

# THE EFFECTS OF 3B FORCES ON ODD-ODD $^{136}\text{Sb}$ NEUTRON RICH NUCLEI STRUCTURE IN THE VICINITY OF $^{132}\text{Sn}$ CORE

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## Abstract

These The take into account of the three-body monopole effects can solve the spectroscopic problems involved in certain nuclear properties, for which the two body realistic interactions failed. In order to study and to understand the role of these effects, calculations in the shell model of nuclear structure were carried out to develop the two body matrix elements of N-N interaction. Their application on  $^{136}\text{Sb}$  nucleus with four valence particles in addition to the inert core  $^{132}\text{Sn}$  has a great importance. This nucleus with  $N \sim 82$  is currently the most experimentally studied in this mass region. The first excited state, dominated by  $\pi(1g_{7/2})^1\nu(2f_{7/2})^3$  configuration, obtained in this work reproduces the experimental one.

**Keywords:** Nuclear Structure, Monopole Hamiltonian, Three Body Effects, Odd-Odd Nuclei.

## Résumé

La prise en compte des effets monopolaires à trois corps peut mener à la résolution de problèmes spectroscopiques liés à certaines propriétés nucléaires, pour lesquelles les interactions réalistes à deux corps ont échoués. Afin d'étude et de comprendre du rôle de ces effets, des calculs dans le modèle en couches de structure nucléaire ont été réalisé pour développer les éléments de matrice à deux corps de l'interaction N-N. Leur application sur le noyau  $^{136}\text{Sb}$  à quatre nucléons de valence en plus du cœur inerte  $^{132}\text{Sn}$  a une grande importance. Ce noyau est l'un des isotopes avec  $N \sim 82$  actuellement les plus étudiés expérimentalement dans cette région de masse. Le premier état excité, dominé par la configuration  $\pi(1g_{7/2})^1\nu(2f_{7/2})^3$ , obtenu dans cette étude reproduit celui expérimental.

**Mots clés :** structure nucléaire, hamiltonien monopolaire, effet à trois corps, interactions effectives, noyaux impair-impairs, astrophysique nucléaire.

## ملخص

إن الأخذ في الحسبان الفعل أحادي القطب قد يؤدي إلى حل بعض المعضلات المطيافية المرتبطة ببعض الخصائص النووية، والتي فشلت التفاعلات الواقعية الثنائية في إعادة الوصول إليها. من أجل دراسة وفهم الدور الذي تلعبه هذه الظاهرة، تم تحقيق بعض حسابات النموذج الطبقي لتفسير وتطوير عناصر المصفوفة الثنائية للتفاعل نووية-نووية N-N. تطبيقاتها على النواة  $^{136}\text{Sb}$ ، مع أربع نويات تكافؤ إضافة إلى قلب القصدير  $^{132}\text{Sn}$ ، له أهمية كبيرة. هذه النواة، التي تملك عدد نيوترونات في جوار  $N=82$ ، تحظى حالياً باهتمام تجريبي كبير في هذه المنطقة الكتلية. أول حالة مثارة، والتي تتولد من التمثيل  $\pi(1g_{7/2})^1\nu(2f_{7/2})^3$ ، المتحصل عليها في هذا العمل متوافقة مع النتيجة التجريبية لهذا المستوى.

**الكلمات المفتاحية :** البنية النووية، الهاملتوني أحادي القطب، المفعول ثلاثي الأجسام، التفاعلات الفعالة، الأنوية فردي- فردي.

## I. INTRODUCTION

The description of the existing nuclei, based on the understanding of the resulting forces between nucleons, present the main topic of the nuclear physics [1]. This may be realized by studying nuclear properties of nuclei from the proton to the neutron drip-lines, in order to examine and develop the existing theoretical nuclear models [2]. The neutron drip-line evolves regularly from light to medium-mass nuclei except for a striking anomaly in the oxygen isotopes. This anomaly is not reproduced in shell-model calculations derived from microscopic two-body forces [3].

The two body realistic interactions derived from the N-N force fail to reproduce some nuclear properties [3]. This problem can be solved by the consideration of the monopole interaction [4]. Two body interaction hypotheses suggest that the cores are inert enough to isolate the contribution of two valence particles. However, the monopole term is the result of the interaction between an inert core and these two addition nucleons in the valence space [5].

## II. THE MONOPOLE INTERACTION

The Hamiltonian can be separated into an unperturbed and a residual parts,  $H=H_0+Hr$ , where  $H_0$  is one body (1b) single-particle field [6]. However, this Hamiltonian contains two and three-body (2b and 3b) components, which makes this separation not clear mathematically. It is E. Caurier et al. [6], which propose another separation to solve this problem.

They propose that the Hamiltonian contains monopole and multipole parts.

$$H = H_m + H_M \quad (1)$$

Where  $H_M$  and  $H_m$  denote respectively the multipole and the monopole Hamiltonians. This later involves single particle energies  $\varepsilon_s$  and all quadratic and cubic (2b and 3b) forms in the scalar products of fermion operators  $a_{rx}^+ \otimes a_{sy}$  [4]-[6].

$$H_m = \sum_s \varepsilon_s n_s + \sum_{s \leq t} (a_{st} n_{st} + b_{st} T_{st}) \quad (2)$$

$T_{s/t}$  and  $n_{s/t}$  are, respectively, isospin and number operators for  $s/t$  orbitals.

The diagonal part of this Hamiltonian reproduces the average energies of configurations at fixed number of particles and isospin in each orbit ( $JT$  representation) [6]. It can be expressed by

$$H_{mjt}^d = k^d + \sum_{s \leq t} (a_{st} n_{st} + b_{st} T_{st}) + V_m^{ds} \quad (3)$$

$V_m^{ds}$  is the 2b part easily extended to include the 3b term. Note that a 3b potential will produce both 1b and 2b terms [4].

In  $JT$  representation, the centroids are associated to the 2b quadratics in number ( $n_s$ ) and isospin operators ( $T_s$ ), to define the diagonal 2b part of the monopole Hamiltonian [4]. By

fitting only the centroid TBMEs of realistic interactions, one can ameliorate our spectroscopic methods [4].

Their components of a general two body interaction  $V$  are given by an angular average over all possible orientations of the two nucleons in  $s$  and  $t$  orbitals [1]:

$$V_{st}^T = \frac{\sum V_{sst}^T (2J+1) (1 - (-1)^{J+T}) \delta_{st}}{\sum_{st} (2J+1) (1 - (-1)^{J+T}) \delta_{st}} \quad (4)$$

$$a_{st} = \frac{1}{4} (3V_{st}^1 + V_{st}^0), \quad b_{st} = \frac{1}{4} (V_{st}^1 - V_{st}^0) \quad (5)$$

$$n_{st} = \frac{1}{1 + \delta_{st}} n_t (n_s - \delta_{st}) \quad (6)$$

$$T_{st} = \frac{1}{1 + \delta_{st}} \left( T_t T_s - \frac{3}{4} n_{st} \delta_{st} \right) \quad (7)$$

$$\text{Here } \delta_{st} = \begin{cases} 1 & \text{if } s = t \\ \text{else} & \\ 0 & \end{cases}$$

The sum over quantum numbers  $J$  can be restricted by antisymmetry, and  $V_{sst}^T$  stands for the matrix elements of a two-body interaction [7]. The SPE of the orbital  $J$  is effectively shifted by  $V_{st}^T$  multiplied by the occupation number of the orbital  $J$ . This leads to the change in the SPE and determines shell structure and the location of the drip-line [1]. The SPE and the three body part 3b will transform the realistic interaction ( $R$ ) 2b centroids  $V_{st}^T(R)$  into [4]:

$$V_{st}^T(R1) = V_{st}^T(R) + \chi_{st}^T \quad (8)$$

$\chi_{st}^T$  is a corrective term.

## III. RESULTS AND DISCUSSION

The  $^{136}\text{Sb}$  with a proton and three neutrons in addition to the tin core is at present the most exotic open-shell nucleus beyond  $^{132}\text{Sn}$ . His lowest excited states are dominated by the configuration  $(\pi 1g_{7/2})^1 (\nu 2f_{7/2})^3$  [8]-[9].

In principle, spectroscopic information about these states can be obtained by studying the  $\beta$ -decay of  $^{136}\text{Sn}$ . Indeed, first results from the  $\beta$ -decay of very neutron-rich Sn isotopes have been obtained in a laser ion source experiment at CERN/ISOLDE [10]. The microscopic calculations are carried out by means of Oxbash code, in  $z50n82$  space model [11]. Which contains four orbitals of  $N=4$  harmonic oscillator major shell, and one orbital of  $N=5$  of opposite parity.  $\pi$  ( $1g_{9/2}$ ,  $2d_{5/2}$ ,  $2d_{3/2}$ ,  $3s_{1/2}$  and  $1h_{11/2}$ ) for protons and five orbitals of  $N=5$  harmonic oscillator major shell, and one orbital of  $N=6$  of opposite parity.  $\nu$  ( $1h_{9/2}$ ,  $2f_{7/2}$ ,  $2f_{5/2}$ ,  $3p_{3/2}$ ,  $3p_{1/2}$  and  $1i_{13/2}$ ), for neutrons.

Basing on three body effects [3]-[4], we carry out some modifications on the  $kh5082$  [12] original interaction and a new interaction named  $kh3$  is introduced. The  $TBME$  of this original interaction are modified taking in consideration the monopole effect for odd-odd nuclei in the  $^{132}\text{Sn}$  region.

$$\langle j_1 j_2 | V | j_2 j_4 \rangle_{kh3} = \{ \langle j_1 j_2 | V | j_2 j_4 \rangle_{kh5082} + \text{Monopole effect} \} \quad (9)$$

These modifications are based on the use of the factors -74 keV, -208 keV and 201 keV for  $\langle ff | V | ff \rangle^{T=1, J=0, 4 \text{ and } 6}$ ,  $\langle ff | V | ff \rangle^{T=1, J=2}$  and  $\langle fr | V | fr \rangle^{T=1, J=2, 3, 4 \text{ and } 5}$  respectively. Here  $f$  and  $r$  refer to  $v2f_{7/2}$  and  $v3p_{3/2}$  respectively. Noted that the corrected factors is taken from Ref.[3], and they will be effective for nuclei in which the valence nucleon number  $n \geq 3$ .

Using our new interaction, the calculations of some nuclear properties for  $^{136}\text{Sb}$  nucleus are developed in the framework of the nuclear shell model by means of *Oxbash* nuclear structure code. The recent experimental values of single particle energies, effective charges  $e_p=1.35e$  and  $e_n=0.9e$ , that reproduce the electromagnetic properties of  $^{134}\text{Sb}$  nucleus [13]-[14], and the effective factors giving by V. I. Isakov are used to evaluate different nuclear properties.

The Fig.1 shows the calculated spectra using *kh5082* [12], and *kh3* interaction in comparison with the experimental and the spectrum calculated using *SMPN* for  $^{136}\text{Sb}$ .

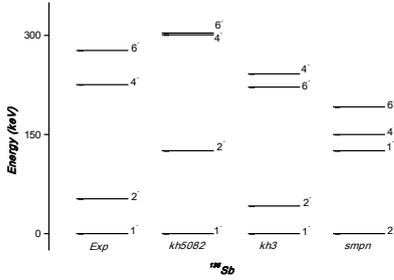


Fig.1: Calculated energies by means of *kh5082* [12] and *kh3* interactions in comparison with the experimental ones and those calculated by means of *SMPN* for  $^{136}\text{Sb}$ .

Our new interaction *kh3* reproduce the experimental energetic sequence, the ground and the first excited state of  $^{136}\text{Sb}$ . However, the  $6^-$  state is decreased under  $4^-$  one. The original interaction reproduce the energetic sequence, but the energetic values are different from the experimental ones. The *SMPN* interaction cannot reproduce the experimental ground state as well as the energetic sequence.

The reduced electromagnetic transition probabilities are calculated using:

$$B(M_{\sigma\lambda} : J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_f | M_{\sigma\lambda} | J_i \rangle \right|^2 \quad (10)$$

As the half-life values in  $^{136}\text{Sb}$  are as yet unknown experimentally, they are calculated by means of  $\gamma$  branching ratios from different levels, in order to obtain the experimental  $B(E2)$  values. The calculation results are presented in the Fig.2.

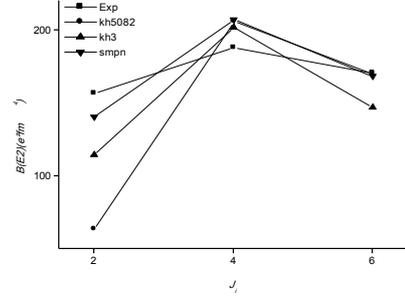


Fig.2: Calculated reduced electric transition probabilities  $B(E2)$  by means of *kh5082* [12] and *kh3* interactions in comparison with the experimental ones and those calculated by means of *SMPN* [9] for  $^{136}\text{Sb}$ .

The calculated values of reduced electric transition probabilities  $B(E2)$  are obtained using the effective charges, these values are taking from Ref. [13]-[14].

The interaction *kh3* gives  $147.3 e^2fm^4$  for the reduced electric transition probability for the transition  $6^- \rightarrow 4^-$  this value is close to the experimental one that has the value of  $170.4 e^2fm^4$ .

The lowest negative parity states of  $^{136}\text{Sb}$  have the  $\{\pi 1g_{7/2}, v2f_{7/2}; J\}$  configuration with  $j(p) = j(n) = j = 7/2$ . The value of the magnetic moment of a state with the spin  $j = l \pm 1/2$  and  $t_z = \pm 1/2$  for a proton or a neutron is calculated by [8]:

$$\mu_J = \mu_N \frac{J}{2} \left[ \frac{\mu_j(p) + \mu_j(n)}{j} \right] \quad (11)$$

The calculated electric quadrupole and magnetic dipole moments, by means of *kh5082*, *kh3* and *SMPN* interactions, are illustrated in Table.1.

TABLE I: CALCULATED ELECTRIC QUADRUPOLE AND MAGNETIC DIPOLE MOMENTS, BY MEANS OF *kh5082*, *kh3* INTERACTION IN COMPARISON WITH THOSE GETTING USING *SMPN* ONE.

$J$	$\mu_p (\mu_N)$			$\mu_n (\mu_N)$			$Q (efm^2)$		
	<i>kh5082</i>	<i>kh3</i>	<i>SMPN</i>	<i>kh5082</i>	<i>kh3</i>	<i>SMPN</i>	<i>kh5082</i>	<i>kh3</i>	<i>SMPN</i>
1	0.426	0.494	0.594	-0.459	-0.366	-0.307	9.72	7.59	6.22
2	1.073	1.153	1.161	-0.165	-0.153	-0.147	11.25	11.33	13.35
4	1.672	1.647	1.671	-0.362	-0.422	-0.421	-14.29	-10.13	-15.37
6	2.292	2.349	2.320	-0.593	-0.647	-0.672	-44.10	-46.80	-50.22

The obtained result for the neutron magnetic dipole moment using the three interactions are close, however they are very different for the proton dipole and the electric quadrupole moments.

#### IV. CONCLUSION

In this work, we calculate the excitation energies, reduced electric transition probabilities  $B(E2)$ , and electric quadrupole moment, for odd-odd nucleus with four valence particles  $^{136}\text{Sb}$ . The calculations are carried out in the framework of the shell model by means of Oxbash nuclear structure code. Basing on *kh5082* [12] interaction, we carry out some modifications used the monopole correction to get *kh3* interaction. The new experimental values of the single particle energies were used.

Our new interaction reproduce the experimental ground and the first excited states of  $^{136}\text{Sb}$ , however we get an inversion between  $4^-$  and  $6^-$  states. The original interaction *kh5082* [12] reproduce the energetic sequence, but the energetic values are different from the experimental ones. The *SMPN* interaction cannot reproduce the experimental ground state as well as the energetic sequence.

For the electric reduced transition probabilities  $B(E2)$ , the *kh3* interaction gives close results to the experimental ones, in the case of pure transitions. In the electromagnetic multipole moment calculations, the obtained result using the three interactions are close.

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