge with nonlinear model of HDR bearing at left abutment are approximated analytically by the 3-D FEM model presented in Fig. 2

Table 1 - Physical and Mechanical properties of the Mila cable-stayed bridge components.

Materials	Deck, piers and stiffening girder	Cable
Weight density, (ρ), KN/m^3	25	80
Poisson's ratio, (ν)	0.20	0.3
Modulus of elasticity (E) KN/m^2	3.9×10^7	1.95×10^8
Tensile stress, (σ), KN/m^2 in cable		1.24×10^6
Equivalent modulus of elasticity of cable , (E_{eq})KN/m^2		1.94×10^8

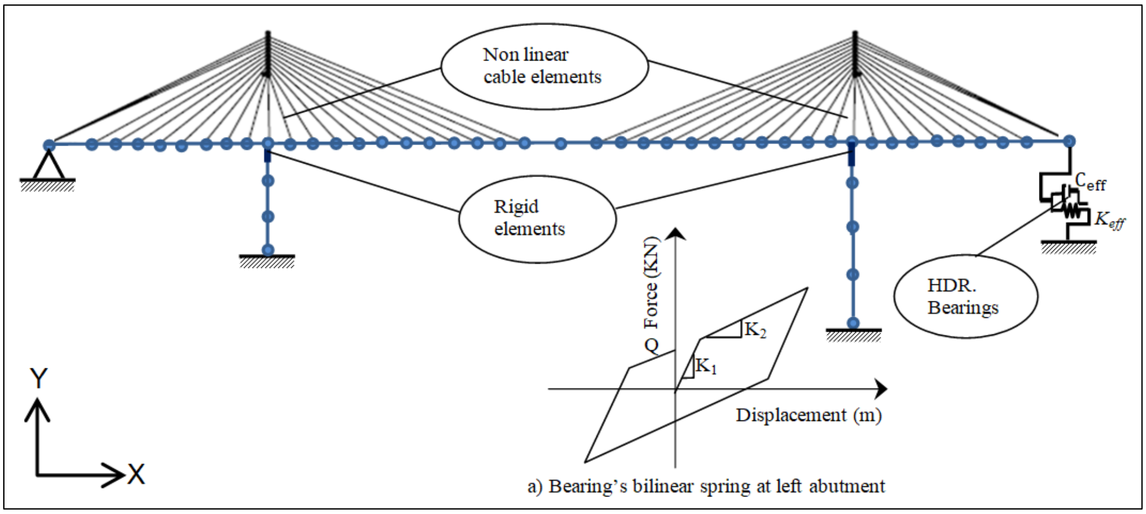

Fig. 2 –Analytical model of Mila cable-stayed bridge

The bridge superstructure consists of rigid abutments and is connected at lower columns by rigid elements. The two rectangular High Damping Rubber Bearings (HDRB) used in the present study consist of alternate layers of rubber and steel plates. Due to the presence of alternate steel plates, these bearings are very stiff in the vertical direction but flexible in the others directions. In the present study, the support provided by the abutment where located the HDR bearings is assumed to be fixed against lateral and vertical directions. It also is fixed against rotation about the longitudinal axis of the superstructure but free in longitudinal direction and about the vertical and lateral directions.

The stiffness and damping parameters of the High Damping Rubber Bearings (HDRB) are characterized by the stiffness (K_{eff}) and viscous damping (C_{eff}) in the longitudinal direction, as shown in Figure 2, and can be expressed as:

$$K_{eff} = \frac{4\pi^2 W}{g T_b^2} \tag{2}$$

$$C_{eff} = 2\xi \sqrt{\left(\frac{W}{g}\right) K_{eff}} \tag{3}$$

Where W is the weight acting on an individual bearing; g is the gravitational acceleration constant and T_b is the time period of the bearing. The rubber bearings are modelled using a bilinear hysteretic model as shown in Figure 2(a) with effective stiffness in the longitudinal direction e.g. [15].

The shear degree-of freedom used for each of the two rectangular, HDRB bearings at left abutment is modelled by a bilinear model, as depicted in Figure 2(a) and based on three parameters, namely initial stiffness K1, post-yield stiffness K2 and characteristic strength Q. The parameters values adopted in this study, in terms of effective stiffness K_{eff} and equivalent damping ratio ξ_{eff} for the pair of bearings in the longitudinal direction were defined for a design displacement of 0,15 musing procedures given in FEMA [15] and are given in Table 2.

Table 2 - Bearing device properties.

Direction	K_1 (KN/m)	K_2 (KN/m)	K_{eff} (KN/m)	Q (KN)	ξ_{eff}
Longitudinal	11922	1992	2607	125	0,16
Vertical direction	1790×10^3	-	-	-	-

The sensitivity study of Load-Dependant Ritz vectors (LDR) on modal and seismic responses of Mila cable stayed bridge were carried out using general purpose FEM computer program [11] as it is the most common and user-friendly software by practicing engineers for linear and nonlinear analyses.

2 Description of traditional modal superposition and Ritz vectors techniques

Dynamic properties have been extensively using numerical or experimental techniques. More recently, however, operational modal analysis has been utilized to extract structural dynamic characteristics from ambient vibrations [16] and forced vibrations [17]. Various numerical techniques [3] have been improved to solve eigenvalue problems resulting from the free vibration response of structures. Alternatively, Load-Dependent Ritz vectors (LDR) have also been used [1, 2, 4, 5] to carry out the analysis of the free vibration response of structures.

2.1 Traditional modal superposition

The solution of the classical eigenvalue problem of the free vibration of bridge structure system can be expressed in form given by Eq. (4).

$$([A] - \lambda[I])\phi = 0 \tag{4}$$

Where ϕ denotes the eigenvectors and λ the corresponding eigenvalues. In the Eq. (4), I and A designed the identity and dynamic matrices of bridge structure (e.g. [3]).

The eigenvectors (ϕ) can be grouped into a modal matrixΦ, where $\Phi = [\phi_1, \phi_2, \ \phi_3, \dots \dots \phi_n,]$. The orthogonality property can be expressed in the following form.

$$\Phi^T M\Phi \ = 1 \qquad \text{and} \qquad \Phi^T K\Phi \ = \Lambda \tag{5}$$

Where Λ is the spectral matrix which is a diagonal matrix formed by the eigenvalues $\boldsymbol{\lambda}$. In above equation [K] and [M] represent respectively the assembled stiffness and mass matrices of bridge system obtained by assembling the structural elementary matrices, e.g., [18].

$$[K_e] = \iiint [B]^T[D][B]dV \tag{6}$$

$$[M_e] = \iiint \rho[N]^T[N]dV \tag{7}$$

Where [B] represents the derivative matrix of shape functions and N the shape functions matrix e.g. [19]. ρ and D denote mass density and elasticity matrix respectively. The number of vibration modes to be retained in modal analysis is generally determined by using an effective modal mass equal at least to 90% or 95% of the total mass corresponding to a given direction.

The modal participation factor (Γ_n) representing a measure of the degree to which the nth mode participates in the response can be determined from, e.g., [3, 18].

$$\Gamma_n = \frac{\emptyset_n^T mr}{\emptyset_n^T m\emptyset_n} \tag{8}$$

In which $\emptyset_n^T$ is called the nth mode shape constituting the displacement vector associated with the n^{th} mode of vibration, m is the structure mass matrix and r is a displacement transformation vector that expresses the displacement of each structure degree of freedom due to static application of a unit support displacement.

2.2 Load-Dependent Ritz vectors technique

The Load-Dependent Ritzvectors (LDR) are generated by consideration of the spatial distribution of the dynamic loading F(t) which is neglected in the generation of Traditional modal vectors. The dynamic loading F(t) can be written as a product of spatial load vector f and a time function $u(t)$.

$$F(t) = f \times u(t) \tag{9}$$

Since the stiffness matrix $\boldsymbol{K}$ is positive definite, it can be decomposed into the multiplication of a lower triangular matrix L and its transposed matrix L^T (upper triangular matrix) by the Cholesky's factorization, e.g., [1, 3].

$$K = L \times L^T \tag{10}$$

L is lower triangular matrix, defined as:

$$L = \begin{bmatrix} L_{11} & 0 \ \dots & 0 \\ L_{21} & L_{22} \ \dots & 0 \\ \vdots & \vdots \quad \ddots & \vdots \\ L_{n1} & L_{n2} \ \dots & L_{nn} \end{bmatrix} \tag{11}$$

The first Ritz vector is a static deformation caused by the spatial distribution of the dynamic load vector f.

$$KX_1^* = f \tag{12}$$

Then, a M-ortho normalization is performed which provides the first Ritz vector(X_1).

$$X_1 = \frac{X_1^*}{\sqrt{(X_1^{*T} M X_1^*)}} \tag{13}$$

The following Ritz vectors are generated using the recurrence expression.

$$KX_i^* = MX_{i-1}^* \tag{14}$$

The linear independence of Ritz vectors is achieved using the Gram-Schmidt orthogonalization. That is, the current Ritz vector is mass-orthogonalized with respect to all the previous Ritz vectors:

$$X_i^{**} = X_i^* - \sum_{j=1}^{i-1} \left(X_j^T M X_i^*\right) X_j \tag{15}$$

Finally, the Ritz vector is mass-normalized:

$$X_i = \frac{X_i^{**}}{\sqrt{X_i^{**T} M X_i^{**}}} \tag{16}$$

3 Numerical results and discussion

3.1 Sensitivity analyze of Ritz vector on critical modal dynamics of study bridge

This section demonstrates the potential applicability of Load-Dependent Ritz vectors (LDR) on modal and seismic responses of studied cable stayed bridge. The results obtained from the analysis of free vibration behaviour and the seismic response along the superstructure of the studied bridge will be compared to those obtained from traditional modal superposition. The variation of the modal frequencies of the bridge corresponding to the Traditional Modal Superposition (TMS) and number of LDR vectors techniques as a function of modal order are plotted in the Figure 3.

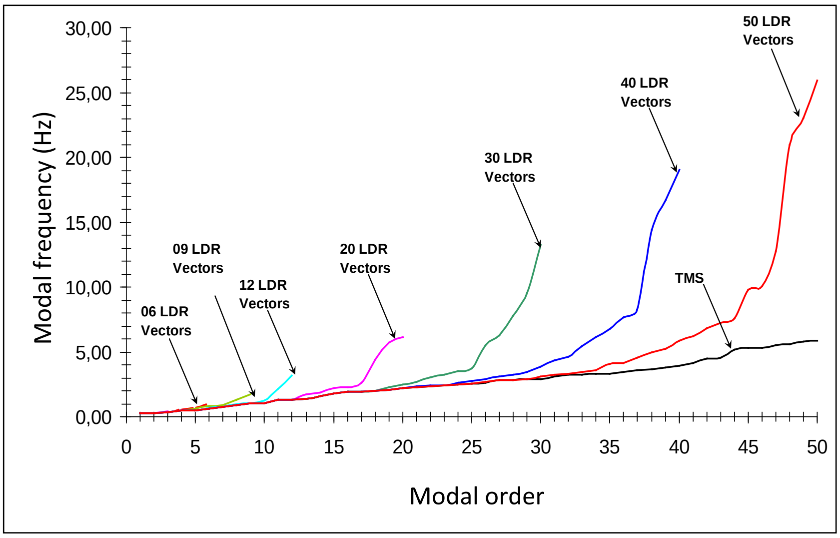

Fig. 3 –Effect of number of LDR vectors on fundamental frequencies for Study Bridge.

From this Figure, it is clearly seen that frequency curves plotted for various number of LDR vectors are generally above those plotted by the traditional modal superposition technique on the one hand, and that frequency differences between the two techniques grow slightly with the increase in the number of LDR vectors other hand.

In addition, for illustration purposes, Table 3 below, summarizes the effective modal mass values of longitudinal (X-X), lateral (Y-Y), vertical (Z-Z) and torsional vibrations of study using traditional modal superposition and (L) of LDRV methods**.**

Table 3 - Effective modal mass values (in %) calculated with TMS and LDR vectors techniques.

Number of (L) of Ritz vector	Ritz vectors				Eigenvectors			
	X-X	Y-Y	Z-Z	RX	X-X	Y-Y	Z-Z	RX
3	76	63	17	66	74	55	9	64
6	79	78	40	67	78	72	9	74
9	82	82	45	77	78	72	37	74
12	84	84	59	78	78	81	37,	77
20	87	88	82	85	79	82	41	80
50	93	93	98	91	85	86	51,	87
100					90	91	94	92

It is showed in Table 3 that the effective modal mass values for 100 eigenvectors (TMS) are 90%, 91%, 94% and 92% respectively in the X, Y, Z directions and about X axis. In comparison with Ritz vector technique, these effective modal mass values are similar when using only 50 Ritz vector. It is also observed from the table 2 that the difference arises from the comparison between the two techniques is mainly related to the Z-Z direction (vertical vibration). In fact, the effective modal mass at the fiftieth mode (50) is only about 51% in the case of the classical technique. In others directions, the effective modal mass values are quite similar between the two techniques.

For illustration purposes : 3-D modal characteristics of Lateral (L), Longitudinal (Lg.), Vertical (V), and Torsional (T) vibrations of dominant modes of both symmetrical and unsymmetrical higher modes of the bridge are identified and compared for traditional modal superposition and Load-Dependent Ritz (LDR) vectors. A 3-D graphical representation of the corresponding mode shapes are visualized in Figures 4 and 5 for traditional modal superposition with 100 eigenvectors and LDR with 50 vectors respectively.

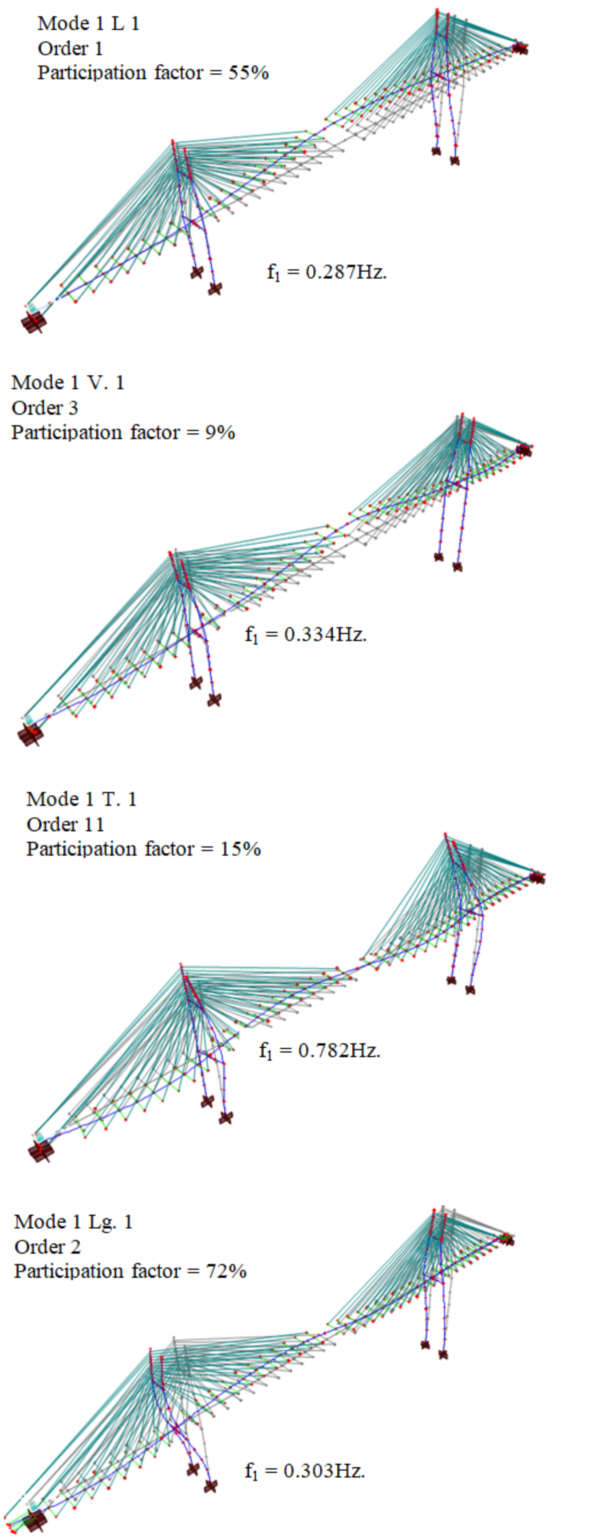

Fig. 4 –Mode Shapes of the Bridge and Corresponding Participation Factors for TMS (100 Eigenvectors)

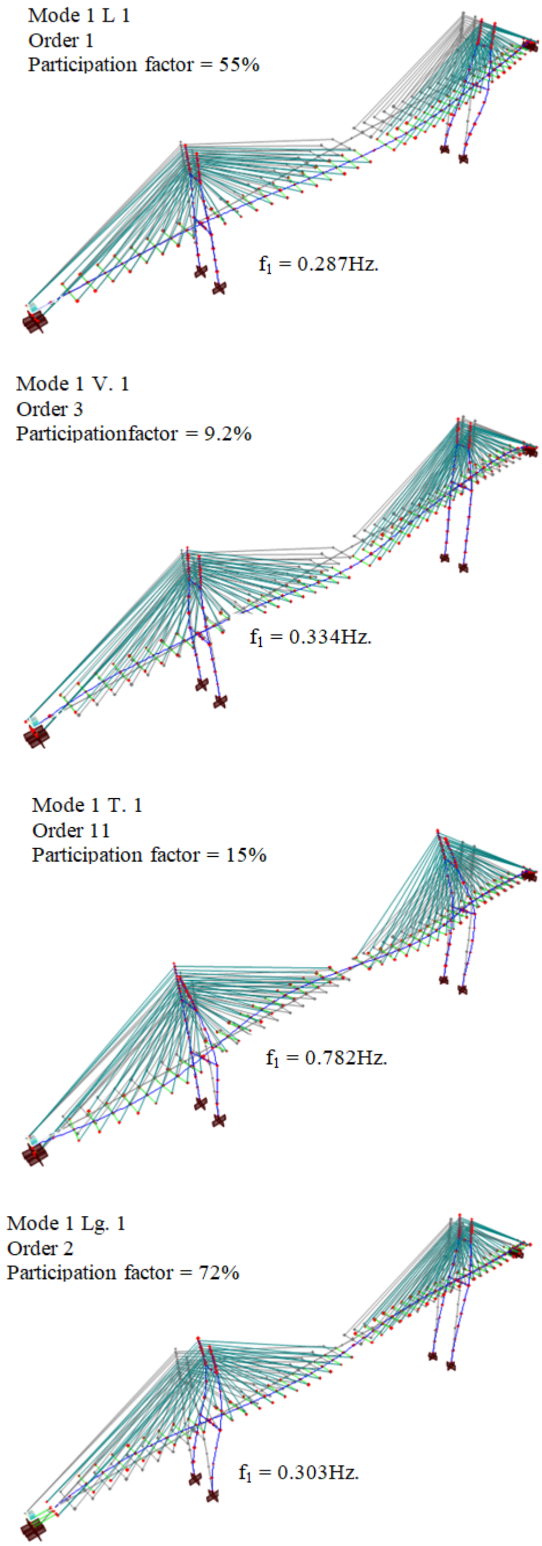

Fig. 5 –Mode Shapes of the Bridge and Corresponding Participation Factors for LDR vectors (50 Ritz vectors)

From the two Figs. 4 and 5, it is clearly seen that the dynamic characteristics (modal frequencies) and modal response parameters of the cable stayed bridge are similar for Traditional Modal Superposition (MS) and Load-Dependent Ritz vectors (LDR). The first three modes of each model are dominated by global flexural behaviour acting simultaneously in the Lateral (Z-Z), Longitudinal (X-X) and Vertical (Y-Y) directions. Higher modes have generally small participation factors and included local deformations as well as "breathing" type of behaviour.

4 Sensitivity analyze of LDR vectors on seismic responses of study bridge

For illustration purposes in order to further assess sensitivity analysis of LDR vectors on seismic responses of cable stayed bridge, the 3-D finite element model of the bridge (see Fig. 2) is now subjected to earthquake loadings in three orthogonal directions (longitudinal, transversal and vertical) in accordance with Algerian seismic design code [13] with Peak Ground Acceleration (PGA) equal to 2.27m/s^2) as shown in Figure 6. More details on simulation techniques for the generation of spectrum compatible earthquake motions can be found in reference [19].

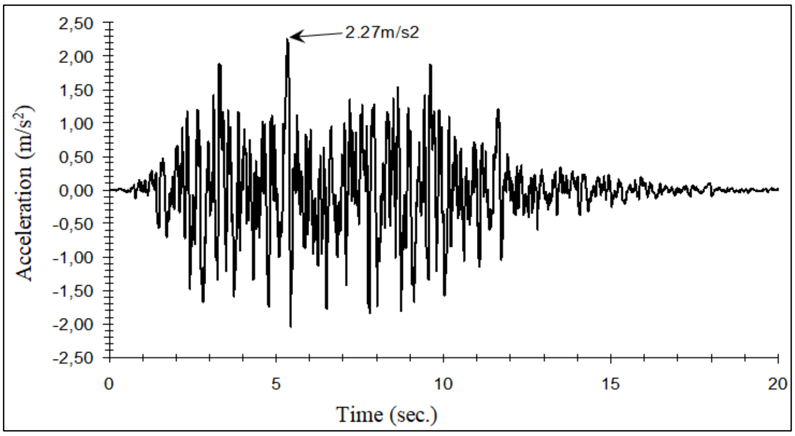

Fig. 6 –Simulated earthquake accelerogram Compatible with RPOA Spectrum

Time history bridge responses in terms of deck displacement at mid-central span, HDR bearings shear strain at left abutment and pier base shear acting at base of footing for the study bridge are investigated using both the TMS and the LDRV techniques. Bridge responses have been determined using the Fast Nonlinear Analysis (FNA) method developed by Ibrahimbegovic and Wilson [20]. This method as compared to that time integration represents an efficient computational technique for the dynamic analysis of large linear structural systems with local non-linearities. The earthquake response evaluation for many practical structures belongs to this class of problems. The technique provides a rational approach to the earthquake-resistant design of structure-foundation systems with predetermined non-linearities occurring along the structure-foundation interface.

Newmark direct time integration algorithm Chopra [5] with parameters α= 0.5 and β = 0.25 (unconditionally stable average acceleration method) as implemented in the code SAP2000, has been applied to the analytical model of Mila cable-stayed bridge under major simulated earthquake accelerogram compatible with RPOA spectrum [13].Time histories components in lateral and vertical directions are reduced by 70% in accordance with RPOA guidelines [13].

Time variation of deck displacement at mid-central span, HDR bearings shear strain at left abutment and pier base shear acting at base of short footing for the study bridge are depicted in Figs 7, 8 and 9 respectively.

It can be noticed from all above Figures that a relatively small number of low-frequency modes calculated with Ritz vectors technique is thus able to represent adequately the dynamic response of the study bridge.

Further and for the sake of clarify, Table 4 summarizes the maximum values of the absolute maximum seismic responses of cable-stayed bridge determined using the TMS and TDRV methods and comparison of CPU time between the two methods, using a laptop with the following characteristics: Intel (R) Core TM i3-5005U CPU@ 2.00 GHz.

It is seen that values of maximum seismic responses of cable-stayed bridge using (12) vectors of LDRV are in excellent agreement with those of (100) eigenvectors of TMS technique. It is also most significant that the CPU time required for maximum seismic responses of cable-stayed bridge calculated with only (12) vectors of LDRV (22 sec) was 1.41 times that needed for generating (100) eigenvectors of TMS technique, using a laptop with the following characteristics: Intel (R) Core TM i3-5005U CPU@ 2.00 GHz.

It concluded that a relatively small number of low-frequency modes with the Load Dependant Ritz Vectors (LDRV) compared to Traditional Modal Superposition (TMS) is thus able to represent adequately the non-linear dynamic response of the bridge.

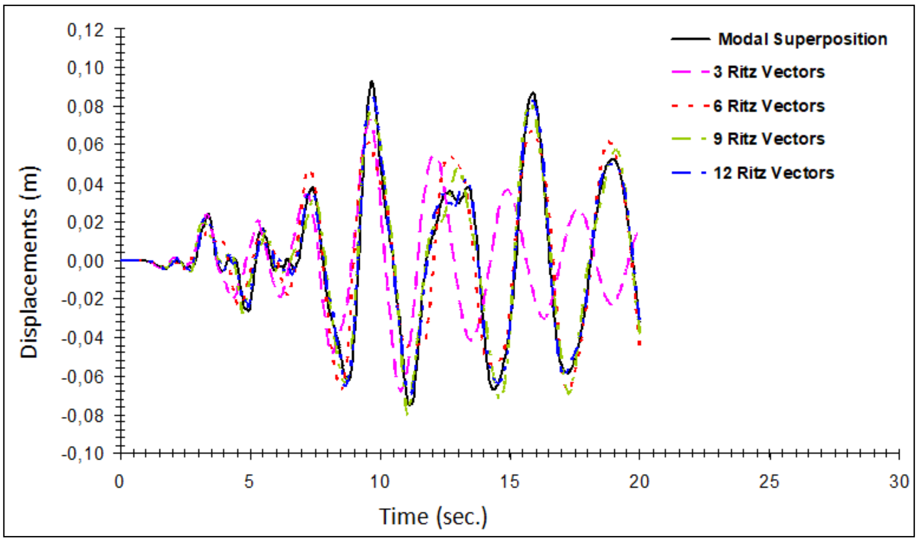

Fig.7– Time variation of deck displacement at mid-central span for Study Bridge

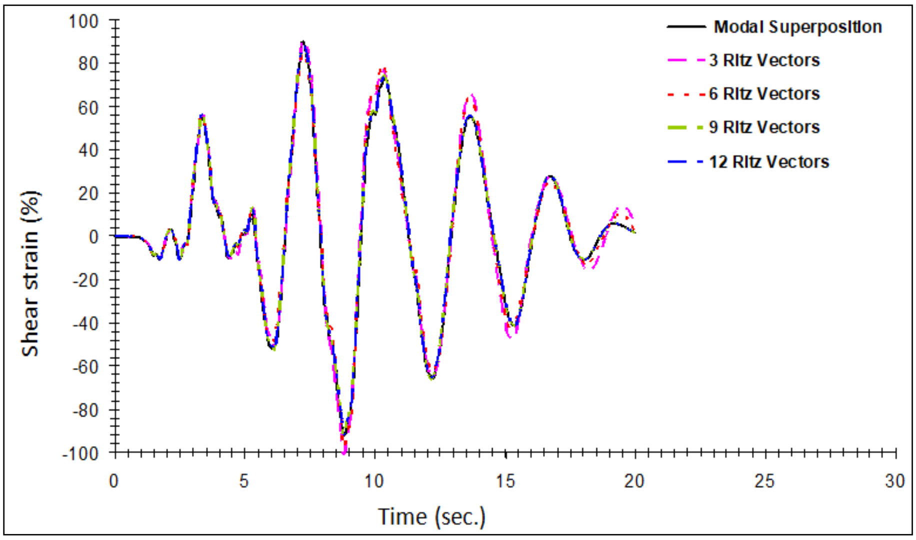

Fig.8 – Time variation of HDR bearings shear strain at left abutment for study bridge

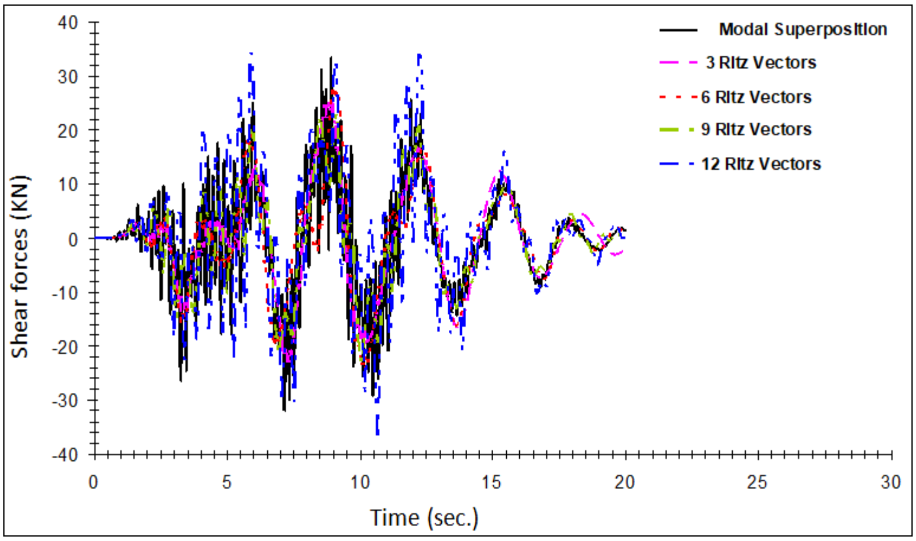

Fig.9 – Time variation of pier base shear at base of footing for Study Bridge

Table 4 - Maximum seismic responses of cable-stayed bridge calculated with TMS and LDR vectors techniques and corresponding CPU time

	Number of vectors	Pier base shear, (KN)	Deck displacement at mid-central span, (m)	Bearings shear strain at abutments, (%)	Central Processing Unit, (CPU) Time (sec.)
TMS	100	33570	0.093	92	31 sec.
LDRV	3	25291	0.073	101	21 sec.
	6	27960	0.068	98	21 sec.
	9	23012	0.081	92	21 sec.
	12	34447	0.085	92	22 sec.

5 Summary and conclusion

A sensitivity study of the traditional modal superposition technique and Load-Dependant Ritz vectors (LDR) technique on critical dynamic parameters and seismic responses of cable stayed bridges using finite element method have been assessed. The Load-Dependant Ritz vectors (LDR) technique based on a very efficient algorithm allowing the systematic generation of LDR orthogonal Vectors have been investigated. Theoretical developments and computational procedures to earthquake loading in three orthogonal directions (longitudinal, transversal and vertical) and nonlinear dynamic problems using the two methods were investigated. The cable-stayed bridge connecting two districts in eastern Algeria has been selected in order to perform critical modal properties such as, frequencies, shapes of the required vibration modes and effective mass participation as well as the dynamic response of the cable stayed bridge under simulated earthquake loading in three orthogonal directions in accordance with Algerian seismic design code with Peak Ground Acceleration (PGA) equal to 2.27m/s^2.

Based on the results obtained from the sensitive study, the following main conclusions can be drawn:

Similar 3-D dynamic characteristics and modal response parameters of the cable stayed bridge for Traditional Modal Superposition (TMS) and small number of low-frequency modes with the Load Dependant Ritz Vectors.

The significance of LDR vectors technique as compared to expensive traditional modal superposition on effective modal mass in three orthogonal directions of bridge and on the number of higher modes required to reach 95% of effective modal mass in a given direction have been assessed.

Numerical results show clearly the importance of The LDR vectors technique on critical dynamic response parameters and seismic responses of cable stayed bridges.

A relatively small number of low-frequency modes with the Load Dependant Ritz Vectors (12 LDRV) compared to Traditional Modal Superposition (TMS) with 100 eigenvectors is thus able to represent adequately the non-linear dynamic response of the bridge.

The Load Dependant Ritz Vectors (LDRV) represent a significant computational advantage short CPU time as compared to traditional modal technique if nonlinear time history analysis is to be carried out from a reduced system of dynamic equilibrium equations expressed in generalized coordinates.

The Load Dependant Ritz algorithm can potentially be extended to compute complex Ritz shapes directly for structures build on flexible foundation soil or in interaction problems such as fluid-structure systems.

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