

Interval estimate of the threshold and the poverty rate by the methods of the simulation

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Abstract

So far, the work and research on the evaluation of the threshold and the poverty rate were satisfied with obtaining a point estimator of these parameters. This assessment is obtained always from a sample of the population whose parameters are under study. In Statistics, such an approach raises the issue of confidence which may be linked to estimators from a sample and used in place of the unknown population parameters. Statisticians rightly consider that no confidence can be given to the point estimators and propose instead using interval estimates.

A problem arises if one was to calculate confidence intervals for the parameters that are the poverty threshold and rate: the data. We need a fairly large number of samples from which we calculate the estimators for both parameters. It is only then that we can make a thorough statistical study to estimate confidence intervals for each of the two unknown parameters. Although theoretically this approach does not suffer from any criticism by statisticians, it could prove costly since it requires taking several samples. Moreover, it could be simply unenforceable because requiring probing a large number of individuals to uncover their financial situation. In some societies, it is seen to be awkward to reveal one's income. Such mentality is current in the Algerian society.

This paper proposes a new methodology that solves the problem of the data requirement. This methodology uses the theory of simulation and has the advantage of requiring taking only a single sample. From this base sample we derive the laws of probability needed for the generation of several samples by a computer simulation. Each resulting sample will be used to compute the poverty threshold and the poverty rate attached to it. The collection of these estimators will be used for statistical analysis to evaluate a confidence interval for the threshold and the poverty rate of the original population from which the sample was taken.

Keywords: poverty, poverty rate, poverty threshold, confidence interval, simulation.

1.Introduction

The economic literature is replete with research papers on the topic of measuring poverty. We can mention without being exhaustive the works of Foster, Greer and Thorbecke [10], Ravallion [19], Sen [21, 22], Gordon and Townsend [24]. In Algeria this topic has been treated by Labes [15] and several master theses [3] [4].

The assessment of poverty requires the resolution of several issues including the definition of poverty. It is generally accepted that an individual is considered to be poor if his/her income does not allow him/her the satisfaction of basic needs. Already, This definition raises two questions:

- What is an income?
- What is a basic need?

The income considered is the annual cash income net of tax and social security. It include earned income such as salaries or fees of the liberal professions, property income (income from apartment rentals etc..) and income as social pensions. Income covers therefore tangible returns easily translated into monetary terms. Intangible income received in kind, such as the fact of living in free housing offered by the employer are often not taken into account. However, some researchers do not stop at the mere notion of monetary income and consider any income which could improve the lives of those who benefit; be it tangible or intangible. In this case, we will talk of an expanded income [19]. From this perspective, the benefits such as free rent housing are evaluated in monetary terms.

The definition of poverty as advanced in the preceding paragraph considers individuals. However, the income, is often given or evaluated for the whole household. If young children generally do not receive income, they still remain poor if the parents are actually poor as we cannot consider the children as poor if their parents are wealthy. Hence the adoption of a simple principle: if a household is poor, all of its individuals are poor and are at the same level [11]. Now, since the expenditure of a household is not sustained in the same way by all its members, it is necessary to introduce an equivalence scale. Thus the Oxford scale [11] considers that the head of the household counts as one unit of consumption, the spouse if there is one accounts for 0.7 units and children account for 0.5 units. The INSEE, the French institute for statistics and economic studies has adopted a different scale [11].

Once a consensus is reached on the definition of the income to be considered, we have yet to find a critical value of the income below which we can say that the most basic needs are not met and therefore the individual who receives such an income is to be considered as poor. In other words, we are posing the problem of assessing the poverty threshold. There are three types of poverty thresholds: absolute, relative or subjective [12,13].

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A poverty threshold is called absolute if its determination involves the assessment of the costs of basic needs. The cost of food expenditure needed for survival in a healthy condition is estimated. To this estimator, we add the cost of non-food expenditures that are considered necessary for a decent life. Non-food expenditures include the cost of housing, health, education, transport etc... It goes without saying that determining the absolute threshold of poverty can be quite difficult. In addition, an absolute threshold of poverty for each country makes comparison between different countries impossible. For these reasons, other types of thresholds were developed. Note that the concept of absolute poverty line was developed by the U.S. [18] and is mainly used by them alone.

A variant of the absolute poverty threshold was developed by the United Nations Program for Development in order to facilitate comparisons between different countries. A person is considered poor if his/her daily income is less than 1.25 dollars expressed in purchasing power parity.

Often, the poverty threshold is defined as a percentage of a parameter of the concentration of the income distribution. The concentration parameter most used is the median income and the percentage most often used is 50%, although Europe intend to use 60% of the median as the poverty threshold [5].

A threshold is called subjective if its determination depends on the answers made by a sample of the population [17].

To estimate the poverty rate in a given population we take a representative sample of this population and we compute the percentage of people whose income is below the poverty threshold.

Once, the poverty rate measured, a question arises: given that this rate is a statistic derived from a sample to estimate the actual population rate of poverty, what is our confidence in this point estimator? Statistical theory answers this question: confidence is null as a statistic may take any value in its domain of variation depending on our choice of individuals to be entered in the sample. For our confidence to grow, we should take as an estimator, not a single value of the poverty rate but a range of values for this variable.

In order to assess a range that ensures a certain level of confidence, we must study the law of distribution of the random variable rate of poverty. This study requires to have at our disposal a number of realizations of this variable. However, the evaluation of only one value of the poverty rate demand an entire sample of observations of the individual incomes. In the case of poverty measurement, it is very expensive not to say impossible to obtain several samples of information as critical as income. This is true for any society but it is even more true in our Algerian society. Hence the question to know whether there is another method to obtain more evaluations of the poverty rate without resorting to a multitude of samples.

To this question the simulation theory responds positively. Thanks to the simulation, we can continue our study of the poverty rate using only one sample of observations.

This paper develops a methodology based on simulation and statistical analysis to evaluate a confidence interval for the threshold and the poverty rate. The methodology is based on the derivation of the laws of probability distribution of the following variables:

- Income,
- The number household individuals,
- And the presence or absence of a spouse.

This derivation is made from a sample of observations of the income. Once defined the laws of distribution, we generate by simulation a large number of samples in accordance with these laws. Each of these samples is used to evaluate a threshold and a poverty rate. A statistical study can finally be realized on the collection of statistics that are the threshold and the rate. From the study we can estimate confidence intervals for these statistics.

2. Simulation theory

This section lays the theoretical foundations needed to write computer programs to perform the simulation. First, we must calculate the probabilities that underlie the generation of the observations. By observation we mean a ordained triplet: the total income of the household; the number of people in the household and last the marital status of the household head . For this, we group the observations of the sample by class of income and the number of individuals in the household into two separate tables. The first table holds the data relative to the households whose heads are married, the second one for the unmarried ones. For each table, the rows represent the income classes while the columns represent the number of people dependent on these heads of families. Table 1 gives an example of such a classification.

We will use the following notation:

i: Index of an income class.

m: Number of income classes.

n: Number of columns or maximum number of dependent people.

j: index of a column representing a number of dependents.

O_{ij} : number of heads of households whose income is included in class **i** and their number of dependents is equal to **j**.

$O_{i.}$: Total number of heads of households whose income is included in class **i**. It is obtained by summing over all columns **j** the O_{ij} 's for the **i**th class.

$$O_{i.} = \sum_{j=0}^n O_{ij} \quad \text{for } i=1,2,\dots,m \quad (\text{eq. 1})$$

$O_{.j}$: Total number of heads of families with dependents **j**. It is obtained by summing over all rows **i** the O_{ij} 's of a column **j**.

$$O_{.j} = \sum_{i=1}^m O_{ij} \quad \text{for } j=0,1,2,\dots,n \quad (\text{eq. 2})$$

$O_{..}$. Grand total can be calculated as the sum of all $O_{.j}$ or the sum of all $O_{i.}$.

$$O_{..} = \sum_{i=1}^m O_{i.} = \sum_{j=0}^n O_{.j} \quad (\text{eq. 3})$$

For the rest of our development, we shall note with a prime the variables relative to the table 1 concerning the married household heads while the variables of unmarried ones shall be noted normally. Thus we will note O_{ij} , $O_{i.}$, $O_{.j}$ and $O_{..}$ to refer to the table of unmarried heads. We note O'_{ij} , $O'_{i.}$, $O'_{.j}$ and $O'_{..}$ to refer to the same variables in the table of married heads.

Once these two tables achieved, the probability theory necessary for the conduct of the simulation can be developed.

2.1 Calculating the probability of the income class

For a given class i , we have $O_{i.}$ Heads of families who are unmarried and $O'_{i.}$ who are married. The total number of heads of families in the sample is equal to:

$$N = O_{..} + O'_{..} \quad (\text{eq. 4})$$

p^c_i the marginal probability of class i object of our attention is thus equal to:

$$p^c_i = P[\text{class } i] = (O_{i.} + O'_{i.})/N = (O_{i.} + O'_{i.})/(O_{..} + O'_{..}) \quad i=1,2,\dots,m \quad (\text{eq. 5})$$

2.2 Calculation of the probability of the marital status

The marginal probability of the marital status knowing the class i of income can be calculated as follows:

$$p^s_i = P[\text{married}|\text{class } i] = O'_{i.}/(O'_{i.} + O_{i.}) \quad i=1,2,\dots,m \quad (\text{eq. 6})$$

$$p^s_i = P[\text{non married}|\text{class } i] = O_{i.}/(O'_{i.} + O_{i.}) \quad i=1,2,\dots,m \quad (\text{eq. 7})$$

Equation 7 above can also be deduced as:

$$p^s_i = P[\text{non married}|\text{class } i] = 1 - P[\text{married}|\text{class } i] = 1 - p^s_i$$

2.3 Calculation of the probability of the number of dependents

for the unmarried heads of household, p^e_{ij} is the probability that the number of dependents is equal to j given that the income of the household lies in class i . Similarly, let p^e_{ij} be the probability of the same event when the marital status of household head is married.

$$p^e_{ij} = O_{ij}/O_{i.} \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \quad (\text{eq. 8})$$

$$p^e_{ij} = O'_{ij}/O'_{i.} \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \quad (\text{eq. 9})$$

2.4 Methods for generating different probabilities

Having determined how to calculate the probabilities p^c_i , p^s_i , p^s_i , p^e_{ij} and p^e_{ij} , we will see now how we can generate random variables that follow these laws of distribution. Suppose, we have chosen a person to enter the sample to be used to estimate the threshold and the poverty rate. For this, we need to know the income, the marital status and number of dependents of that person. We begin with the generation of income.

2.5 Income generation

Income generation will be done in two steps. First we generate the income class in which the income will fall, then in a second phase we generate the income itself between the lower and upper bounds of the generated class.

2.5.1 Determination of the class

To determine the simulated income class, we use the tabular method of generating a random number [9]. First we calculate the cumulative probabilities for each class of income. Let P^c_i be the cumulative probability of class i :

$$P^c_i = \sum_{j=1}^i p^c_j \quad i=1,2,\dots,m \quad \text{where } p^c_j \text{ is as given by equation 5.} \quad (\text{eq. 10})$$

Table 2 shows how the required computations are to be performed.

Let X be the random number given by the random number generator. X is uniformly distributed on the interval $[0,1]$. The tabular method considers that the value of X is actually the value of the cumulative probability of the class of income. So, the class i^* selected by the random number generator is the first class that satisfies the following condition:

$$P^{c_{i^*-1}} < X \leq P^{c_{i^*}} \quad (\text{eq. 11})$$

Example: Let's suppose the value of X is 0.657. i^* is the index of the first class of income whose cumulative probability is greater than or equal to the value of X .

2.5.2 Income Determination

Once the income class known, we will have to assess more accurately the income of the household currently under study. We assume that the incomes in any class are uniformly distributed over the interval of that class. Since there are m income classes, each class interval is equal to:

$$\text{Interval} = (\text{maximum income} - \text{minimum income})/m \quad (\text{eq. 12})$$

Let a_i be the lower limit of class i and b_i its upper limit.

$$a_i = \text{revenu minimum} + (i-1) * \text{interval} \quad (\text{eq. 13})$$

$$b_i = a_i + \text{interval} \quad (\text{eq. 14})$$

A random variable X follows the uniform distribution on the interval $[a_i, b_i]$ which we note $X \sim U(a_i, b_i)$ if its probability density function (PDF) is:

$$P(X = R) = 1/(b_i - a_i) \quad \text{if } a_i \leq R \leq b_i; 0 \text{ otherwise} \quad (\text{eq. 15})$$

$$\text{And } P(X \leq R) = \int_{a_i}^R dx / (b_i - a_i) = (R - a_i) / (b_i - a_i) \quad (\text{eq. 16})$$

The uniform law is one of the few laws of probability that can be easily simulated by inversion [9]. So, we will use this method to generate the income. The inversion method involves generating a random number and to equalize the number to the right side of the equation 16. Let X be a random number generated by the computer. Solving the following equation gives the value of R

$$\mathbf{X}=(R-\mathbf{a}_i)/(\mathbf{b}_i-\mathbf{a}_i) \quad (\text{eq. 17})$$

That is:

$$R=\mathbf{a}_i+\mathbf{X}^*(\mathbf{b}_i-\mathbf{a}_i) \quad (\text{eq. 18})$$

R is the desired income. Equation 18 gives the formula that translates into income the computer generated random number following the uniform distribution on the interval $[\mathbf{a}_i, \mathbf{b}_i]$.

Let's review how the income generation has been achieved according to the law of distribution as determined from the sample. We proceed in two steps:

1. First we determine randomly a class of income. For this we generate a random number \mathbf{X} which will be used to decide the class in which the income will fall. The class chosen is that which satisfies equation 11. Let i^* be the index of that particular class.
2. Once the class is known, we compute its lower and upper limits \mathbf{a}_{i^*} and \mathbf{b}_{i^*} according to equations 13 and 14. A second random number \mathbf{X} is obtained and the desired income R is estimated using the equation 18.

Next, we need to evaluate the marital status of the household head.

2.6 The determination of the marital status.

In probability theory, a random variable that can take a value among two values is said to follow Bernoulli's law of probability. Usually one of the two values is called a success and the other one is considered a failure. The probability of success is denoted p, that of the failure is denoted q with $q = 1-p$.

The most appropriate method to generate random numbers that follow the Bernoulli distribution with p as the probability of success is the so-called rejection method [9]. A random number \mathbf{X} is generated by the computer. If \mathbf{X} is less than or equal to the probability p, we recognize that we are dealing with a success. Otherwise we are dealing with a failure.

In our case, if we consider the fact of being married as a success, the probability of success p is simply \mathbf{p}_i^s as calculated by equation 6. Hence, if the random number \mathbf{X} given by the computer is less than or equal to \mathbf{p}_i^s the marital status obtained is "married", otherwise the status is "unmarried".

Finally, we need to see how to generate the number of people in the household.

2.7 Generation of the number of dependents

Equations 8 and 9 show us how to derive the probability associated with the number of dependent people knowing the class of income and the marital status of the head of a household. Again, We use the tabular method that we used to determine the income class.

Knowing the income class i^* and the marital status that have been selected by the computer, we can calculate the cumulative probabilities associated with the number of dependents of the household. Table 3 shows how to compute these cumulative probabilities for the married household heads and table 4 does the same thing for the unmarried household heads.

Having calculated the cumulative probabilities associated with the number of dependents, generating a number of such individuals comes down to asking the computer to obtain a random number \mathbf{X} and finding the value of \mathbf{j}^* which satisfies the following condition:

$$\mathbf{P}_{i^*(j^*-1)}^e < \mathbf{X} \leq \mathbf{P}_{i^*j^*}^e \quad \text{If the marital status is « married »} \quad (\text{eq. 19})$$

$$\mathbf{P}_{i^*(j^*-1)}^e < \mathbf{X} \leq \mathbf{P}_{i^*j^*}^e \quad \text{If the marital status is « unmarried »} \quad (\text{eq. 20})$$

Armed with these theoretical definitions, we can proceed to one or more simulations. It is the possibility of proceeding to more than one simulation that interests us in particular. For each simulation we obtain an estimate of the poverty threshold and an estimate of the poverty rate associated to this threshold. Allowing us to make a statistical analysis of the probability distribution of these variables. If a distribution law can be determined for each of the two variables, we will be able to establish a confidence interval for each with a desired level of confidence. With the use of simulation, we need no longer take several samples of observation: one sample will do.

3 Application

To start the simulation, we obtained a sample of 5374 observations. These observations were collected in a very large institutions that operate in Algeria and that employs more than 11000 agents. They are intent for example purpose only. Each observation is a triplet consisting of the marital status of the household head, the number of dependents and the household income. Table 5 shows a sample of the original data.

Having obtained the original data, we start the process by classifying the observations according to the marital status. Once the data divided into 2 categories, we need to group them by income class and marital status in order to achieve the tables for the \mathbf{O}_{ij} and the \mathbf{O}'_{ij} as defined by equations 1, 2 and 3. We chose to take a number of classes equal to 25 which ensures that the interclass interval is approximately 2000.00.

Tables 6 and 7 are the basis of the probabilities evaluation. First we calculate the probabilities of the income classes \mathbf{p}_i^c according to equation 5. In table 8 are given the probabilities of the different income classes.

Using equations 6 and 7, we evaluate the estimators of the marginal probabilities of the marital status. This is done in table 9.

The final step in the derivation of the probability estimates is to evaluate the marginal probability estimates of the number of dependent people knowing the class of income and the marital status of the household head in conformity with to equations 8 and 9. Table 10 gives the probability distribution of the number of dependents per household whose heads are unmarried, while table 11 gives the same for married household heads

Using the different estimators of the probabilities presented in Tables 8 to 11, we can generate a sample of virtual observations in accordance with the laws of probability distribution that underlie the original sample of 5374 observations insight of which is given in Table 5. We chose to include in this computer-generated sample 5000 observations each consisting of the triplet (marital status, number of dependents, income).

for the sake of the application, we use the Oxford scale to assess the consumption units. The household head counts as one unit, the spouse if any accounts for 0.7 units. Each of the other dependents in the household account for 0.5 units of

consumption. The poverty line is set at 60% of the median income. It should be noted that the simulation does not impose any assumption related to the consumption units or the definition of the poverty line. Any other hypothesis can be considered.

4 Simulation algorithm

1. For each household head in the simulation (there are 5000)
 - Generate income class;
 - Generate income in this class;
 - Generate marital status;
 - Generate the number of dependents;
 - Calculate the number of consumer units according to the Oxford scale;
 - Calculate the income per consumption unit.
2. After completing the generation process above for all the individuals, calculate the median income per consumption unit. The poverty line is equal to 60% of the median.
3. For each pair (consumption units, income), compare the income poverty line calculated in step 2 above. If the income per consumption unit is below the poverty line, add the number of consumption units to the total of poor people.
4. The poverty rate is the ratio of the number of poor people to the total number of consumption units in the sample.

Table 12 gives a sample of the simulated observations. Now that we are able to do a simulation, it is easy to assess the threshold and the poverty rate associated with the values generated by the simulation. However, there is no reason to perform only a single simulation. In fact, we can do more in order to generate multiple values for the threshold and the poverty rate. This generation allows us to do a statistical analysis of these two variables. Do the threshold and the poverty rate follow a particular distribution law? With this in mind we conducted 500 simulations, each requiring the realization of 5000 observations. This collection of simulations resulted in a sample of 500 poverty thresholds and poverty rates insight of which is given in the table 13, while table 14 summarizes some of the statistics relative to the threshold and the poverty rate.

We use the test of the goodness of fit which is based on the chi-squared distribution. If we denote by f_i the i th observed frequency of a random variable X , and by e_i the corresponding expected value, then the new variable X_c^2 (chi-square calculated) = $(f_i - e_i)^2 / e_i$ follows the chi square distribution. e_i represent the expected values if the variable X under study (the threshold or the poverty rate) is following a definite law. We start from the null hypothesis that the variable X under consideration follows the normal distribution and we verify if indeed the sample data support or refute this hypothesis. The value of the χ_c^2 is compared to χ_t^2 whose value is the theoretical value of a random variable following the chi square distribution. All calculations are given in tables 15 and 16. We suggest to the reader reference [16].

Statistical analysis on the poverty threshold:

Number of observations	:	500
Average observation	:	5125.401753
Variance of the observations	:	585.2856749
Variance estimator	:	586.4585921
Grouping interval width	:	6.715185547
Estimator of the standard deviation	:	24.21690715

The chi-squared value calculated in Table 15 is 26.6540752 and the tabulated chi-squared for 14 degrees of freedom is 26.873 [16] for a 2% risk, thus, we cannot reject the hypothesis of normality of the threshold distribution. Following this, the distribution of the threshold can be completely described by the normal distribution with an expected value equal to 5125.401753 and a variance of 586.4585921.

Statistical analysis of the poverty rate:

Number of observations	:	500
Average observations	:	11.5690
Variance of the observations	:	0.1839
Variance estimator	:	0.1843
Grouping interval width	:	0.1295
Estimator of the standard deviation	:	0.4292

The Chi-squared value calculated in Table 16 being 8.8316, and the tabulated chi-squared for 13 degrees of freedom being 22.362 [16] for a risk of 5%, we cannot reject the assumption of normality of the poverty rate distribution. Thus, the distribution of the poverty rate can be completely described by the normal distribution with an expected value equal to 0.1843 and a variance of 11.5690.

If the assumption of the normal distribution for both the threshold and the rate must be accepted, then we can calculate a confidence interval for the two of them. Let's assume that the desired degree of confidence is c with $0 \leq c \leq 100\%$. α , the risk is $1-c$, and therefore $0 \leq \alpha \leq 100\%$. In accordance with the desired degree of confidence, we get $z_{1-\alpha/2}$ from the table of the standard normal distribution. The confidence interval for the poverty threshold would be:

$$\text{Average threshold} \pm z_{1-\alpha/2} * \text{standard deviation of the threshold.} \quad (\text{eq. 21})$$

The confidence interval for the poverty rate is calculated in the same way:

$$\text{Average rate} \pm z_{1-\alpha/2} * \text{standard deviation of the rate.} \quad (\text{Eq. 22})$$

Thus, for a confidence level of 95%, $z_{1-\alpha/2} = 1.96$ [16]. So we can estimate the confidence interval at 95% for the threshold and the poverty rate.

For the poverty threshold the confidence interval is according to the equation 21:

$$5125.401753 \pm 1.96 * 24.21690715 = 5125.401753 \pm 47.465138$$

That is from 5077.93662 to 5172.86689.

Similarly, using equation 22, the confidence interval for the poverty rate can be estimated to be from 10.727768 to 12.410232.

5. Conclusion

This paper develops a new methodology to provide robust interval estimates of the poverty threshold and its rate in a society. The estimate is robust because we can decide a priori of an acceptable level of confidence. Furthermore, the developed methodology is not expensive because it requires taking a single sample from which will be derived the laws of probabilities that control the variables needed to assess the threshold and the rate. After this assessment, the simulation takes over for the development of a number of samples that meet the laws of probability distribution derived from the initial sample. The statistical study of the results of these samples can then lead to evaluate confidence intervals for the threshold and rate.

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Annex: tables and figures

Classes	0 dépendent	1 dép.	2 dép.	n dép.	class Total
Class 1	O_{10}	O_{11}	O_{12}		O_{1n}	$O_{1.}$
Class 2	O_{20}	O_{21}	O_{22}		O_{2n}	$O_{2.}$
Class 3	O_{30}	O_{31}	O_{32}		O_{3n}	$O_{3.}$
.....						
Class m	O_{m0}	O_{m1}	O_{m2}		O_{mn}	$O_{m.}$
Total	$O_{.0}$	$O_{.1}$	$O_{.2}$		$O_{.n}$	$O_{..}$

Table 1: classification of the observations by class of income and number of dependents

Class index i	Class Probability	Cumulative Probability
1	p^c_1	$P^c_1 = p^c_1$
2	p^c_2	$P^c_2 = p^c_1 + p^c_2$
...	...	
m	p^c_m	$P^c_m = p^c_1 + p^c_2 + \dots + p^c_m = 1$

Table 2: Cumulative probabilities of the income classes

Number of Dependents j	0	1	n
$p^{e,j}_{i^*}$	p^{e,i^*0}	p^{e,i^*1}	p^{e,i^*n}
$P^{e,j}_{i^*}$	p^{e,i^*0}	$p^{e,i^*0} + p^{e,i^*1}$	$p^{e,i^*0} + p^{e,i^*1} + \dots + p^{e,i^*n} = 1$

Table 3: Calculation of cumulative probabilities. Case of a married head.

Number of dependents j	0	1	n
$p^{e,j}_{i^*}$	p^{e,i^*0}	p^{e,i^*1}	p^{e,i^*n}
$P^{e,j}_{i^*}$	p^{e,i^*0}	$p^{e,i^*0} + p^{e,i^*1}$	$p^{e,i^*0} + p^{e,i^*1} + \dots + p^{e,i^*n} = 1$

Table 4: Calculation of cumulative probabilities. Case of an unmarried head.

marital status = 0 for unmarried, 1 for married

Table 5: Some of the original data

Class Limits		Number of dépendent individuals													
lower	upper	0	1	2	3	4	5	6	7	8	9	10	11	total	
5.50	2025.91	0	1	7	11	0	0	0	0	0	0	0	0	19	
2025.91	4046.32	0	0	0	0	7	2	1	0	0	0	0	0	10	
4046.32	6066.73	0	0	0	0	0	0	0	0	0	0	0	0	0	
6066.73	8087.15	0	0	0	0	0	0	0	0	0	0	0	0	0	
8087.15	10107.56	0	2	1	0	1	0	0	0	0	0	0	0	4	
10107.56	12127.97	0	26	1	3	1	1	0	0	0	0	0	0	32	
12127.97	14148.38	0	42	28	15	12	3	2	0	0	0	0	0	102	
14148.38	16168.79	0	25	71	86	50	25	2	2	0	0	0	0	261	
16168.79	18189.21	0	15	56	64	64	65	14	3	0	0	0	0	281	
18189.21	20209.62	0	19	52	78	73	39	11	2	1	0	0	0	275	
20209.62	22230.03	0	9	22	27	40	41	6	2	0	0	0	0	147	
22230.03	24250.44	0	14	32	34	16	18	5	1	0	0	0	0	120	
24250.44	26270.85	0	16	36	37	55	21	2	1	0	0	0	0	168	
26270.85	28291.26	0	28	78	67	47	40	16	2	0	0	0	0	278	
28291.26	30311.67	0	20	127	171	111	39	9	3	2	0	1	0	483	
30311.67	32332.09	0	48	88	142	224	120	22	5	1	0	0	0	650	
32332.09	34352.50	0	14	119	240	266	129	36	13	2	0	0	1	820	
34352.50	36372.91	0	22	48	91	147	166	91	20	6	0	0	0	591	
36372.91	38393.32	0	15	74	104	102	63	25	11	2	0	0	0	396	
38393.32	40413.73	0	7	24	86	91	88	27	7	0	0	0	0	330	
40413.73	42434.14	0	6	5	10	21	13	13	4	0	0	0	0	72	
42434.14	44454.55	0	4	10	7	2	2	6	0	0	0	0	0	31	
44454.55	46474.96	0	2	1	5	10	1	1	0	0	0	0	0	20	
46474.96	48495.38	0	0	2	2	5	3	0	0	0	0	0	0	12	
48495.38	50515.79	0	0	0	0	1	2	0	0	0	0	0	0	3	
Total		0	335	882	1280	1346	881	289	76	14	0	1	1	5105	

Table 6: Distribution of married household heads O_{ij}

Class limits		Number of dépendent individuals									
lower	upper	0	1	2	3	4	5	6	7	8	total
5.5	2025.912	0	0	11	3	0	0	0	0	0	14
2025.912	4046.323	0	0	0	0	2	1	1	0	0	4
4046.323	6066.735	0	0	0	0	0	0	0	0	0	0
6066.735	8087.146	0	0	0	0	0	0	0	0	0	0
8087.146	10107.56	0	0	0	0	0	0	0	0	0	0
10107.56	12127.97	0	0	1	0	0	0	0	0	0	1
12127.97	14148.38	0	2	2	1	2	0	0	0	0	7
14148.38	16168.79	0	0	16	8	1	0	0	0	0	25
16168.79	18189.21	0	0	3	2	2	1	0	0	0	8
18189.21	20209.62	0	0	3	1	0	0	0	0	0	4
20209.62	22230.03	0	1	2	0	0	1	0	0	0	4
22230.03	24250.44	0	0	6	0	0	0	0	0	0	6
24250.44	26270.85	0	1	7	1	2	0	0	0	0	11
26270.85	28291.26	0	0	9	4	0	1	0	0	0	14
28291.26	30311.67	0	0	17	12	0	1	0	0	0	30
30311.67	32332.09	0	0	24	8	2	1	0	0	0	35
32332.09	34352.5	0	0	25	19	3	4	1	1	0	53
34352.5	36372.91	0	2	9	7	3	3	0	1	1	26
36372.91	38393.32	0	0	7	5	3	2	0	0	0	17
38393.32	40413.73	0	1	1	3	0	1	0	0	0	6
40413.73	42434.14	0	0	1	1	0	0	0	0	0	2
42434.14	44454.55	0	0	1	0	0	0	0	0	0	1
44454.55	46474.96	0	0	0	0	1	0	0	0	0	1
46474.96	48495.38	0	0	0	0	0	0	0	0	0	0
48495.38	50515.79	0	0	0	0	0	0	0	0	0	0
Total		0	7	145	75	21	16	2	2	1	269

Table 7: Distribution of the number of the unmarried household heads O_{ij}

Class limits		Class probability c
lower	upper	
5.5	2025.912	6.14E-03
2025.912	4046.323	2.61E-03
4046.323	6066.735	0
6066.735	8087.146	0
8087.146	10107.56	7.44E-04
10107.56	12127.97	6.14E-03
12127.97	14148.38	2.03E-02
14148.38	16168.79	5.36E-02
16168.79	18189.21	5.36E-02
18189.21	20209.62	5.17E-02
20209.62	22230.03	2.81E-02
22230.03	24250.44	2.34E-02
24250.44	26270.85	3.41E-02
26270.85	28291.26	5.38E-02
28291.26	30311.67	9.53E-02
30311.67	32332.09	0.1276517
32332.09	34352.5	0.1622628
34352.5	36372.91	0.1148121
36372.91	38393.32	7.69E-02
38393.32	40413.73	6.27E-02
40413.73	42434.14	1.36E-02
42434.14	44454.55	5.95E-03
44454.55	46474.96	3.91E-03
46474.96	48495.38	2.23E-03
48495.38	50515.79	5.58E-04

Table 8: Probability of classes.

Class Limits		Marital status Probability	
lower	upper	unmarried	married
5.5	2025.912	0.4242424	0.5757576
2025.912	4046.323	0.2857143	0.7142857
4046.323	6066.735	0	0
6066.735	8087.146	0	0
8087.146	10107.56	0	1
10107.56	12127.97	3.03E-02	0.969697
12127.97	14148.38	6.42E-02	0.9357798
14148.38	16168.79	8.68E-02	0.9131944
16168.79	18189.21	2.78E-02	0.9722222
18189.21	20209.62	1.44E-02	0.9856115
20209.62	22230.03	2.65E-02	0.9735099
22230.03	24250.44	4.76E-02	0.952381
24250.44	26270.85	6.01E-02	0.9398907
26270.85	28291.26	4.84E-02	0.9515571
28291.26	30311.67	5.86E-02	0.9414063
30311.67	32332.09	5.10E-02	0.9489796
32332.09	34352.5	6.08E-02	0.9392202
34352.5	36372.91	4.21E-02	0.9578606
36372.91	38393.32	4.12E-02	0.9588377
38393.32	40413.73	1.78E-02	0.9821959
40413.73	42434.14	2.74E-02	0.9726027
42434.14	44454.55	0.03125	0.96875
44454.55	46474.96	4.76E-02	0.952381
46474.96	48495.38	0	1
48495.38	50515.79	0	1

Table 9: Probability P_i^* and P_i^*

Class		Number of dependent individuals									
limits	lower	upper	0	1	2	3	4	5	6	7	8
5.5	2025.91	4046.32	0.000	0.000	0.786	0.214	0.000	0.000	0.000	0.000	0.000
2025.91	4046.32	6066.73	0.000	0.000	0.000	0.000	0.500	0.250	0.250	0.000	0.000
4046.32	6066.73	8087.14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6066.73	8087.14	10107.55	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8087.14	10107.55	12127.97	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
10107.5	12127.97	14148.38	0.000	0.286	0.286	0.143	0.286	0.000	0.000	0.000	0.000
12127.97	14148.38	16168.79	0.000	0.000	0.640	0.320	0.040	0.000	0.000	0.000	0.000
14148.38	16168.79	18189.20	0.000	0.000	0.375	0.250	0.250	0.125	0.000	0.000	0.000
16168.79	18189.20	20209.61	0.000	0.000	0.750	0.250	0.000	0.000	0.000	0.000	0.000
18189.20	20209.61	22230.02	0.000	0.250	0.500	0.000	0.000	0.250	0.000	0.000	0.000
20209.62	22230.02	24250.44	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
22230.03	24250.44	26270.85	0.000	0.091	0.636	0.091	0.182	0.000	0.000	0.000	0.000
24250.44	26270.85	28291.26	0.000	0.000	0.643	0.286	0.000	0.071	0.000	0.000	0.000
26270.85	28291.26	30311.67	0.000	0.000	0.567	0.400	0.000	0.033	0.000	0.000	0.000
28291.26	30311.67	32332.09	0.000	0.000	0.686	0.229	0.057	0.029	0.000	0.000	0.000
30311.67	32332.09	34352.5	0.000	0.000	0.472	0.358	0.057	0.075	0.019	0.019	0.000
32332.09	34352.5	36372.91	0.000	0.077	0.346	0.269	0.115	0.115	0.000	0.038	0.038
34352.5	36372.91	38393.32	0.000	0.000	0.412	0.294	0.176	0.118	0.000	0.000	0.000
36372.91	38393.32	40413.73	0.000	0.167	0.167	0.500	0.000	0.167	0.000	0.000	0.000
38393.32	40413.73	42434.14	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000
40413.73	42434.14	44454.55	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
42434.14	44454.55	46474.96	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
44454.55	46474.96	48495.38	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
46474.96	48495.38	50515.79	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
48495.38	50515.79		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 10: Probability p_{ij}^e

Class Limits		Number of dependent individuals											
lower	upper	0	1	2	3	4	5	6	7	8	9	10	11
5.5	2025.91	0	5.26E-02	0.36842	0.579	0	0	0	0	0	0	0	0
2025.91	4046.32	0	0	0	0	0.7	0.2	0.1	0	0	0	0	0
4046.32	6066.73	0	0	0	0	0	0	0	0	0	0	0	0
6066.73	8087.14	0	0	0	0	0	0	0	0	0	0	0	0
8087.14	10107.56	0	0.5	0.25	0	0.25	0	0	0	0	0	0	0
10107.56	12127.97	0	0.8125	0.03125	0.094	0.03125	0.03125	0	0	0	0	0	0
12127.97	14148.38	0	0.41176	0.27451	0.147	0.11765	2.94E-02	1.96E-02	0	0	0	0	0
14148.38	16168.79	0	9.51E-02	0.26996	0.327	0.19772	9.51E-02	7.60E-03	7.60E-03	0	0	0	0
16168.79	18189.21	0	5.36E-02	0.20357	0.229	0.22143	0.23214	0.05	1.07E-02	0	0	0	0
18189.21	20209.62	0	6.93E-02	0.18613	0.285	0.26642	0.14234	4.01E-02	7.30E-03	3.65E-03	0	0	0
20209.62	22230.03	0	6.12E-02	0.14966	0.184	0.27211	0.27891	4.08E-02	1.36E-02	0	0	0	0
22230.03	24250.44	0	0.11667	0.26667	0.283	0.13333	0.15	4.17E-02	8.33E-03	0	0	0	0
24250.44	26270.85	0	9.30E-02	0.2093	0.221	0.33721	0.12209	1.16E-02	5.81E-03	0	0	0	0
26270.85	28291.26	0	0.10182	0.28727	0.24	0.16	0.14545	5.82E-02	7.27E-03	0	0	0	0
28291.26	30311.67	0	4.15E-02	0.26141	0.355	0.23029	8.09E-02	0.018672	6.22E-03	4.15E-03	0	2.07E-03	0
30311.67	32332.08	0	7.37E-02	0.13518	0.218	0.34409	0.18433	3.53E-02	7.68E-03	1.54E-03	0	0	0
32332.08	34352.5	0	1.71E-02	0.1453	0.293	0.32479	0.15751	4.27E-02	1.59E-02	2.44E-03	0	0	1.22E-03
34352.5	36372.91	0	3.72E-02	8.12E-02	0.154	0.24873	0.28088	0.153976	3.38E-02	1.02E-02	0	0	0
36372.91	38393.32	0	3.79E-02	0.18687	0.263	0.25758	0.15909	6.31E-02	2.78E-02	5.05E-03	0	0	0
38393.32	40413.73	0	2.11E-02	7.25E-02	0.263	0.27492	0.26586	0.081571	2.11E-02	0	0	0	0
40413.73	42434.14	0	8.45E-02	7.04E-02	0.127	0.29577	0.1831	0.183099	5.63E-02	0	0	0	0
42434.14	44454.55	0	0.12903	0.32258	0.226	6.45E-02	6.45E-02	0.193548	0	0	0	0	0
44454.55	46474.97	0	0.1	0.05	0.25	0.5	0.05	0.05	0	0	0	0	0
46474.97	48495.38	0	0	0.16667	0.167	0.41667	0.25	0	0	0	0	0	0
48495.38	50515.78	0	0	0	0	0.33333	0.66667	0	0	0	0	0	0

Table 11: Calculation of p_{ij}^e

Marital Status	Number of dependents	Income
0	2	24535.97
1	3	31209.5
1	6	35876.65
1	3	33630.55
1	1	23520.31
1	4	15906.58
1	3	30220.16
0	2	16450.08
1	4	29688.18
1	4	31402.74

1	2	26605.43
1	5	27768.1

Table 12: Sample of simulated observations

Threshold	Rate
5157.979	11.55
5092.476	11.51
5130.993	11.26
5156.487	10.91
5121.879	11.19
5134.684	12.12
5162.195	11.03
5112.277	11.57
5113.53	11.63
5146.335	11.77
5122.213	12.08
5102.487	11.46
5071.576	12.21
5158.719	10.91

Table 13: Some of the generated thresholds and rates.

Statistics	Origin of Statistics	
	Threshold	Poverty Rate
Max	5195.903	12.89
Min	5061.599	10.3
Average	5125.401746	11.56904
Stan. Dev	24.21691016	0.429249565

Table 14: Statistics of the threshold and rate.

Class limit	Lower	Class Upper Limit	f_i	f_i	prob upper limit	Interval prob	e_i	e_i	$(f_i - e_i)$	$(f_i - e_i)^2$	$(f_i - e_i)^2 / e_i$
0	5061.60	0	9	0.0042	0.00421	2.1057	9.3803	-0.3803	0.1447	0.0154	
5061.60	5068.31	4		0.0092	0.00499	2.4959					
5068.31	5075.03	5		0.0188	0.00956	4.7787					
5075.03	5081.74	8		0.0357	0.01695	8.4764		-0.4764	0.2270	0.0268	
5081.74	5088.46	12		0.0636	0.02786	13.9294		-1.9294	3.7225	0.2672	
5088.46	5095.18	19		0.1060	0.04241	21.2064		-2.2064	4.8684	0.2296	
5095.18	5101.89	30		0.1658	0.05982	29.9104		0.0896	0.0080	0.0003	
5101.89	5108.61	36		0.2440	0.07817	39.0836		-3.0836	9.5086	0.2433	
5108.61	5115.32	75		0.3386	0.09463	47.3137		27.6863	766.5327	16.2011	
5115.32	5122.04	46		0.4447	0.10613	53.0638		-7.0638	49.8972	0.9403	
5122.04	5128.75	49		0.5550	0.11027	55.1353		-6.1353	37.6419	0.6827	
5128.75	5135.47	57		0.6611	0.10615	53.0739		3.9261	15.4142	0.2904	
5135.47	5142.18	36		0.7558	0.09466	47.3317		-11.3317	128.4074	2.7129	
5142.18	5148.90	37		0.8340	0.07821	39.1059		-2.1059	4.4350	0.1134	

5148.90	5155.61	29	0.8939	0.05987	29.9332		-0.9332	0.8708	0.0291	
5155.61	5162.33	23	0.9363	0.04245	21.2266		1.7734	3.1448	0.1482	
5162.33	5169.04	13	0.9642	0.02789	13.9453		-0.9453	0.8936	0.0641	
5169.04	5175.76	7	0.9812	0.01698	8.4878		-1.4878	2.2134	0.2608	
5175.76	5182.47	10	14	0.9908	0.00957	4.7860	9.3962	4.6038	21.1949	4.4285
5182.47	5189.19	1		0.9958	0.00500	2.5002				
5189.19	25000	3		1.0000	0.00422	2.1100				
total		500				500				26.6540752

Table 15: Calculation of the goodness of fit test for the threshold.

Lower limit	Upper Limit	f_i	f_i	Upper lim probability	Interval probability	e_i	e_i	$f_i - e_i$	$(f_i - e_i)^2$	$(f_i - e_i)^2 / e_i$
0	10.3	0	7	0.0016	0.0016	0.77809	10.0583	-3.0583	9.3532	0.9299
10.30	10.43	3		0.0040	0.0024	1.2062				
10.43	10.56	3		0.0093	0.0053	2.6709				
10.56	10.69	1		0.0201	0.0108	5.4031				
10.69	10.82	10		0.0401	0.0200	9.9861		0.0139	0.0002	0.0000
10.82	10.95	17		0.0738	0.0337	16.8621		0.1379	0.0190	0.0011
10.95	11.08	24		0.1258	0.0520	26.0135		-2.0135	4.0540	0.1558
11.08	11.21	33		0.1992	0.0733	36.6651		-3.6651	13.4327	0.3664
11.21	11.34	59		0.2936	0.0944	47.2146		11.7854	138.8955	2.9418
11.34	11.47	59		0.4047	0.1111	55.5483		3.4517	11.9143	0.2145
11.47	11.60	67		0.5241	0.1194	59.7084		7.2916	53.1670	0.8904
11.60	11.72	52		0.6414	0.1173	58.6370		-6.6370	44.0495	0.7512
11.72	11.85	47		0.7466	0.1052	52.6112		-5.6112	31.4858	0.5985
11.85	11.98	38		0.8329	0.0863	43.1277		-5.1277	26.2928	0.6097
11.98	12.11	30		0.8975	0.0646	32.3001		-2.3001	5.2902	0.1638
12.11	12.24	26		0.9417	0.0442	22.1014		3.8986	15.1990	0.6877
12.24	12.37	15		0.9693	0.0276	13.8167		1.1833	1.4002	0.1013
12.37	12.50	7		0.9851	0.0158	7.8915		-0.8915	0.7947	0.1007
12.50	12.63	7	9	0.9933	0.0082	4.1179	7.4582	1.5418	2.3772	0.3187
12.63	12.76	1		0.9972	0.0039	1.9632				
12.76	12.89	1		0.9990	0.0017	0.8551				
12.89	∞	0		1.0000	0.0010	0.5221				
total		500			1.0000	500				8.8316

Table 16: Calculation of the goodness of fit test for the rate.

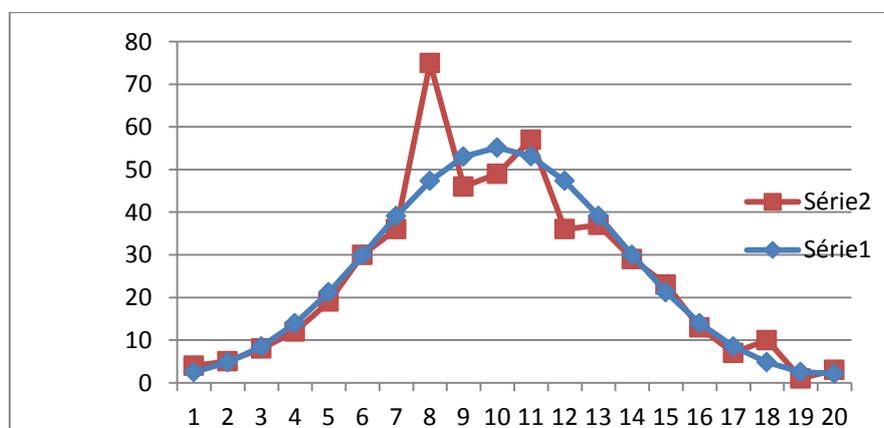


Figure 1: Distribution of the poverty threshold and normal distribution.

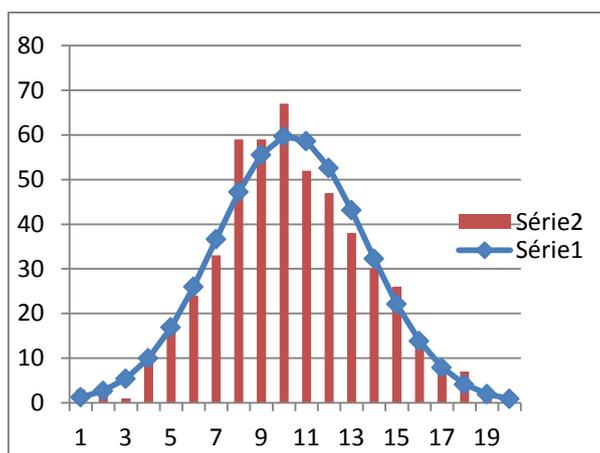


Figure 2: Distribution of the rate and the normal distribution.