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DISCHARGE COEFFICIENT FOR A TRIANGULAR NOTCH WEIR THEORY AND EXPERIMENTAL ANALYSIS

COEFFICIENT DE DÉBIT DU DEVERSOIR TRIANGULAIRE THEORIE ET ANALYSE EXPÉRIMENTALE

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ABSTRACT

The study deals with the thin-plate triangular notch weir as a flow measurement device. Unlike previous studies on this subject, a theoretical development is proposed which takes into account the effect of the approach flow velocity. Between two well-chosen sections, the energy equation is applied with certain simplifying assumptions 1) the head loss and the effect of both viscosity and surface tension are neglected, 2) Assuming a hydrostatic distribution of the pressure. 3) The effect of the flow streamlines curvature over the weir is neglected. All the parameters influencing the discharge coefficient are well defined in the theoretical equation, as the experiment predicts so well. The theoretical equation of the discharge coefficient is adjusted to be in conformity with the experimental data. The adjusted relationship is a function exclusively of the relative weir height ratio P/h_1 and dimensionless number $M_1=mh_1/B$, where m is the side slope of the notch, i.e. m horizontal to 1 vertical, h_1 corresponds to the upstream water depth measured above the vertex of the notch, and B is the width of the rectangular channel of approach. The corrected theoretical relationship causes a maximum deviation of 6% on the calculation of the discharge coefficient, knowing that the average relative error is less than 1.88%. It is worth noting that among the 168 calculated values of the relative deviation on the computation of the discharge coefficient, 94% are less than 5% meaning that 6% only are greater than 5%, 87% are less than 4%, while 76.2% are less than 3%.

Keywords: V-notch, weir, discharge coefficient, weir height, approach velocity.

RESUME

L'étude porte sur le déversoir à échancrure triangulaire à plaque mince comme appareil de mesure de débit. Contrairement aux études précédentes sur ce sujet, un développement théorique est proposé qui prend en compte l'effet de la vitesse d'approche de l'écoulement. Entre deux sections bien choisies, l'équation de l'énergie est appliquée avec certaines hypothèses simplificatrices 1) La perte de charge et l'effet à la fois de la viscosité et de la tension superficielle sont négligés, 2) Il est supposé une distribution hydrostatique de la pression. 3) L'effet de la courbure des filets liquide sur le déversoir est négligé. Tous les paramètres influençant le coefficient de débit sont bien définis dans l'équation théorique, comme l'expérience le prédit si bien. L'équation théorique du coefficient de débit est ajustée pour être conforme aux données expérimentales. La relation ajustée est fonction exclusivement du rapport de hauteur relative du déversoir P/h_1 et du nombre sans dimension $M_1 = mh_1/B$, où m est la pente latérale de l'échancrure, c'est-à-dire m horizontal à 1 vertical, h_1 correspond à la profondeur d'eau en amont mesurée au-dessus du sommet de l'échancrure, et B est la largeur du canal d'approche rectangulaire. La relation théorique corrigée occasionne un écart maximum de 6% sur le calcul du coefficient de débit, sachant que l'erreur relative moyenne est inférieure à 1,88%. Il est à noter que parmi les 168 valeurs calculées de l'écart relatif sur le calcul du coefficient de débit, 94% sont inférieurs à 5%, ce qui signifie que 6% seulement sont supérieures à 5%, 87% sont inférieurs à 4%, tandis que 76,2 % sont inférieurs à 3%.

Mots clés : Echancrure en V, déversoir, coefficient de débit, hauteur de pelle, vitesse d'approche.

INTRODUCTION

Weirs are commonly used to measure the volumetric rate of water flow (Achour et al., 2003; Bos, 1976; Bos, 1989). The triangular shaped-weir is actually a thin plate from which a triangular notch located at the top edge of the plate is cut (Fig. 2). The notch is in the center of the upper part of the plate. This device consisting of the thin plate provided with its V-notch is installed perpendicularly to the flow in a supply channel whose water nappe flows over the notch. This enables to accurately measure the flow rate by measuring the depth upstream of the V-notch (Fig. 2). The V-notch sharp-crested weir is one of the most precise discharge measuring devices suitable for a wide range of flow, used in laboratories and industry. In international literature, the 90° V-notch sharp-crested-weir is frequently referred to as the 'Thomson weir'. Two types of V-notch weirs can be considered: (1) a fully contracted weir, and (2) a partially contracted weir. A fully contracted weir is a weir which has an approach channel whose bed and sides are sufficiently remote from the edges of the V-notch to allow for a sufficiently great approach velocity component parallel to the weir face so that the contraction is fully developed. It is defined as that for which the ratio $h/B \leq 0.2$, where h is the water depth above the vertex of the V-notch and B is the approach channel width (Fig. 2). A partially contracted weir is a weir the contractions of which along the sides of the V-notch are not fully developed due to the proximity of the walls and/or bed of the approach channel. A partially contracted weir is that for which the ratio $h/B \leq 0.4$.

The flow parameters of a triangular notch thin-plate weir in a rectangular channel (Fig. 2) are namely: the flow rate Q; the width B of the approach rectangular channel; the height P which represents the vertical distance between the vertex of the triangular notch and the bottom of the approach channel; the water depth h_1 measured above the vertex of the weir; the apex angle α, i.e. the opening angle of the notch; the density ρ of the liquid; the dynamic viscosity μ of the liquid; the surface tension σ of the liquid; and the specific weight γ of the liquid which is expressed as $\gamma = \rho g$, where g is the acceleration due to gravity. Assuming that the discharge is the dependent variable, this one can be written according to the following functional relationship:

$$Q = f(B, P, h_1, \alpha, \rho, \mu, \sigma, \gamma) \tag{I}$$

Intuitively, one can form a dimensionless parameter related to the flow rate depending on some dimensionless variables such as:

$$\frac{Q}{h_1^2\sqrt{gh_1}} = f(h_1/B, h_1/P, \alpha, R, W) \tag{II}$$

The function f represents the discharge coefficient C_d of the weir. The first three ratios on the right-hand side of Eq. (II) are closely related to the geometry of the weir, approach channel, and the state of the flow particularly the depth h_1 . The last two ratios are the Reynolds number R and the Weber number W. The depth h_1 represents the reference length that can be used in both the Weber number and the Reynolds number. The influence represented by the Weber number can be expressed in terms of h_1, σ, and ρ, while the influence represented by the Reynolds number can be expressed in terms of h_1, μ, and ρ. For a given liquid and for a reduced temperature range, μ, σ, and ρ can be considered as constants. Thus, the depth h_1 alone can represent the effects of both dynamic viscosity and surface tension. Whence, one can write:

$$\frac{Q}{h_1^2\sqrt{gh_1}} = f(h_1/B, h_1/P, \alpha, h_1) \tag{III}$$

Or:

$$C_d = f(h_1 / B, h_1 / P, \alpha, h_1) \tag{IV}$$

According to Kindsvater and Carter (1959), the combined effects of viscosity and surface tension on a notch weir are proportional to the fictitious increases in notch depth and width. The discharge coefficient can be expressed in terms of the geometric ratios alone if the effects of viscosity and surface tension are accounted for by an adjustment of the measured values of depth and notch width. For triangular-notch weirs, for which notch width is a function of h (Fig. 2), only the measured values of depth need to be adjusted. The adjustment of measured value of h_1 is translated through the following simple equation:

$$h_{1e} = h_1 + k_h \tag{V}$$

where h_{1e} is the adjusted depth called effective depth. The k_h factor must be determined by laboratory tests. For a given liquid and a reduced temperature range, and for a given notch angle, k_h is assumed to be a constant. The international literature gives a graph showing the variation of k_h as a function of the apex angle of the weir (Fig. 1) (National Bureau of Standards, Kulin and Compton, 1975).

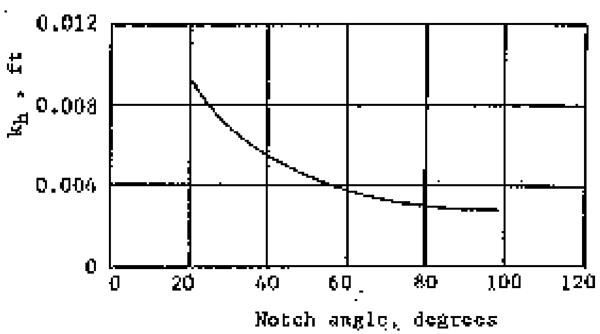

Figure 1: Depth correction factor k_h for V-notches of any angle (National Bureau of Standards, Kulin and Compton, 1975).

Using h_1 instead of h_{1e} to compute the discharge Q can lead to significant errors depending however on the opening angle of the notch. The more the opening angle of the notch decreases, the more the error in the flow rate calculation increases for the same value of h_1. On the other hand, the error is important for low values of h_1.

Thus, the discharge Q as well as the effective discharge coefficient C_e are written respectively as (Kulin and Compton, 1975; Shen, 1981):

$$Q = C_{d,e} \frac{8}{15} \sqrt{2g}\, m\, h_{1e}^{5/2} \tag{VI}$$

$$C_d = f(h_1 / B, h_1 / P, \alpha) \tag{VII}$$

The h_1/B ratio expressed by Eq. (VII), in combination with α, is a dimensionless parameter characterizing the width contraction which approximates the meaning of the *b/B* ratio for rectangular-notch weirs (Kindsvater and Carter, 1959). In order to play the same role of contraction effect, the *P/B* ratio is more suitable because it is a constant for a given weir installation. This ratio is obtained by dividing h_1/B by h_1/P, which is one of the three influencing dimensionless parameters included in Eq. (VII). The ratio h_1/P is a measure of the depth-contraction characteristic and reflects the magnitude of the velocity in approach channel. Therefore, the ratio h_1/B can be replaced with *P/B*. Thus, one may write Eq. (VII) as:

$$C_d = f(h_1 / P, P / B, \alpha) \tag{8}$$

For the 90° notch, Thomson (1858, 1861) recommended a constant value of C_d such that $C_d = 0.593$.

In the present study, a theoretical development is proposed to deduce the discharge coefficient relationship for a triangular-notch thin-plate weir. The theoretical approach is based on the energy equation (Henderson, 1966; Bos, 1989; Achour, 1989) applied between two well chosen sections. The first section is located upstream of the device, while the second section is at the location of the weir assumed to be crossed in a critical flow regime. Installing a weir in an open channel causes critical depth to form over the weir. It should be noted that, unlike previous studies, the approach velocity of the flow is taken into account. The theoretical equation of the discharge coefficient is contrasted to the available experimental data.

Note that this study is dedicated to the triangular weir with an opening angle other than 90 °. Regarding this one, a special study will be devoted to it.

DESCRIPTION OF THE DEVICE

Fig. 2 shows an overview of the rectangular channel of approach of width B and the triangular weir of apex angle α. It is a symmetrical, V-shaped notch in a vertical thin plate. The weir is characterized by a height P, i.e. the height of the notch vertex with respect to the floor of the channel of approach. The side wall of the device is inclined at an angle θ with respect to the horizontal. The depth h_1 corresponds in fact to the upstream water depth measured above the vertex of the notch. The depth measuring section 1-1 (Fig. 3) is located a sufficient distance upstream of the device in order to avoid the surface drawdown zone. On the other hand, the depth measuring section must be close enough to the device so that the energy loss between sections 1-1 and 2-2 can be neglected. The discharge flowing through the approach rectangular channel is *Q*. One may define *m* as the side slope of the notch such that *m* = cotg (θ), i.e. *m* horizontal to 1 vertical.

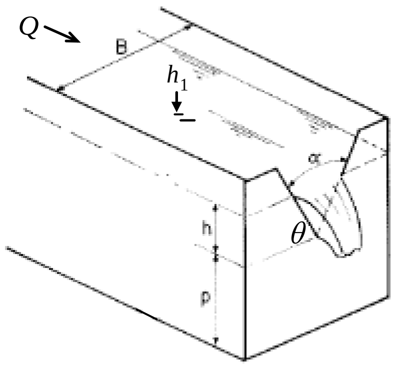

Figure 2: Definition sketch of the studied contracted weir

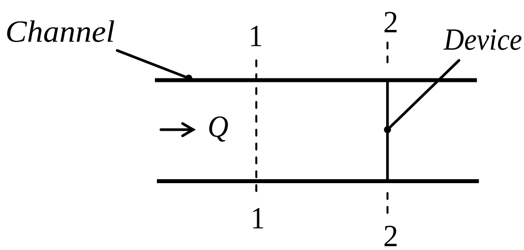

Figure 3: Plan view of the channel and the device

THEORETICAL DISCHARGE COEFFICIENT RELATIONSHIP

The critical depth in the rectangular cross-section 1-1 (Fig. 3) is written as:

$$h_{1c} = \left(\frac{Q^2}{gB^2} \right)^{1/3} \quad (1)$$

where the subscript « *c* » denotes the critical conditions.

On the other hand, the critical depth in the triangular cross-section 2-2 (Fig. 3) is as:

$$h_{2c} = \left(\frac{2Q^2}{g m^2} \right)^{1/5} \tag{2}$$

where $m = \mathrm{cotg}\theta$ is the side slope of the notch: *m* horizontal to 1 vertical.

Eliminating the discharge Q^2 between Eqs. (1) and (2) results in:

$$h_{2c} = \left(\frac{2B^2}{m^2} \right)^{1/5} h_{1c}^{3/5} \tag{3}$$

Assume that there is no head loss between sections 1-1 and 2-2. Equal total heads between sections 1-1 and 2-2 translates into:

$$H_1 = H_2 = \frac{5}{4} h_{2c} \tag{4}$$

Combining Eqs. (3) and (4) results in:

$$H_1 = \frac{5}{4} \left(\frac{2B^2}{m^2} \right)^{1/5} h_{1c}^{3/5} \tag{5}$$

Taking into account the approach flow velocity, total head H_1 can be written as:

$$H_1 = h_1 + \frac{Q^2}{2gB^2 \left(h_1 + P \right)^2} \tag{6}$$

where h_1 is the upstream depth above the apex of the weir, and *P* is the weir height (Fig. 2). Eq. (6) can be rewritten as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 \left(h_1 + P \right)^2 h_{1c}} \tag{7}$$

Eq. (7) can be written as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 h_1^2 \left(1 + P / h_1 \right)^2 h_{1c}} \tag{7a}$$

Eq. (1) allows writing that:

$$\frac{Q^2}{gB^2} = h_{1c}^3 \tag{1a}$$

Combining Eqs. (7a) and (1a) yields:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} \tag{8}$$

Eqs. (5) and (8) give what follows:

$$\frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} = \frac{5}{4}\left(\frac{2B^2}{m^2}\right)^{1/5}\frac{1}{h_{1c}^{2/5}} \tag{9}$$

Eq. (9) can be rewritten as:

$$\frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} = \frac{5}{4}\left(\frac{2B^2}{m^2 h_1^2}\right)^{1/5}\frac{h_1^{2/5}}{h_{1c}^{2/5}} \tag{9a}$$

Let us adopt the following non-dimensional parameters:

$$h_1/h_{1c} = h_1^* \tag{10}$$

$$M_1 = \frac{mh_1}{B} \tag{11}$$

$$P^* = P/h_1 \tag{12}$$

Inserting Eqs. (10), (11) and (12) into Eq. (9a) results in:

$$h_1^* + \frac{1}{2h_1^{*2}(1+P^*)^2} = \frac{5}{4}\left(\frac{\sqrt{2}}{M_1}\right)^{2/5} h_1^{*2/5} \tag{13}$$

After some rearrangements Eq. (13) reduces to:

$$h_1^{*3} - \frac{5}{4}\left(\frac{\sqrt{2}}{M_1}\right)^{2/5} h_1^{*12/5} + \frac{1}{2(1+P^*)^2} = 0 \tag{14}$$

In practice, the known parameters are M_1 and P^* which will be used to deduce h_1^* by solving the implicit Eq. (14). Note that the flow in the section 1-1 is subcritical, meaning that $h_1 > h_{1c}$ or $h_1^* > 1$.

Eq. (1) gives the discharge Q through the rectangular channel of approach as:

$$Q = \sqrt{g} B h_{1c}^{3/2} \tag{1b}$$

which can be rewritten as:

$$Q = \sqrt{g} B \frac{h_1^{3/2}}{h_1^{*\,3/2}} \tag{1c}$$

or as:

$$Q = \sqrt{g} \frac{B}{h_1} \frac{h_1^{5/2}}{h_1^{*\,3/2}} \tag{1d}$$

On the other hand, the discharge Q flowing through the V-notch (Fig. 2) is given by Shen's formula (1981). Neglecting the effect of viscosity and surface tension, this formula can be written as:

$$Q = C_d \frac{8}{15} \sqrt{2g}\, m h_1^{5/2} \tag{15}$$

That is:

$$C_{d,Exp} = \frac{15}{8} \frac{Q}{\sqrt{2g}\, m h_1^{5/2}} \tag{15a}$$

Eliminating the discharge Q between Eqs. (1d) and (15) results in the following theoretical discharge coefficient relationship:

$$C_{d,Th} = \frac{15}{8\sqrt{2}} \frac{1}{M_1 h_{1,Th}^{*\ \ 3/2}} \tag{16}$$

where the subscript "*Th*" denotes "Theoretical". This is the general theoretical discharge relationship valid for all angles of the V-notch. However, the literature indicates that the 90 ° V-notch is a rather special case for which a study should be devoted.

Thus, the theoretical development shows that the discharge coefficient depends on P / h_1 and mh_1 / B.

According to Eq. (16), it is obvious that the experimental discharge coefficient can be written as:

$$C_{d,Exp} = \frac{15}{8\sqrt{2}} \frac{1}{M_1 h_{1,Exp}^{*\ 3/2}} \qquad (16a)$$

This relationship will be very useful for the rest of the study.

EXPERIMENTAL ANALYSIS

The experimental data were taken from the literature (Vatankhah and khamisabadi, 2019). The theoretical parameters indicated by the subscript "*Th*" were calculated according to the appropriate relationships described above. During the tests, 173 measurement points of flow rate Q and depth h_1 were considered in a rectangular channel of width B = 0.25 m. The depth h_1 (cm) has been varied in the range [3.15; 20.1] while the discharge Q (l/s) was in the range [0.13; 10]. Three values of $m = \text{cotg}\,\theta$ of the considered triangular weirs were taken, namely 0.75 (θ = 53.13°), 0.5 (θ = 63.43°) and 0.375 (θ = 69.4°). Three weir crest heights P were considered corresponding to P (cm): 5.2; 10.2 and 15.3. Consequently, the relative weir crest height P/h_1 has been varied in the range [0.263; 4.857].

Knowing experimentally Q, B and h_1, Eq. (1) gives $h_{1,c}$ and therefore the dimensionless parameter $h_{1,Exp}^{*}$ is easily worked out since $h_{1,Exp}^{*} = h_1 / h_{1,c}$. The dimensionless theoretical parameter $h_{1,Th}^{*}$ was calculated by applying the fundamental equation (14) using the "TI-84 Plus" handheld calculator solver, knowing both $M_1 = m_{h1}/B$ and P/h_1 values. The experimental measurements allowed calculating $M_1 = m_{h1}/B$ values between 0.05355 and 0.3042.

The processing of the experimental data given by the study of Vatankhah and khamisabadi (2019) allowed graphically representing the variation between the dimensionless parameters $h_{1,Exp}^{*}$ and $h_{1,Th}^{*}$ as shown in the Fig. 4.

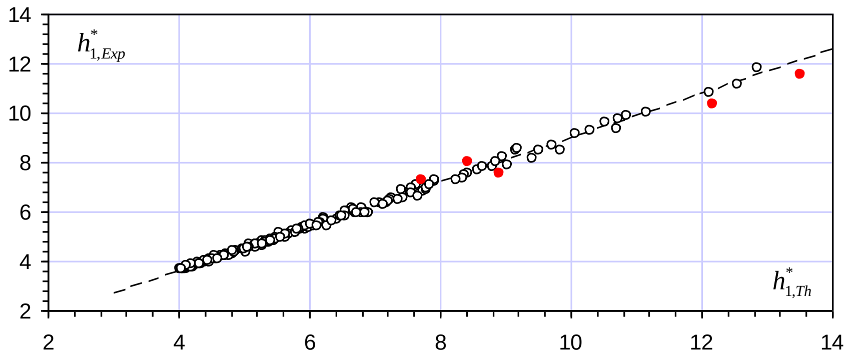

Figure 4: Variation of h_1^* experimental versus h_1^* theoretical

It is obvious that $h^*_{1,Th}$ is different from $h^*_{1,Exp}$ due to the simplifying assumptions adopted during the study. Moreover, one can observe in Fig. 4 that $h^*_{1,Th}$ is greater than $h^*_{1,Exp}$ meaning that $C_{d,Th}$ is less than $C_{d,Exp}$ according to both Eqs. (16) and (16a). Fig. 4 also highlights an almost linear trend for the variation of $h^*_{1,Exp}$ versus $h^*_{1,Th}$ represented by the broken line, with the exception of five measurement points which appear to deviate from this trend. These points are represented in red color and appear to be marred by a measurement error. Therefore, they are excluded from the calculation. In total, a sample of (173-5) = 168 experimental points seem to be significant and which will serve as the basis for the analysis. The calculations showed that the linear trend line is governed by the following equation:

$$h^*_{1,Exp} = 0.890 h^*_{1,Th} \tag{17}$$

with a coefficient of determination $R^2 = 0.9964$.

Inserting Eq. (17) into Eq. (16a) results in:

$$C_{d,Exp} = \frac{1.579}{M_1 \, h^{*\,3/2}_{1,Th}} \tag{18}$$

Eq. (18), along with Eq. (14), allows computing the discharge coefficient for a triangular weir provided M_1 and P/h_1 are given. The relative deviation between Eq. (15a) and (18) varies in the range [0.0039 %; 6 %] with an average relative error less than 1.9%. It is worth noting that among the 168 calculated values of the relative deviation on the computation of the discharge coefficient, 94% are less than 5% meaning that 6% only are greater than 5%, 87% are less than 4%, while 76.2% are less than 3%.

CONCLUSIONS

The study was devoted to the triangular-notch thin-plate Weir. The main objective was to define the discharge coefficient relationship by a theoretical approach. By choosing two sections, one upstream of the device and the other at the location of the weir crossed in a critical flow regime, the energy equation was applied, assuming some simplifying assumptions, in particular the head loss and the effect of both viscosity and surface tension have been neglected, the pressure distribution was assumed to be hydrostatic. It should be noted that the approach flow velocity is taken into account in the theoretical development which led to the establishment of Eq. (14), applicable to any triangular weir. The relationship contains the dimensionless parameters P/h_1 and $M_1 = mh_1/B$. The equality of discharges in the approach channel and crossing the weir allowed identifying the theoretical discharge coefficient governed by Eq. (16). This was compared to available experimental data from which experimental discharge coefficient was derived. The results of this comparison allowed establishing a linear relationship between the theoretical and experimental dimensionless parameter h_1^* which is closely related to the theoretical and experimental discharge coefficients [Eqs. (16) and (16a)]. After that, it was easy to derive the experimental discharge coefficient as a function of the theoretical h_1^* given by a polynomial equation provided P/h_1 and $M_1 = mh_1/B$ are known. Calculations have shown that the maximum deviation between the theoretical and experimental discharge coefficients is of the order of 6% with an average value less than 1.9%. This maximum error of 6% is not excessive in the field of discharge flow measurement using weirs as shown by a recent study (Vatankhah and khamisabadi, 2019) where maximum relative errors of 9% were observed.

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