COMPARATIVE SURVEY OF THE DCT AND THE WAVELET TRANSFORMS FOR IMAGE COMPRESSION

KAMMOUN Fahmi, FOURATI Wiem, and BOUHLEL Mohamed Salim
Research Unit: Sciences and Technologies of Image and Telecommunications
Higher Institute of Biotechnology, Sfax Road of Soukra km3, 3038 Sfax Tunisia
fahmi_kammoun@yahoo.fr medsalim.bouhlel@enis.rnu.tn

1. Introduction

The data compression is a domain that knew a renewal of interest from the nineteen's. It follows the apparition of the multimedia that has brought an important flux of audio and video information. This new rise, generated by the technical evolution and the increasing demand of different organisms, allow to the members of the International Consultative Committee of the Telegraph and the Telephone (CCITT) [1] and the International Organization of Standardization (ISO) [2] to elaborate a new compressive international norm of the gray levels and color images. This group is called JPEG (Joined Photographic Expert Group) and the compressive norm carries the same name [3-5]. The members of the JPEG opted for a method based on the Discrete Cosine Transform (DCT) [6-7]. The JPEG2000 norm [8-12] focuses the research undertaken after the standard JPEG toward a new norm. Its goal is not limited to create new algorithms but also to achieve accommodating architectures and compression formats. Indeed, the achievement of efficient and adapted techniques of compression is imperative because of the development of the exchanges, transmission and storage of important volumes of data. The goal of this paper is to compare the Discrete Cosine Transform (DCT) and the Wavelet Transforms (WT), used respectively in the JPEG's norms and JPEG2000's. We compare the performances of five wavelets: the Haar wavelet and the Daubechies wavelets of order 2, 4, 10 and 20. On the level of the quantization, we verify the influence of the report of the retained details (not truncated to zero) for the WT and the one of the quality factor for the DCT. We use three entropy encoders: the arithmetic encoder [13], the Huffman encoder [14] and the Run Length Encoding (RLE).

2. Different types of compression

The compression reduces the physical size of a block of information by removing the redundancy that exists between its elements. This operation permits to reduce the memory space of storage, the read and transmission times, what allows the real-time working. Inversely, the decompression aims to recover the initial information block. The compressive methods are valued by their compression rate, by the quality (or accurateness) of the reconstruction and by their execution time. After the compression, the initial data are not more directly accessible as coherent information. It is the stage of decompression that permits to recover them. The conservative methods restore an exact copy of the initial data. The other methods are non-conservative techniques. They generate a light modification but they succeed elevated compression rate. The choice of the method depends on the nature of the processing data. On the other hand, the compressive techniques can be predictive or transform coding. On the level of the first family, only the difference (innovation) between two successive samples is coded. This classic technique of removing redundancy is applied directly on the picture or the spatial domain. Inversely, the second family transforms the picture in a different space of representation. These methods use some linear transformations in a first step and code the transformed coefficients in a second step.

3. Conservative compression

The redundancy of the image information generally allows compression without losing information. The measure of the quantity of information by the entropy of Shannon [15] proves that in a coded image on 8 bits/pixel, the entropy is generally of 3 or 4 bits/pixel. This result leads to a compression rate roughly equal to 2. For this aim, one can uses an entropy encoder (Huffman [14], arithmetic coding [13], Ziv-Lempel coding [16], RLE etc.). This kind of coding suits for the data that don't tolerate any modification, particularly the computer files. The JPEG norm [3-5] includes a method without loss. It includes a first step that makes a linear prediction followed of an entropy encoder.

4. Compression with loss

The compressive methods with loss eliminate some information which are non useful after the compression-decompression cycle. In imagery, the suppressed details are not noticeable for the visual appreciation of the hunan eye. The compression with loss has like main advantage the possibility to have elevated and adjustable compression rates. The deterioration of the picture is evidently function of the gotten compression rate. Generally, a compression of factor 10 doesn't generate any perceptible deterioration. For compression rates of more than 100, the deterioration of the picture appears by an aspect of fuzzy around the edges and on the textured zones. For the JPEG algorithm, an artifact of the blocks 8×8 appears. It is due to the decomposition of the picture in blocks for the application of the Discrete Cosine Transform. The art of the compression consists therefore to find the algorithms that insert an acceptable distortion, that is to say imperceptible (or little visible) in the normal conditions of observation of the picture. This topic has been studied since several years and many methods [17-21] have been proposed to answer to the compromise that exists between the compression rate and the distortion. Some of these algorithms are already standardized [22], [23], [4], [5], [12]. The compression with loss includes in the first stage a transformation, generally linear, to remove the components redundancy of the signal. The quantization is applied in second stage. This non-reversible step truncates the components precision and rounded the low values to zero. The compression rate depends strongly on parameters of this stage. The result of the quantization is finally coded. This third stage is conservative.

Figure 1 : the stages of the image compression with loss

5. The jpeg coder

The baseline JPEG coder [3-5] decomposes the initial picture in 8×8 blocks. Every block is transformed by the DCT. This decomposition avoids the variable size DCT application and the too important execution times. The expression of the DCT [24] is:

$$\mathrm{DCT}(i,j)=\frac{2}{N}c(i)c(j)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}I(x,y)\cos\left[\frac{(2x+1)i\pi}{2N}\right]\cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

$$c(i)=c(j)=c(k)=\begin{cases}\frac{1}{\sqrt{2}} & \text{if } k=0\\ 1 & \text{if } k>0\end{cases}$$

Where $I_{(N,N)}$ is the pixels matrix of size N×N. A more efficient way to represent the DCT is to use the matrix notation. The matrix definition of the DCT is the following:

$$DCT_{(N,N)}=C_{(N,N)}\otimes I_{(N,N)}\otimes C^t_{(N,N)}$$

Where $C_{(N,N)}$ is the DCT matrix. It is defined as following:

$$C_{(N,N)}(i,j)=\begin{cases}\frac{1}{\sqrt{N}} & \text{if } i=0\\ \sqrt{\frac{2}{N}}\times\cos\left(\frac{(2j+1)i\pi}{2N}\right) & \text{if } i>0\end{cases}$$

Where C^t is the transposed matrix of C. The DCT is applied on a square matrix representing the values of the pixels, and restitute the coefficients associated to the spatial frequencies. These coefficients are quantified. The quantization reduces the number of bits necessary for the storage of the amplitude of every spatial frequency. It is an important stage for the images compression. It introduces an unavoidable distortion in the signal. The research of the quantifier that minimizes this distortion was the object of several studies [25-26]. In the context of the measure of the distortion introduced by the quantization, several functions have been proposed in the literature [27-29]. However, the most used function is the one that measures the mean square error. In our case, the quantization is done according to a matrix. This matrix remains the same for all resulting blocks of the DCT. The basic idea is that the reduction of the precision is more important when one moves away of the continuous coefficient of the origin. The quality factor determines the difference between the adjacent strips of the quantization matrix. Increasing the quality factor involves a decrease of the coefficients precision, a more important compressions rates and a lower reconstruction quality. The entropy coder constitutes the final stage of the compression. The continuous components, which contain a significant fraction of the total image energy, are differentially encoded. The rest of the matrix is ordered into the zigzag sequence [30] and coded separately. Baseline coder use Huffmann and RLE coding [14].

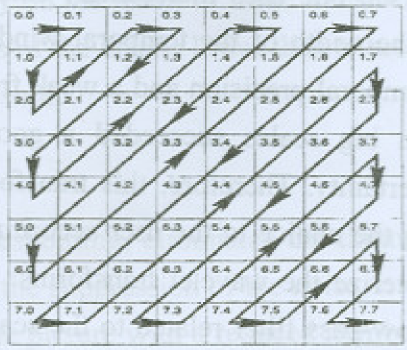

Figure 2 : the zigzag sequence

6. Wavelet Transforms

Wavelet Transforms constitute a cutting edge technology for signal processing and particularly for image processing. It gives a new description of the spectral decomposition via the concept of scale [31]. In the framework of the analysis of the seismic signals, the paper of Goupillaud and al. [32] is among the first to describe clearly the linear decomposition time-scale of a signal by wavelets. This concept is interesting because it is not about a classic transformation time-frequency but of a new formalism in which the basic operator acting on the signal depends of the time and the scale. Although the first use of the wavelets was the analysis of the earth waves with a high resolution [32-33], one also uses them in quantum mechanics, in the survey of the turbulences, in analyses of sound, speech and cardiac signals [34]. In imagery, the wavelets are used mainly in compression, but other applications have been presented [35]. Grossmann and Morlet developed the mathematical formalism related to the wavelet transforms in the beginning while using the language of the quantum mechanics [36]. Meyer [37] proposes the discrete-time wavelets and leads to an orthogonal transformation that is assured by two quadratic mirrors filters. This transformation is iterative and relatively easy to put dawns. This allows several applications as the image compression. Mallat [38] established the relation between the multi-resolution analysis and the wavelet transforms. Daubechies [39-40] was the first to establish the link between discrete wavelet transforms and filterbank.

To achieve a fine analysis, one uses increasingly small scales. It corresponds to the highs frequencies, and the width of the temporal window decreases. So for the small scales one has a good temporal precision and a weak frequency precision. On the other hand, the analysis with big scales succeeded a good frequency precision and an important temporal uncertainty. Therefore, this transformation offers a large range of wavelets mothers letting the choice to the user according to the considered application [41]. Two filters characterize the wavelet transforms: a high-pass filter related to the mother function and a low-pass filter related to the scale function. At two dimensions, these two filters are applied horizontally and vertically. The resulting image is composed of four quadrants: the first block, noted LL results from the application of the low-pass filter horizontally and vertically. The second and the third blocks, noted HL and LH result of an application of the high-pass filter according to a direction and the low-pass filter according to the other direction. The last block corresponds to the application of the high-pass filter according to the two directions.

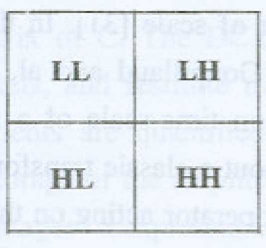

Figure 3 : decomposition of the image after an application of the filters

The elements of the first block are called approximations and concentrate the major part of the energy of the signal. The three other blocks are constituted of details and include the high frequencies and the noise. This procedure is applied in an iterative way on the block of the approximations.

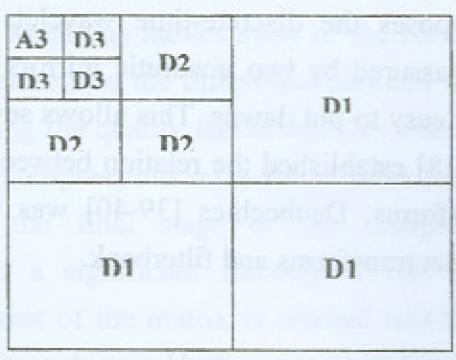

Figure 4 : decomposition of the picture in three iterations

In our survey, we are interested by the Haar wavelet and the Daubechies wavelets of order 2, 4, 10 and 20. We apply on the result of these transformations a truncation: this procedure puts at zero the terms of the details lower then a threshold. The retained detail percentage depends on this threshold.

7. The Truncation

The truncation, also called thresholding, eliminates the thinnest details of the picture. Besides the fact that it permits the suppression of the noise, it will assure a compression of the pictures. We can code the picture with a reduced number of bits because we only preserve the most important wavelet coefficients. Two types of thresholding exist:

The first is the hard thresholding. It is the most "intuitive": it sets a threshold T>0 and it only preserves the wavelet coefficients superior to T. The other coefficients are putted at zero:

$$\theta^{H}_{(T)}(x) = \begin{cases} 0 & \text{if } |x| < T \\ x & \text{if } |x| \geq T \end{cases}$$

The second is the soft thresholding, it always puts at zero the coefficients lower then T. For those superior to T, it attenuates their amplitudes by the value of the threshold in order to remove the noise effect even for the elevated values coefficients. In our survey, we use the hard thresholding.

$$\theta^{S}_{(T)}(x) = \begin{cases} 0 & \text{if } |x| < T \\ x - \operatorname{sign}(x).T & \text{if } |x| \geq T \end{cases}$$

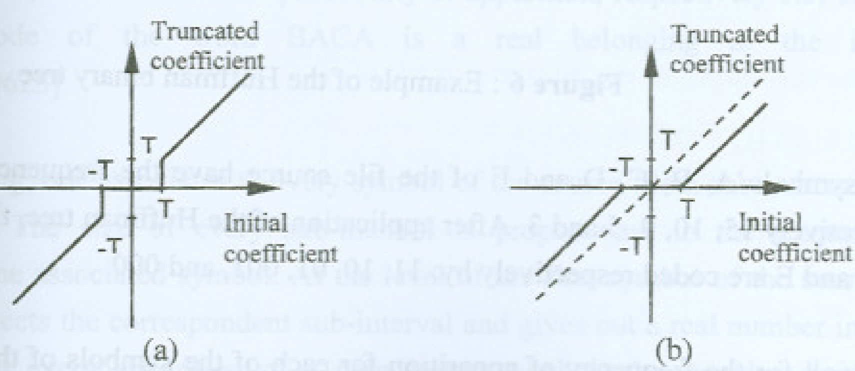

Figure 5 : hard thresholding (a) and soft thresholding (b)

8. The Entropy encoder

THE RLE ENCODER

The Run Length Encoder or RLE includes a simple and fast algorithm. Its principle consists to detect the information having a number of consecutive apparition and to replace this sequence by three data: an identification character for the counter, the counter that is the number of apparitions of the symbol to repeat and the symbol. The identification character of the counter must not appear in the file source to avoid all confusion when decoding.

THE HUFFMAN ENCODER

The Huffman encoder is based on the reduction of the medium length of the coding of an alphabet. More the frequency of apparition of the symbol is important, more the code that is associated him is short. Moreover, every code cannot be the prefix of another code. These two properties are assured by the use of the Huffman binary tree for the assignment of the codes.

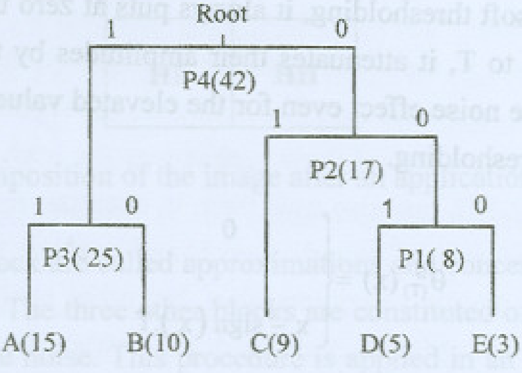

Figure 6 : Example of the Huffman binary tree

The symbols A, B, C, D and E of the file source have the frequencies of apparition respectively 15, 10, 9, 5 and 3. After application of the Huffman tree, the symbols A, B, C, D and E are coded respectively by: 11, 10, 01, 001, and 000.

We look for the frequency of apparition for each of the symbols of the file source and we classify them by order of decreasing probability. Every symbol constitutes a free node of the tree having a weight equal to the frequency of apparition (or probability of occurrence). We regroup the two weakest weight nodes and we create a related node having for weight the sum of the weights of the two son's nodes. The related node is added to the list of the free nodes and the two son's nodes are removed. One of the two sons is designated to be the path took from the parent to code a bit 0; the other son is associated to the bit 1. This procedure is repeated until it only remains one free node. To determine the code of a given symbol, one browses the tree of Huffman from the root until the symbol, while accumulating the bits at every passage by a related node.

THE ARITHMETIC ENCODER

The arithmetic coding is based on the fact that a compact interval on the set of the real includes infinity of elements. This encoder doesn't replace each symbol of the file source by a code, but replaces a stream of symbols by a real number included between 0 and 1.

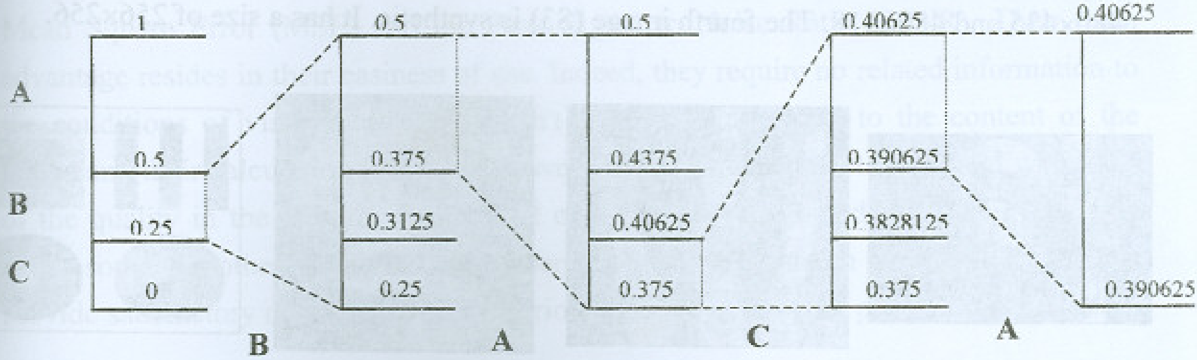

Figure 7 : Example of arithmetic coding

One looks for the code of the word "BACA". To each of the three symbols A, B and C of the source file, one looks for the probability of apparition, respectively 0.5, 0.25 and 0.25. The code of the word BACA is a real belonging to the interval [0.390625,0.40625].

At the beginning, one associates to every symbol of the source file a sub-interval of the interval [0,1]. The size of every sub-interval is proportional to the frequency of apparition of the associated symbol. At the level of the first symbol of the source file, the encoder selects the correspondent sub-interval and gives out a real number included in this last. At every apparition of a new source symbol, the encoder divides the precedent sub-interval with identical proportions, select the new sub-interval and gives out a new real belonging to this sub-interval. At the end of the file source, the encoder gives out the last real.

Theoretically, to code and to decode a stream of symbols by using the arithmetic coding is not very complicated. Practically, it seems to be completely unfeasible. Most computers support floating-point numbers of 80 bits. These 80 bits permit to code only the messages of few symbols. The idea is therefore to use the integers of 16 or 32 bits [42].

9. The images of test

We achieved our tests on four images of 256 grey levels, of different natures and different sizes. The first is the image portrait (clown). It has a size of 320×200. The second and the third image (Q1 and Q6) are medicals. They have the respective sizes of 480×435 and 480×418. The fourth image (S3) is synthetic. It has a size of 256×256.

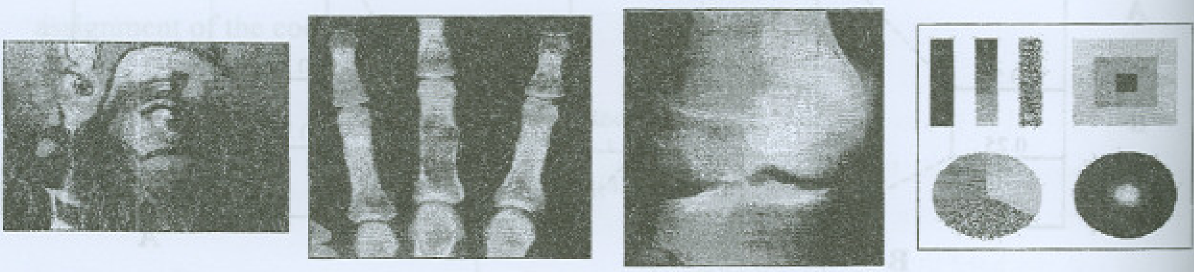

Image clown [320×200] Image Q 1 [480×435] Image Q 6 [480×418] Image S3 [256×256]

Figure 8 :The images of test

10. The standards of evaluation

EVALUATION OF IMAGES

There is no perfect and universal metrics of the picture's quality at the present time. Indeed, the processes of evaluation of a human observer are currently unknown. One can think that to establish a judgment, the observer makes call to an internal models of representation of the content of the picture that he compares to the image brought by the visual system. The quality can be defined according to the faculty of an observer to make detection or an evaluation from the picture. On the other hand, the metrics of fidelity value the difference between an image reference and an image having undergone a treatment. In the compressive phase, it is necessary to have a metrics of fidelity to value the perceptible difference between the original picture and the decompressed picture in order to optimize the compression rate according to the requisite quality. On the other hand, a metrics of fidelity must permit to value the threshold of perception of the deteriorations according to the applications. Besides, it provides a perceptible deterioration scale for the applications that are content with a middle quality.

THE CLASSIFICATION OF THE FIDELITY

One can distinguish two fundamental classes of the fidelity: the first uses a certain number of metrics based on the punctual differences between the original picture and the treated picture [43]. Among the most known metrics of this class, one mentions the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). Their main advantage resides in their easiness of use. Indeed, they require no related information to the conditions of visualization, they don't need any adaptation to the content of the image and their calculation is simple. However, they cannot reflect the spatial variations of the quality in the picture. The second class is oriented toward the model of vision using some hypotheses related to the perception of the human visual system. They provide satisfactory results when the deteriorations are near of the perception threshold.

THE DEFINITION OF THE ERROR

This measure is defined as being the report of the quadratic difference between the original image A_O and the processed one A_P, by the energy of the original picture:

$$\text{Erreur}=\frac{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[A_O(m,n)-A_P(m,n)\right]^2}}{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[A_O(m,n)\right]^2}}$$

Where M and N are respectively the number of lines and columns of the pictures

THE PEAK SIGNAL TO NOISE RATIO

The PSNR is defined as follows:

$$PSNR = -10\log_{10}\left[\frac{MSE}{S^2}\right]$$

It is expressed in decibel. S is the maximal intensity of the pixels. The Mean Square Error MSE is defined as follows:

$$MSE = \frac{1}{M.N}.\sum_{m=1}^{M}\sum_{n=1}^{N}\left[A_O(m,n) - A_P(m,n)\right]^2$$

MSE is the quadratic error between the original image A_O and the rebuilt image A_R. This measure is the quadratic difference average on the set of the pixels of the image. This measure can value the quality of the rebuilt image, but it depends strongly on the scale of the grey levels of the picture.

11. Results

WAVELETS TRANSFORMS

EVALUATION OF WAVELETS AND COMPARISON OF DIFFERENT TRUNCATIONS

We are interested by the survey of the behavior of the filters of Haar and those of Daubechies of order 2, 4, 10 and 20, applied to the four images of test. To evaluate the influence of the truncation of the coefficients of the details on the quality of the compressed images, we calculate the value of the error in percent and the PSNR in decibel for many thresholds. We give for each tested image the PSNR and the error after compression by using the Haar, db2 and db10 filters; and this for some thresholds applied respectively for the corresponding truncations at 30%, 1% and 0% of retained coefficients on the three matrixes of details.

The following figure presents the original image and the three truncated images by the limitation of the retained details at 30%, 1% and 0% respectively for Haar, db2 and db10.

Fig. 9 :The image clown

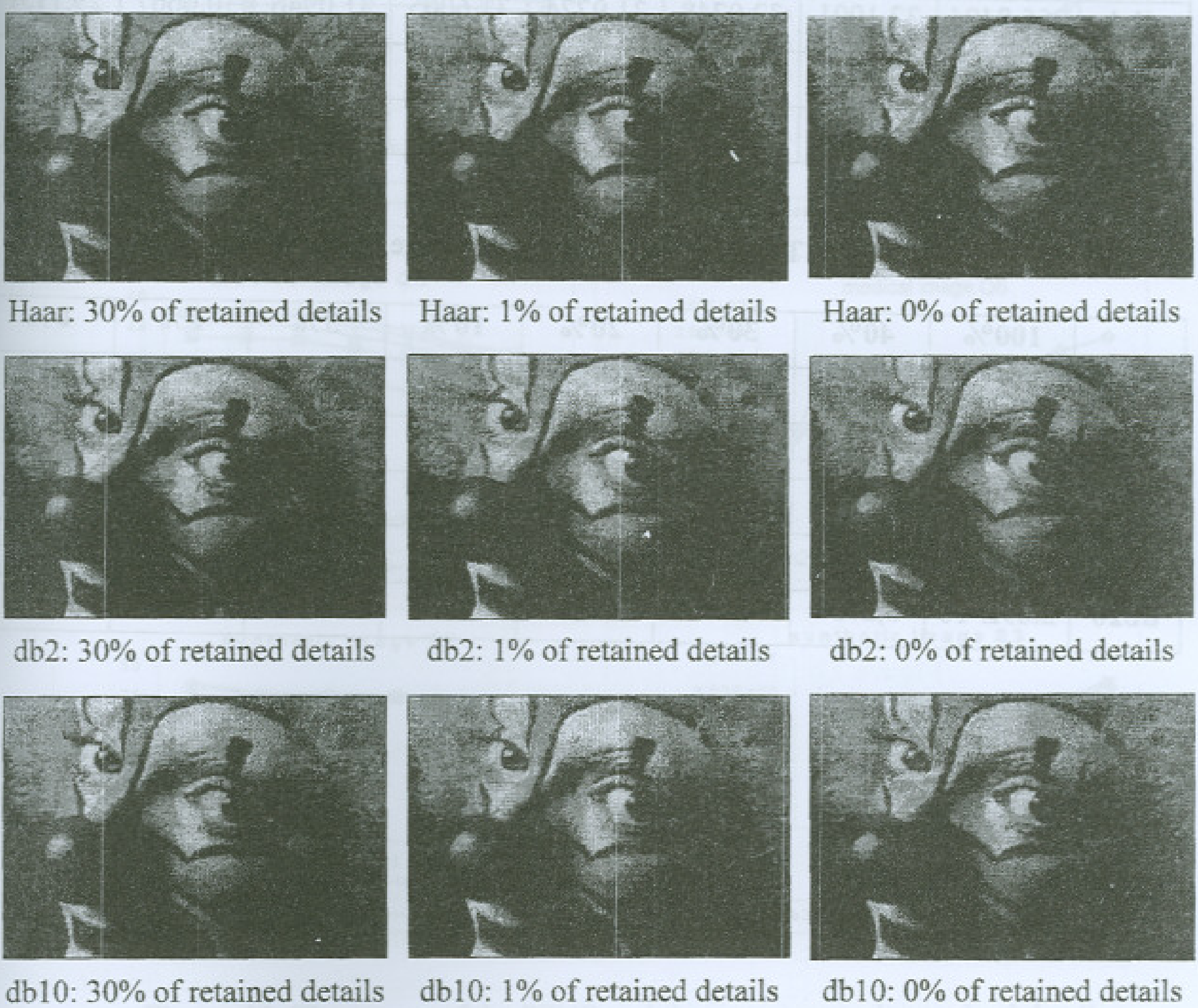

Figure 10 : The transformed images by the wavelet of Haar, db 2 and db10, truncated by the limitation of the retained details at different thresholds

The following tables and curves presents the value of the PSNR in decibel and the error in percent, for the tested images, after compression by using a transformation by the Haars filters, db2, db4, db10 and db20 for eight different retained details percentages:

Table 1 :The PSNR for the image clown

	100%	40%	30%	20%	10%	5%	1%	0%
Haar	318.2746	30,0553	30,0183	29,9897	29,7282	29,3905	28,568	27,1199
db2	280.9765	31,6347	31,6189	31,5527	31,2817	31,0084	29,6187	28,5005
db4	255.8404	32,1001	32,0748	31,9774	31,6005	31,0946	30,0667	29,1151
db10	247.3701	32,3031	32,2801	32,1732	31,8008	31,309	30,084	29,2759
db20	241.8534	32,3258	32,3008	32,1939	31,7897	31,231	30,2292	29,329

Table 2 :The error for the image clown

	100%	40%	30%	20%	10%	5%	1%	0%
Haar	3.16E-14	8,1502	8,185	8,2119	8,463	8,7985	9,6724	11,4271
db2	3.7E-12	6,7951	6,8075	6,8596	7,0769	7,3032	8,5703	9,7479
db4	4.18E-11	6,4406	6,4594	6,5323	6,822	7,231	8,1395	9,0819
db10	1.11E-10	6,2918	6,3085	6,3866	6,6664	7,0548	8,1233	8,9153
db20	2.09E-10	6,2754	6,2935	6,3715	6,6749	7,1184	7,9886	8,861

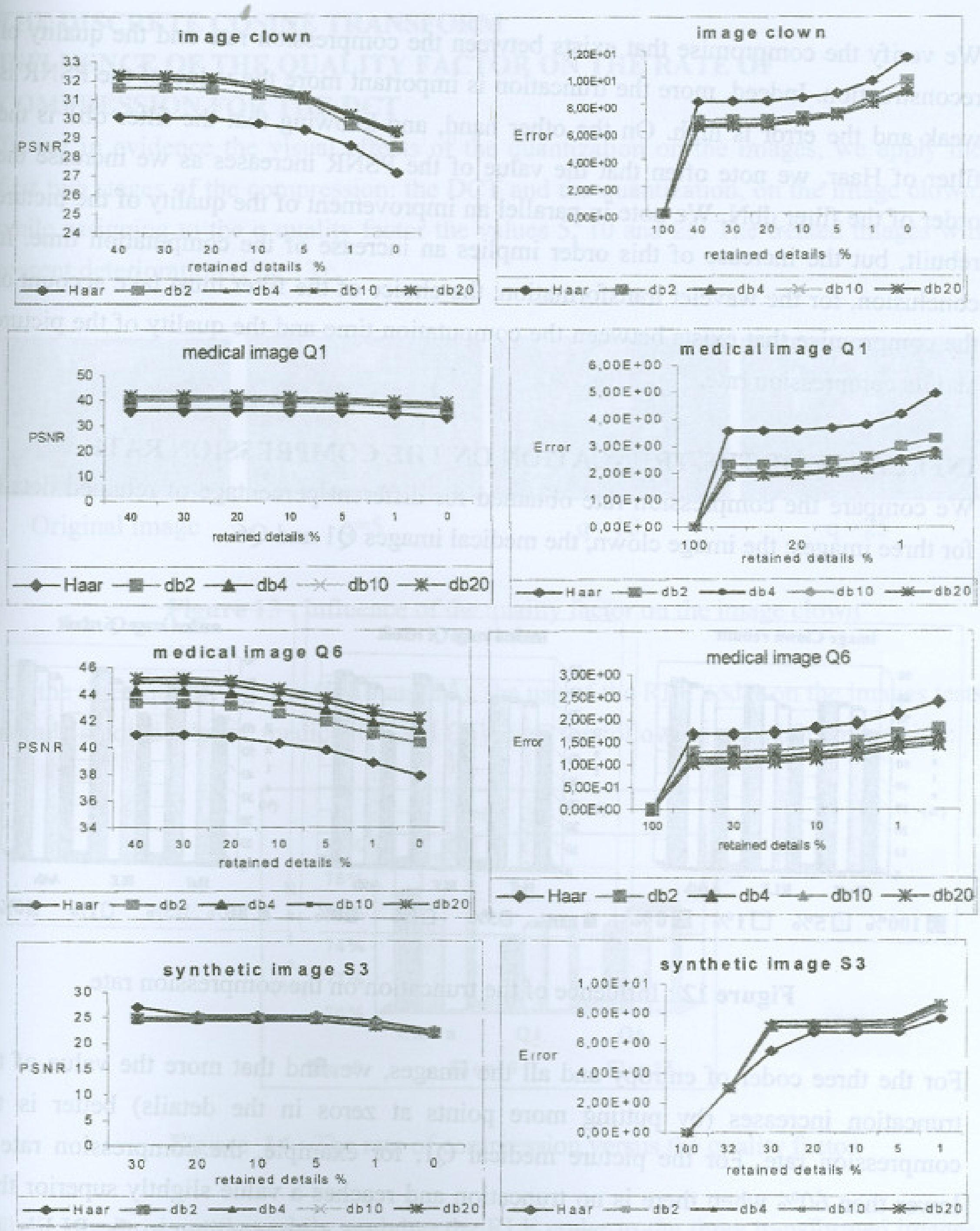

Figure 11 :The curves of the PSNR and the error versus the retained details

THE DISCRETE COSINE TRANSFORM

INFLUENCE OF THE QUALITY FACTOR ON THE RATE OF COMPRESSION FOR THE DCT

To put in evidence the visual effects of the quantization on the images, we apply the first two stages of the compression: the DCT and the quantization, on the image clown, while assigning to the q quality factor the values 5, 10 and 25. The treated images will present deterioration (Fig. 13):

Original image q=5 q=10 q=25

Figure 13 : Influence of the quality factor on the image clown

For the three quality factors (5, 10 and 25), the use of the RLE coder on the images tests already studied (clown, medical Q1 and Q6) gives the following rates of compression:

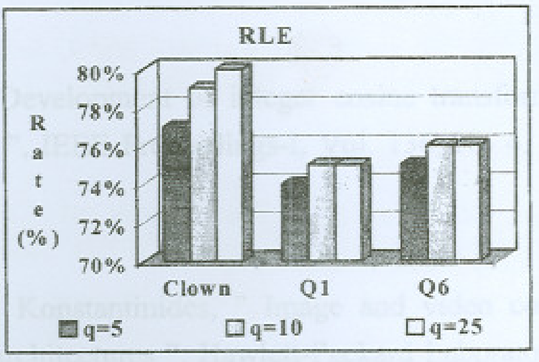

Figure 14 : The rate of compression versus the quality factor

These results show that while applying the RLE coder to the three transformed images, the rate of compression increases with the quality factor q. This observation is also valid for the other coders studied. We note that the user doesn't have interest to increase the q factor to a large extent to benefit from an elevated compression rate because the quality of the image risks an important deterioration.

12. Conclusion

We verify the compromise that exists between the compression rate and the quality of the rebuilt image. Indeed, when we apply the wavelet transform, more we suppress details, more the PSNR is weak and the error is important. We note also that the quality of reconstruction is better as the order of the filter associated to the wavelets of Daubechies increases. On the other hand, the computation time increases because the filters become increasingly complex. When we apply the DCT, we verify that when the step of quantization increases, the compression ratio increases at the detriment of the quality of the picture rebuilt. We note also that for height compression and for very near visual quality, wavelet transform leads to best compression rate. Inversely, the DCT gives better results for the low compression rates.

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