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Buckling analysis of thick plates using refined trigonometric shear deformation theory

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ABSTRACT

In this paper, a refined trigonometric shear deformation plate theory is applied for the buckling analysis of thick isotropic square and rectangular plates. The theory involves only two unknowns, as against three in first order shear deformation theory and other higher order theories. The theory involves sinusoidal function in the in-plane displacement. The transverse displacement involves bending and shear components. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. A simply supported isotropic rectangular plate subjected to uniaxial and biaxial compression is considered for the detailed numerical study. Results of critical buckling load for simply supported isotropic rectangular plates are compared with those of other refined theories.

1 Introduction

The rectangular plates are used in engineering structures are often subjected to normal, compressive and shearing loads acting in the middle plane of plate. Under certain conditions such loads can result in plate buckling. Buckling instability of plate is a grate practical importance. In many cases the failure of plate elements may be attributed to buckling instability and not to the lack of their strength. Therefore plate buckling analysis presents an integral part of general analysis of structure. The importance of buckling is the initiation of deflection patterns which if the loads are further increase above their critical values rapidly leads to vary large lateral deflection. It leads to large bending stresses and eventually to complete failure of plate. The well-known classical plate theory due to Kirchhoff [1] predicts good results for the buckling of thin plates only, because, the transverse shear deformation is neglected. The transverse shear deformation effect is more pronounced in thick plates. The displacement based first order shear deformation theory considering the effect of transverse shear deformation is developed by Mindlin [2] which required shear correction factor. The limitations of classical plate theory and first order shear deformation theory led the development of higher order shear deformation theories. Reddy's

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theory [3] is one of the well-known higher order shear deformation theory. Shimpi and Patel [4] have developed two variable plate theory for the static and dynamic analysis of thick plate. Kim et al. [5] extended this theory for the buckling analysis of isotropic and orthotropic plates using Navier solution technique whereas Thai and Kim [6] used Levy type solution for the buckling analysis of thick plate using two variable plate theory. Ghugal and Pawar [7] applied hyperbolic shear deformation theory for the buckling and vibration analysis of orthotropic plates. Sayyad [8] and Sayyad and Ghugal [9] applied an exponential shear deformation theory for buckling analysis of thick isotropic and orthotropic plates. Recently Sayyad and Ghugal [10] is developed trigonometric shear and normal deformation theory for the uniaxial and biaxial buckling analysis of isotropic and laminated composite plates. Sayyad et al. [11] applied nth order shear deformation theory for the cylindrical bending of orthotropic plates. A trigonometric shear deformation theory used in the present study was developed by Thai and Vo [12] which extended by Shinde et al. [13] for the bending analysis of isotropic and orthotropic plates. The theory involves only two unknown variables which are three in case of first order shear deformation theory and other higher order shear deformation theories cited in the literature. The theory is variationally consistent and does not require shear correction factor. The governing equations and boundary conditions are obtained using the principle of virtual work. The analytical solution for simply supported boundary condition is obtained using Navier solution technique. The numerical results of critical buckling loads are compared with existing literature.

2 Mathematical formulation of the present theory

Consider a rectangular plate made up of isotropic material having length 'a' in x-direction, width 'b' in y-direction, and thickness 'h' in z-direction. The z-direction is assumed positive in downward direction. The plate occupies a region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le x \le h/2$ in Cartesian coordinate systems. The goal of buckling analysis of plates is to determine the critical buckling loads. Following assumptions are made in the present analysis.

1. Prior to loading, a plate is ideally flat and all the applied external loads acts strictly in the middle plane of the plate.
2. State of stress is described by the equation of linear plane elasticity. Any changes in the plate dimension are neglected prior to buckling.
3. All the loads applied to plates are dead loads, they are not change either in magnitude or in direction when the plate deform.
4. The plate bending is described by the refined trigonometric shear deformation theory.

The displacement field of the present theory can be written as:

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_b}{\partial x} - \left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial w_s}{\partial x}, \\ v(x,y,z) &= -z\frac{\partial w_b}{\partial y} - \left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial w_s}{\partial y}, \\ w(x,y) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{1}$$

where u, v and w denotes the displacement in x, y and z-directions respectively. The w_b and w_s are the bending and shear components of the transverse displacement. The trigonometric function is assigned according to the shear stress distribution through the thickness of the plate. The strains associated with the present theory are obtained using strain-displacement relationship from theory of elasticity.

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w_b}{\partial x^2} - \left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial^2 w_s}{\partial x^2}, \\ \varepsilon_y &= \frac{\partial v}{\partial y} = -z\frac{\partial^2 w_b}{\partial y^2} - \left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial^2 w_s}{\partial y^2}, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w_b}{\partial x\,\partial y} - 2\left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial^2 w_s}{\partial x\,\partial y}, \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \cos\frac{\pi z}{h}\frac{\partial w_s}{\partial x}, \quad \gamma_{xz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \cos\frac{\pi z}{h}\frac{\partial w_s}{\partial y} \end{aligned} \tag{2}$$

The strains corresponding to strains are as follows:

$$
\begin{aligned}
\sigma_x &= \frac{E}{1-\mu^2}\left(\varepsilon_x + \mu\varepsilon_y\right) = \frac{E}{1-\mu^2}\left[-z\left(\frac{\partial^2 w_b}{\partial x^2} + \mu\frac{\partial^2 w_b}{\partial y^2}\right) - z\left(\frac{\partial^2 w_s}{\partial x^2} + \mu\frac{\partial^2 w_s}{\partial y^2}\right) + \frac{h}{\pi}\sin\frac{\pi z}{h}\left(\frac{\partial^2 w_s}{\partial x^2} + \mu\frac{\partial^2 w_s}{\partial y^2}\right)\right],\\
\sigma_y &= \frac{E}{1-\mu^2}\left(\mu\varepsilon_x + \varepsilon_y\right) = \frac{E}{1-\mu^2}\left[-z\left(\mu\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right) - z\left(\mu\frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2}\right) + \frac{h}{\pi}\sin\frac{\pi z}{h}\left(\mu\frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2}\right)\right],\\
\tau_{xy} &= \frac{E}{2(1+\mu)}\gamma_{xy} = \frac{E}{(1+\mu)}\left[-z\left(\frac{\partial^2 w_b}{\partial x\,\partial y} + \frac{\partial^2 w_s}{\partial x\,\partial y}\right) + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{\partial^2 w_s}{\partial x\,\partial y}\right],\\
\tau_{xz} &= \frac{E}{2(1+\mu)}\gamma_{xz} = \frac{E}{2(1+\mu)}\cos\frac{\pi z}{h}\frac{\partial w_s}{\partial x}, \quad \tau_{yz} = \frac{E}{2(1+\mu)}\gamma_{yz} = \frac{E}{2(1+\mu)}\cos\frac{\pi z}{h}\frac{\partial w_s}{\partial y}
\end{aligned}
\tag{3}
$$

where *E, G* and μ are Young's modulus, shear modulus and Poisson's ratio respectively. The principle of virtual work is used to obtain the governing equations and boundary conditions associate with the present theory. The analytical version of principle of virtual work is:

$$
\int_{dV}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right)dV + \int_A\left(N_{xx}\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\delta v + N_{yy}\frac{\partial^2 w}{\partial y^2}\right)\delta w\; dA = 0 \tag{4}
$$

where δ be the variational operator. Integration Eq. (4) by parts and setting coefficients of δw_b and δw_s zero, the following governing equations (*Euler-Lagrange* equations) of equilibrium are obtained.

$$
\begin{aligned}
&\frac{Eh^3}{12\left(1-\mu^2\right)}\left[\frac{\partial^4 w_b}{\partial x^4} + \frac{\partial^4 w_b}{\partial x^2\partial y^2} + \frac{\partial^4 w_b}{\partial y^4}\right] + \frac{Eh^3}{12\left(1-\mu^2\right)}\left(1-\frac{24}{\pi^3}\right)\left[\frac{\partial^4 w_s}{\partial x^4} + \frac{\partial^4 w_s}{\partial x^2\partial y^2} + \frac{\partial^4 w_s}{\partial y^4}\right]\\
&+\left(N_{xx}\frac{\partial^2 w_b}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_b}{\partial x\partial y} + N_{yy}\frac{\partial^2 w_b}{\partial y^2}\right) + +\left(N_{xx}\frac{\partial^2 w_s}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_s}{\partial x\partial y} + N_{yy}\frac{\partial^2 w_s}{\partial y^2}\right) = 0
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&\frac{Eh^3}{12\left(1-\mu^2\right)}\left(1-\frac{24}{\pi^3}\right)\left[\frac{\partial^4 w_b}{\partial x^4} + 2\frac{\partial^4 w_b}{\partial x^2\partial y^2} + \frac{\partial^4 w_b}{\partial y^4}\right] + \frac{Eh^3}{12\left(1-\mu^2\right)}\left(1-\frac{48}{\pi^3}+\frac{6}{\pi^2}\right)\left[\frac{\partial^4 w_s}{\partial x^4} + 2\frac{\partial^4 w_s}{\partial x^2\partial y^2} + \frac{\partial^4 w_s}{\partial y^4}\right]\\
&-\frac{Eh}{4(1+\mu)}\left[\frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2}\right] + +\left(N_{xx}\frac{\partial^2 w_b}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_b}{\partial x\partial y} + N_{yy}\frac{\partial^2 w_b}{\partial y^2}\right) + +\left(N_{xx}\frac{\partial^2 w_s}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_s}{\partial x\partial y} + N_{yy}\frac{\partial^2 w_s}{\partial y^2}\right) = 0
\end{aligned}
\tag{6}
$$

3 Stability analysis of simply supported plates using Navier solution

Buckling analysis of simply supported rectangular plate is obtained using Navier solution technique. The plate is subjected to in-plane compressive forces which are uniformly distributed along the edges. The plate subjected to uniaxial and biaxial compression is shown in Fig. 1. All other loads acting on plate are assumed to be zero.

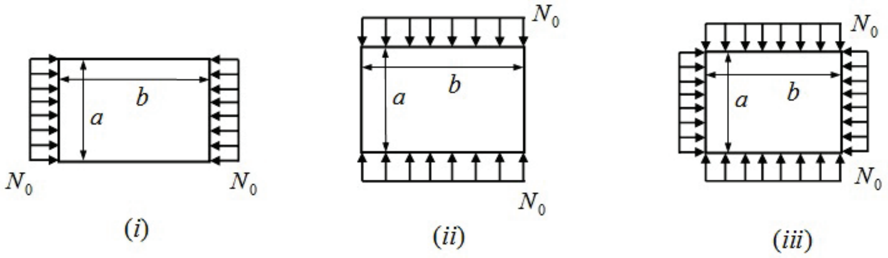

Fig. 1- A simply supported rectangular plate subjected to in-plane compressive forces: (i) uniaxial compression along x-direction (ii) uniaxial compression along y-direction (iii) biaxial compression

The displacement variables $\left(w_b, w_s\right)$ which satisfy the above boundary conditions can be expressed in the double trigonometric forms:

$$w_b\left(x,y\right) = w_{bmn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \text{ and } w_s\left(x,y\right) = w_{smn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{7}$$

In-plane compressive forces are assumed as follows,

$$N_{xx} = k_1 N_0, \quad N_{yy} = k_2 N_0 \quad \text{and} \quad N_{xy} = 0 \tag{8}$$

Substitution of equations (7) into the governing equations (5) - (6) leads to the following equation

$$\left(\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} - N_0 \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}\right)\begin{Bmatrix} w_{bmn} \\ w_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

where elements of matrix $[K_{ij}]$ and matrix $[N_{ij}]$ are as follows:

$$\begin{aligned}
K_{11} &= \frac{Eh^3}{12\left(1-\mu^2\right)}\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4}\right] \\
K_{12} = K_{21} &= \frac{Eh^3}{12\left(1-\mu^2\right)}\left(1-\frac{24}{\pi^3}\right)\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4}\right] \\
K_{22} &= \frac{Eh^3}{12\left(1-\mu^2\right)}\left(1-\frac{48}{\pi^3} + \frac{6}{\pi^2}\right)\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4}\right] \\
&\quad + \frac{Eh}{4\left(1+\mu\right)}\left[\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right] \\
N_{11} = N_{12} = N_{21} = N_{22} &= \left[k_1\frac{m^2\pi^2}{a^2} + k_2\frac{n^2\pi^2}{b^2}\right]
\end{aligned} \tag{10}$$

4 Numerical results and discussion

A simply supported isotropic square/rectangular plate is considered for the detail numerical study. The critical buckling load is obtained for uniaxial compression along x-direction, uniaxial compression along y-direction and biaxial compression. The plate has following material properties.

$$E = 210 \text{ GPa}, \ \mu = 0.3 \quad \text{and} \quad G = \frac{E}{2\left(1+\mu\right)} \tag{11}$$

The following non-dimensional form is used to present critical buckling load.

$$N_{cr} = \frac{a^2 N_0}{Eh^3} \tag{12}$$

The non-dimensional displacement and stresses obtained using present theory are compared and discussed with those obtained by the classical plate theory (CPT) of Kirchhoff [1], first order shear deformation theory (FSDT) of Mindlin [2], higher order shear deformation theory (HSDT) of Reddy [3] and trigonometric shear and normal deformation theory (TSDT) of Sayyad and Ghugal [10]. The non-dimensional critical buckling load in case of uniaxial compression along x-direction is compared in Table 1, in case of uniaxial compression along y-direction is compared in Table 2 and in case of biaxial compression is compared in Table 3. The numerical results are obtained for rectangular plates (b/a = 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0) with various aspect ratios (a/h = 5, 10, 20, 50 and 100). Variation of non-dimensional critical buckling load with respect to 'a/h' is shown in Figs. 2 and 3 whereas with respect to 'b/a' is shown in Figs. 4 through 6. When plate is subjected to uniaxial compression along x-direction, critical buckling load is decreased with increase in 'b/a' ratios whereas increased with increase in a/h ratios. When plate is subjected to uniaxial compression along y-direction, critical buckling load is increased with increase in '*b/a*' ratios and *a*/*h* ratios. It is pointed out from Table 3 that, when plate is subjected to biaxial compression, critical buckling load is exactly half of uniaxial compression for square plate (*b/a* = 1.0).

Table 1- Comparison of non-dimensional critical buckling loads for simply supported isotropic rectangular plates under uniaxial compression along *x* direction.

$(k_1\ k_2)$	*a*/*h*	Theory	*b*/*a*						
			1	1.5	2.0	2.5	3.0	3.5	4.0
(1, 0)	5	Present	2.949	1.662	1.238	1.076	0.992	0.943	0.911
		Sayyad and Ghugal [10]	3.026	1.654	1.259	1.093	1.007	0.957	0.925
		Reddy [3]	2.951	1.621	1.237	1.075	0.991	0.942	0.911
		Mindlin [2]	2.949	1.621	1.237	1.075	0.991	0.942	0.911
		Kirchhoff [1]	3.615	1.885	1.412	1.216	1.115	1.057	1.020
	10	Present	3.422	1.812	1.364	1.178	1.082	1.026	0.991
		Sayyad and Ghugal [10]	3.454	1.825	1.373	1.185	1.089	1.032	0.997
		Reddy [3]	3.422	1.812	1.364	1.177	1.081	1.026	0.990
		Mindlin [2]	3.422	1.811	1.364	1.177	1.081	1.026	0.990
		Kirchhoff [1]	3.615	1.885	1.412	1.216	1.115	1.057	1.020
	20	Present	3.565	1.867	1.400	1.206	1.107	1.049	1.013
		Sayyad and Ghugal [10]	3.582	1.874	1.405	1.211	1.111	1.053	1.016
		Reddy [3]	3.564	1.866	1.399	1.206	1.107	1.049	1.012
		Mindlin [2]	3.564	1.866	1.399	1.206	1.107	1.049	1.012
		Kirchhoff [1]	3.615	1.885	1.412	1.216	1.115	1.057	1.020
	50	Present	3.607	1.883	1.412	1.216	1.114	1.057	1.020
		Sayyad and Ghugal [10]	3.621	1.889	1.415	1.219	1.118	1.059	1.022
		Reddy [3]	3.606	1.882	1.410	1.124	1.114	1.055	1.018
		Mindlin [2]	3.607	1.882	1.410	1.214	1.114	1.056	1.019
		Kirchhoff [1]	3.615	1.885	1.412	1.216	1.115	1.057	1.020
	100	Present	3.613	1.885	1.412	1.216	1.115	1.057	1.020
		Sayyad and Ghugal [10]	3.625	1.891	1.416	1.219	1.119	1.060	1.023
		Reddy [3]	3.613	1.884	1.411	1.215	1.115	1.056	1.019
		Mindlin [2]	3.613	1.885	1.411	1.215	1.115	1.057	1.020
		Kirchhoff [1]	3.615	1.558	1.412	1.216	1.115	1.057	1.020

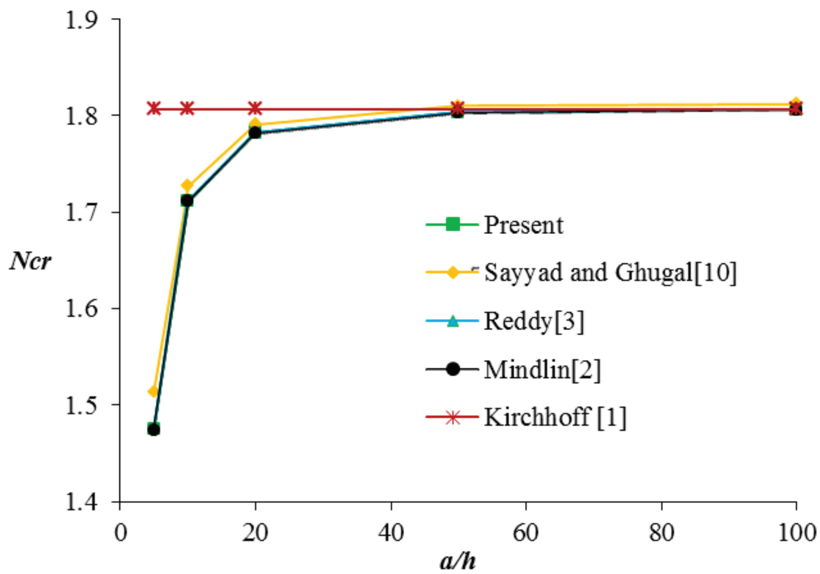

Fig. 2- Variation of non-dimensional critical buckling load with respect to aspect ratios (a/h) when square plate (b/a = 1) is subjected to uniaxial compression along x or y directions

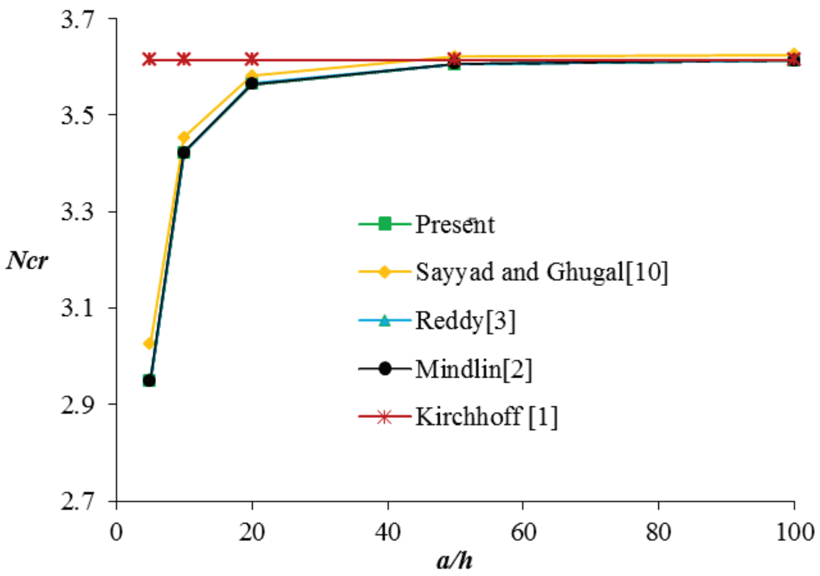

Fig. 3- Variation of non-dimensional critical buckling load with respect to aspect ratios (a/h) when square plate (b/a = 1) is subjected to biaxial compression

Table 2- Comparison of non-dimensional critical buckling loads for simply supported isotropic rectangular plates under uniaxial compression along *y* direction.

			b/a						
(k_1 k_2)	*a/h*	Theory	1	1.5	2.0	2.5	3.0	3.5	4.0
(0, 1)	5	Present	2.949	3.650	4.953	6.724	8.927	11.549	14.583
		Sayyad and Ghugal [10]	3.026	3.721	5.039	6.835	9.069	11.729	16.269
		Reddy [3]	2.951	3.649	4.951	6.722	8.925	11.546	14.580
		Mindlin [2]	2.949	3.648	4.950	6.721	8.923	11.545	14.578
		Kirchhoff [1]	3.615	4.242	5.648	7.601	10.042	12.953	16.325
	10	Present	3.422	4.077	5.457	7.361	9.738	12.570	15.851
		Sayyad and Ghugal [10]	3.454	4.108	5.495	7.410	9.802	12.652	15.953
		Reddy [3]	3.422	4.076	5.456	7.360	9.737	12.570	15.850
		Mindlin [2]	3.422	4.076	5.456	7.360	9.737	12.570	15.850
		Kirchhoff [1]	3.615	4.242	5.648	7.601	10.042	12.953	16.325
	20	Present	3.565	4.200	5.600	7.539	9.964	12.855	16.204
		Sayyad and Ghugal [10]	3.582	4.218	5.623	7.570	10.005	12.907	16.269
		Reddy [3]	3.564	4.200	5.599	7.539	9.964	12.855	16.204
		Mindlin [2]	3.564	4.200	5.599	7.539	9.964	12.855	16.204
		Kirchhoff [1]	3.615	4.242	5.648	7.601	10.042	12.953	16.325
	50	Present	3.607	4.236	5.641	7.591	10.030	12.937	16.306
		Sayyad and Ghugal [10]	3.621	4.251	5.660	7.618	10.066	12.984	16.365
		Reddy [3]	3.606	4.235	5.640	2.590	10.028	12.935	16.303
		Mindlin [2]	3.607	4.236	5.641	7.591	10.030	12.937	16.305
		Kirchhoff [1]	3.615	4.242	5.648	7.601	10.042	12.953	16.325
	100	Present	3.613	4.241	5.647	7.599	10.039	12.949	16.320
		Sayyad and Ghugal [10]	3.625	4.255	5.665	7.624	10.073	12.992	16.375
		Reddy [3]	3.613	4.240	5.646	7.595	10.037	12.944	16.312
		Mindlin [2]	3.613	4.241	5.646	7.598	10.037	12.950	16.321
		Kirchhoff [1]	3.615	4.242	5.648	7.601	10.042	12.953	16.325

Table 3- Comparison of non-dimensional critical buckling loads for simply supported isotropic rectangular plates under biaxial compression.

			b/a						
(k_1 k_2)	*a/h*	Theory	1	1.5	2.0	2.5	3.0	3.5	4.0
(1, 1)	5	Present	1.475	1.123	0.991	0.927	0.893	0.872	0.858
		Sayyad and Ghugal [10]	1.513	1.145	1.007	0.942	0.907	0.885	0.871
		Reddy [3]	1.475	1.122	0.990	0.927	0.892	0.871	0.857
		Mindlin [2]	1.474	1.122	0.990	0.927	0.892	0.871	0.857
		Kirchhoff [1]	1.807	1.305	1.129	1.048	1.004	0.977	0.960
	10	Present	1.711	1.254	1.091	1.015	0.974	0.949	0.932
		Sayyad and Ghugal [10]	1.727	1.264	1.099	1.022	0.980	0.954	0.938
		Reddy [3]	1.711	1.254	1.091	1.015	0.973	0.948	0.932
		Mindlin [2]	1.711	1.254	1.091	1.015	0.973	0.948	0.932
		Kirchhoff [1]	1.807	1.305	1.129	1.044	1.004	0.977	0.960
	20	Present	1.783	1.292	1.120	1.040	0.996	0.970	0.953
		Sayyad and Ghugal [10]	1.791	1.298	1.124	1.045	1.000	0.974	0.957
		Reddy [3]	1.782	1.292	1.119	1.039	0.996	0.970	0.953
		Mindlin [2]	1.782	1.292	1.119	1.039	0.996	0.970	0.953
		Kirchhoff [1]	1.807	1.305	1.129	1.048	1.004	0.977	0.960
	50	Present	1.804	1.303	1.128	1.047	1.003	0.976	0.959
		Sayyad and Ghugal [10]	1.810	1.308	1.132	1.050	1.006	0.979	0.962
		Reddy [3]	1.803	1.303	1.128	1.046	1.002	0.976	0.959
		Mindlin [2]	1.803	1.303	1.128	1.046	1.002	0.976	0.959
		Kirchhoff [1]	1.807	1.305	1.129	1.048	1.004	0.977	0.960
	100	Present	1.807	1.305	1.129	1.048	1..004	0.977	0.960
		Sayyad and Ghugal [10]	1.812	1.309	1.133	1.051	1.007	0.980	0.963
		Reddy [3]	1.806	1.304	1.129	1.047	1.003	0.976	0.959
		Mindlin [2]	1.806	1.305	1.129	1.048	1.003	0.977	0.960
		Kirchhoff [1]	1.807	1.305	1.129	1.048	1.004	0.977	0.960

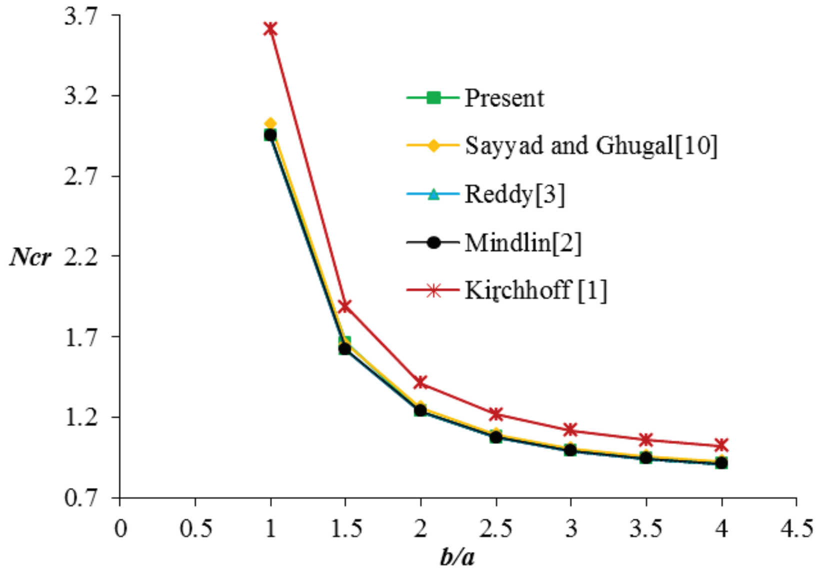

Fig. 4- Variation of non-dimensional critical buckling load with respect to b/a ratios when plate is subjected to uniaxial compression along x-direction for a/h = 5.

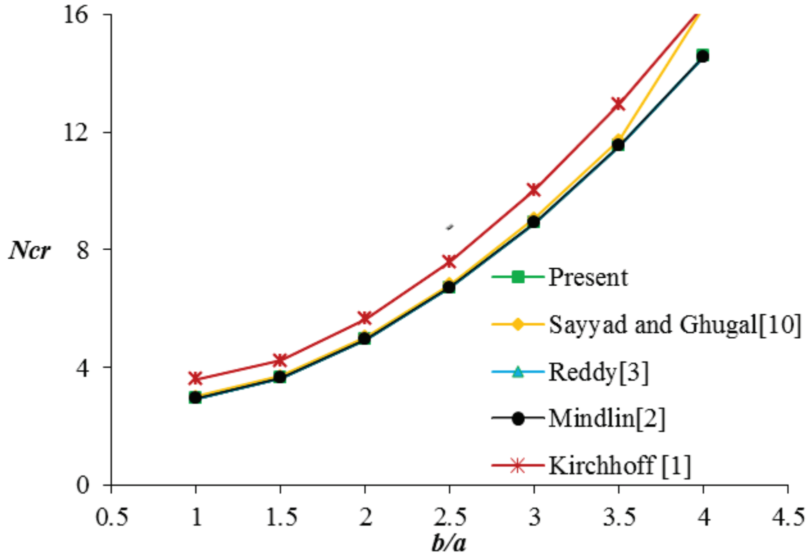

Fig. 5- Variation of non-dimensional critical buckling load with respect to b/a ratios when plate is subjected to uniaxial compression along y-direction for a/h = 5.

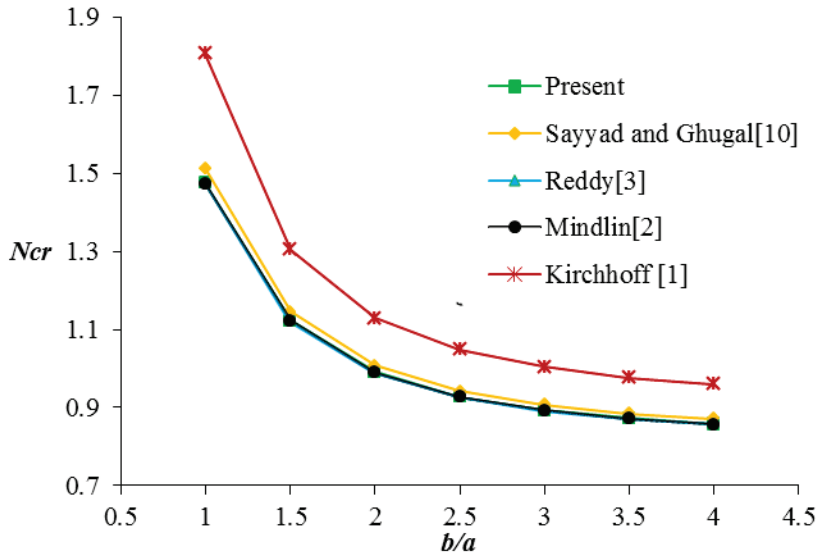

Fig. 6- Variation of non-dimensional critical buckling load with respect to b/a ratios when plate is subjected to biaxial compression for a/h = 5.

5 Conclusions

In this paper, a refined trigonometric shear deformation theory is developed for the buckling analysis of isotropic rectangular plate. The theory involves two unknowns against three in case of first order shear deformation theory. The theory satisfies the shear stress free conditions at top and bottom surfaces of the plate without using shear correction factor. The critical buckling load is obtained for simply supported isotropic rectangular plates subjected in-plane compressive forces. It is concluded from the study that, the present theory is in excellent agreement while predicting the buckling behavior of rectangular plate. The critical buckling load for square plate when plate is subjected to biaxial compression is exactly half when compared to uniaxial compression. It is also observed that the critical buckling load for square plate is same in case of uniaxial compression.

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