BREAK TESTS:
APPLICATION TO EURO-DOLLAR EXCHANGE RATES

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ABSTRACT:

This article describes the procedures of breaks test developed recently and provides an application of these tests to the euro-dollar exchange rate. In particular, in these tests, the dates and the number of breaks in the exchange rate are not assumed to be a priori known and are endogenously determined by a statistical procedure. The tests on the exchange rate are conducted over the period 01/1999-05/2015. The results reveal that three significant breaks have accorded, namely, in 07 /2002, 04/ 2007 and 02/ 2011.

Keywords:
Euro-dollar exchange rates, breaks dates, number of breaks.

I. Introduction

The exchange rate is key factors that influence economic activities. That is why foreign exchange market fluctuations have always attracted considerable attention in both the economics and statistics literature. Several models have been proposed and applied to exchange rate (see Andersen and Bollerslev, 1998a and Kasman and Tunc 2011), and others. Recently, currency debates have taken centre–stage with the euro–zone currency and sovereign debt crises, US dollar volatility, and among others.

Stochastic models for time sequenced data are generally characterized by several unknown parameters. These parameters may change over time, and if the changes, when they occur, do so

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unannounced and at unknown time points, the associated inferential problem is referred to as the change-point problem (breaks).

Traditionally, the number and dating breaks are based on expert opinions, and treated as known a priori the dates of breaks and their number in the application (test Chow 1960). The results thus depend on prior choices and potentially arbitrary.

However, the econometrics of structural change have been enriched in recent years (see Bai (1997); Hansen, 2001, Hawkins (2001); Sullivan (2002); Bai and Perron (2003)) and offers new tools to address the issue of breaks.

To illustrate these tests in unknown date, we use the data of exchange rate euro-dollar. This previous literature was dealt with statistical aspects, the following questions are of interest: Are there breaks in the euro-dollar exchange rate? In what dates did they occur? And in what numbers ?

Our aim is not to develop a particular theoretical model or find a meaning to the detected breaks, rather it is to show in particular the importance of breaks tests (Perron, 2007).

The rest of paper is organized as follows; section (II) presents the theoretical foundations of our approach from which we drive the tests and we will apply the test in section (III) on the exchange rate euro-dollar. Finally, Section (IV) concludes the paper.

II. Methods:

II.1.The model

In the implementation, the authors propose a series of break tests. Our methodology is valid under fairly general assumptions on regressors and disturbances, see e.g. Krämer, Ploberger, Alt (1988), Bai (1997) and Perron, (2007).

Consider the standard linear regression model.

$$y_t = x'_t\beta_j + u_t \quad \left(t = T_{j-1} + 1, \dots, T_j\,, j = 1 \dots, m + 1\right), \tag{1}$$

y_t is the dependent variable observed at time t; x_t is a $(q \times 1)$ vector of regression , and β_j $(j = 1 \dots, m + 1)$ is a $(q \times 1)$ vector of regression coefficients corresponding to each plan, u_t is the disruption at the time t. The dates of breaks are unknown $(T_1, \dots, T_m$, by convention $T_0 = 0, T_{m+1} = T$).

This paper focuses on testing the hypothesis that the coefficients regression is constant. The objective is to estimate the dates and detect the number of breaks from y_t observations.

II.2.The tests

Test 1: No break against a fixed number of breaks or test SupF

The supF test checks the following hypothesis

$$\begin{cases} H_0: m = 0 \\ H_a: m = k \end{cases}$$

The null hypothesis is the estimate of the model with full sample, while the alternative hypothesis is the estimated coefficients of each sub sample size $T_i = \lambda_i T$.

The fractions of the sample λ_i are such that:

$$\Lambda_\epsilon = \{(\lambda_1, \dots, \lambda_k); |\lambda_{i+1} - \lambda_i| \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \geq 1 - \epsilon\}$$

Where ϵ is a positive number close to 0.

For every partition $(T_1, \dots, T_k)$, we calculate the Fisher statistics as follows:

$$F_T^* (\lambda_1, \dots, \lambda_k;\ q) = \frac{1}{T}\left(\frac{T-(k+1)q}{kq}\right) \hat{\beta}' R' (R\hat{V}(\hat{\beta})R')^{-1} R\hat{\beta}$$

(2)

Where

$$\hat{V}(\hat{\beta}) = \text{plim } T(X'X)^{-1} X' \Omega X (X'X)^{-1}$$

(3)

Is the variance covariance matrix of $\hat{\beta}$.

The statistics of test supF is then defined by

$$\sup F_T^* (k;\ q) = \sup_{(\lambda_1, \dots, \lambda_k) \in \Lambda_\epsilon} F_T^* (\lambda_1, \dots, \lambda_k;\ q)$$

(**4**)

The distribution of the critical values of $\sup F_T^*$ dependent on the number of breaksk, the number of variants q regressors. The Tables of critical values have been obtained by simulation. We shall refer to Bai and Perron, 1998, page 58 for the table of critical values where $\epsilon = 0.05$.

Test 2: Test of breaks l against l + 1 :

In the second test procedure, Bai and Perron were interested in the relevance of the $l + 1$ th break knowing that we already considers l. They tested the null hypothesis (H_0) of **l** breaks against the alternative hypothesis (H_a) of $l + 1$ of breaks.

$$\begin{cases} H_0: m = l \\ H_a: m = l + 1 \end{cases}$$

We start from a sample in which breaks have been identified and we note $\widehat{T}_1 \ldots \widehat{T}_l$ the estimated sub-samples, so minimizing the sum of squared residuals.

So we look if a break can be detected on one of the sub-samples, thus leading to $l + 1$ break. To this end, we use the test statistic $F_T\,(l + 1/\,l)$, defined as:

$$F_T\left(l + \frac{1}{l}\right) =$$

$$\left\{S_T\left(\widehat{T}_1 \ldots \widehat{T}_l\right) - \min_{l \le i \le l+1} \inf_{\tau \epsilon \Lambda_{i,\eta}} S_T\left(\widehat{T}_1 \ldots \widehat{T}_{i-1}, \tau, \widehat{T}_i \ldots \widehat{T}_l\right)\right\} / \hat{\sigma}^2 \quad \textbf{(5)}$$

$$\text{And } \Lambda_{i,\eta} = \left\{\tau, \widehat{T}_{i-1} + \left(\widehat{T}_i - \widehat{T}_{i-1}\right)\eta \le \tau \le \widehat{T}_i - \left(\widehat{T}_i - \widehat{T}_{i-1}\right)\eta\right\}$$

where η is the parameter is which defining the fractioning term.

Bai and Perron (1998) present the critical values of this sequential test for a number of l breaks, a number of variables q and $\eta = 0.05$. The implementation of sequential tests leads ordered in following way. First, we try to identify a break. If the Fisher statistic associated with this break is superior to the critical value, we select this break and deduce from the test the date of the corresponding break. Then, the sample is divided into two sub periods, and then we test the presence of any possible additional break. If the Fisher statistic exceeds the critical value of each of the two sub-samples, it is the date that corresponds to the highest value that will represent the second breaking point. The process so repeats until no break date appears significant.

Test 3: The CUSUM test

This test is used to study the stability of an econometric model estimated over time (Ploberger and Krämer 1992). The CUSUM test is based on the cumulative sum of recursive residuals. For this purpose, we denote by $(\tilde{u}_t)$ the normalized residual compared to the standard deviation, i.e. $\tilde{u}_t = \hat{u}_t / \hat{\sigma}_u$, and we denote by k the number of parameters to be estimated in the model. The statistics S_T of CUSUM is defined by

$$S_T = (T - K)\frac{\sum_{i=k+1}^{t} \tilde{u}_i}{\sum_{i=k+1}^{t} \tilde{u}_i^2}\ ,\ t = k + 1, \ldots, T$$

(6)

If the coefficients are variable over time, then recursive residues S_T should remain in the range defined by

$$S_T \in \left[-\frac{\alpha(2t + T - 3k)}{\sqrt{T - k}}\ , + \frac{\alpha(2t + T - 3k)}{\sqrt{T - k}}\right]$$

(7)

where $\alpha = 1.143, 0.918$ or 0.850 corresponding the thresholds 1%, 5% or 10%, respectively.

II.2. Dating breaks

For m partition $T_1, \ldots, T_m$, the estimation method is based on ordinary least squares estimators, the values of B_j are obtained by minimizing the sum of squared residuals.

$$S_T(T_1, \ldots, T_m) = \sum_{i=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_i} (y_t - x'_t \beta_i)^2 \qquad \textbf{(8)}$$

Let $\hat{\beta}(\{T_j\})$ be the results of the estimate based on the m partition. From these estimators gets the estimated dates of breaks, which satisfy

$$(\hat{T}_1, \ldots, \hat{T}_m) = \operatorname{argmin}_{(T_1,\ldots,T_m)} S_T(T_1, \ldots, T_m) \qquad \textbf{(9)}$$

The minimization is performed on all the partitions $T_1, \ldots, T_m$ such that $T_i - T_{i-1} \geq h \geq q$. Finally, the regression parameters are estimated by OLS over the estimated partition $\{T_j\}$, i.e. $\hat{\beta} = \hat{\beta}(\{\hat{T}_j\})$. Bai and Perron (2003) propose an algorithm of resolution based on the principles of the dynamic programming.

II.3. Unknown number of breaks: Approach by the criterion of information

A simpler approach to determining the number of break based on the information criterion BIC (Bayesian Information Criterion). The use of this criterion was proposed by Yao (1988) and Kim (1997). The method consists in estimating the model for all the numbers of break and all possible breaks dates and selects the model for which the criterion BIC is minimal. For a given model,

$$BIC = \ln(SRR/T) + [(m+1)q + m + p] \ \ln(T)/T \qquad \textbf{(10)}$$

Here m is the number of breaks, the parameter q is affected by a break and finally p the number of parameters unaffected by a break.

III. Empirical Results

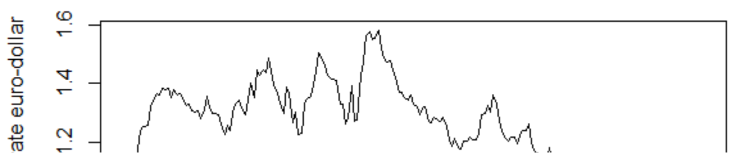
In what follows, we will focus on the exchange rate of the euro - Dollar; the series has been collected from the site of the National

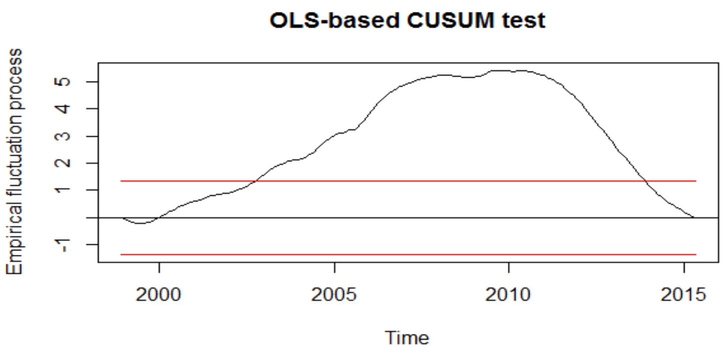
Institute for Statistics and Economic Studies (INSEE)[1]. Our period runs from January 1999 to May 2015 (197 monthly observations), and is depicted in Figure 1.

We applied the procedure tests discussed in section (II), without any x_t variable (we test if the average monthly exchange rate euro-dollar changes over time, i.e., it is not constant). So it is a pure structural change model.

We have incorporated all the above tests described in the package '' strucchange'' R statistical computing software for all the tests [2] .

Figure 1: Exchange Rate Euro-Dollar; 1999: 1-2015:5





To check for structural changes in this model we first use the CUSUM test based on OLS estimation to process (6). The Figure 2 below shows how to fit this fluctuation process and produce the plot; it gives the process together with its boundaries at an (asymptotic) 5% significance level.

Figure 2: CUSUM test processes for the Exchange rate euro-dollar

[1] : *www.bdm.insee.fr*

[2] : The package can be downloaded from *R* Archive Network Software (CRAN) in *http://cran.R-project.org/* .

The process went out of the boundaries in the period around 2002 until 2013 at appears in Figure2 and hence indicates a clear structural shift at that time.

The same conclusion emerges from tests based on F statistics, as shown in Figure 3 below. The plot of the resulting process together with the boundaries corresponding to a supF test at the 5% significance level.

From this sequence of F statistics the optimal breakpoint for a 2-segment partition can be obtained as it is equivalent to minimizing the residual sum of squares (9).

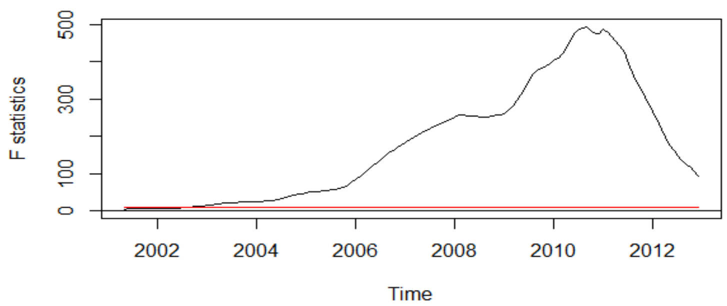


Figure 3: Sup F statistics for the Exchange rate euro-dollar

We also compare it to models with additional breakpoints. The following Figure 4 shows the computation of arbitrary m-segment models based on the S_t (RSS) triangular matrix (with the default trimming of h = 0.15)

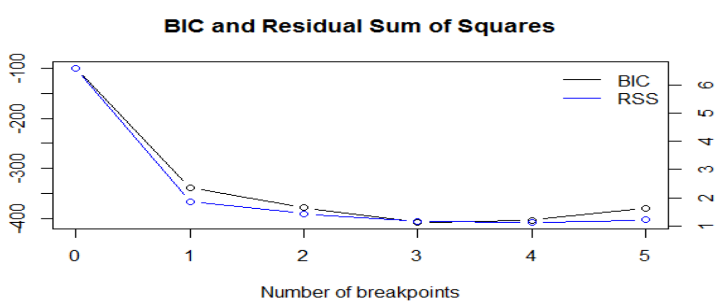


Figure 4: BIC for models with m breakpoints

A summary in Table 1 shows breakpoints for m + 1 segment models with m = 0,. . . , 5 (the maximum possible with h = 0.15) and the associated S_t (RSS) and BIC. Such criteria information is often used to model selection, which is in this case a number of selection means m breakpoints.

Table 1: breaks point (m=5) and the values of BIC

m	0	1	2	3	4	5
BIC	-99.88	-337.99	-378.12	**-407.76**	-403.79	-378.99

Bai and Perron (2003) argue that the AIC generally overestimates the number of break than the BIC which is an appropriate selection procedure in many situations.

Number of breaks	Dates				
1	09/2010				
2	07/2007	02/2011			
3	**07/2002**	**04/2007**	**02/2011**		
4	01/2002	06/2004	03/2007	02/2011	
5	11/2001	04/2004	09/2006	02/2009	07/2011

Table 2: Dating breaks associate to the minimum BIC

For data on Euro-Dollar exchange rate, Figure 4 shows that the BIC selects a model with m = 3 breakpoints (associated to the minimum of BIC), there are dates July 2002, April 2007 and February 2011.(see Table 2)

VI. Conclusion

We have shown how recent methodological advances in testing against and dating multiple structural changes. They allow for visualization and graphical analysis of empirical fluctuation processes and sequences of F statistics which often convey information about the presence and location of breaks in the data. In addition, they provide formal significance tests and a dynamic programming algorithm for computing breakpoint estimates that are global minimizes of the residual sum of squares.

The tools are applied to investigate the breakpoint at an unknown date of the exchange rate euro- dollar, three breaks are found. First at July 2002 associate to the running period of the euro, the second in time April 2007 when has a valuation of the euro and the beginning of the crisis in 2007, and the last one at time February 2011 can be the transitory effect of crash 2008.

Although our empirical results are encouraging, they should not conceal that the difficulties with BIC-based model selection for our data suggest that the problem of determining the number of breakpoints deserves further study. This is currently under investigation.

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